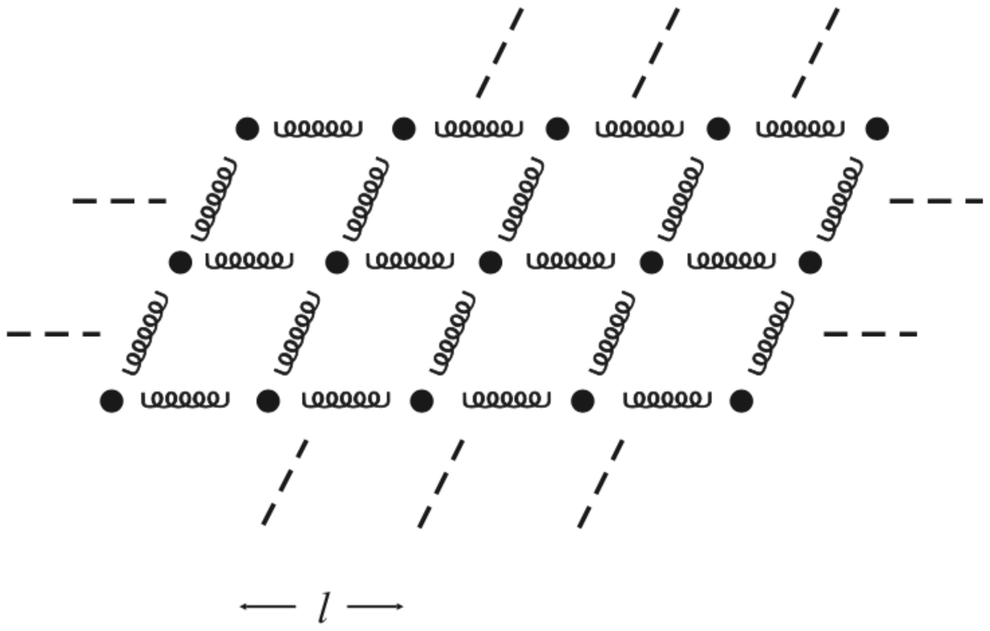
Cosmic Strings and Gravitational Waves from the Early Universe

Saturday Morning of Theoretical Physics, Nov 2024

Edward Hardy

Classical Fields



[Zee, Quantum Field Theory in a Nutshell]

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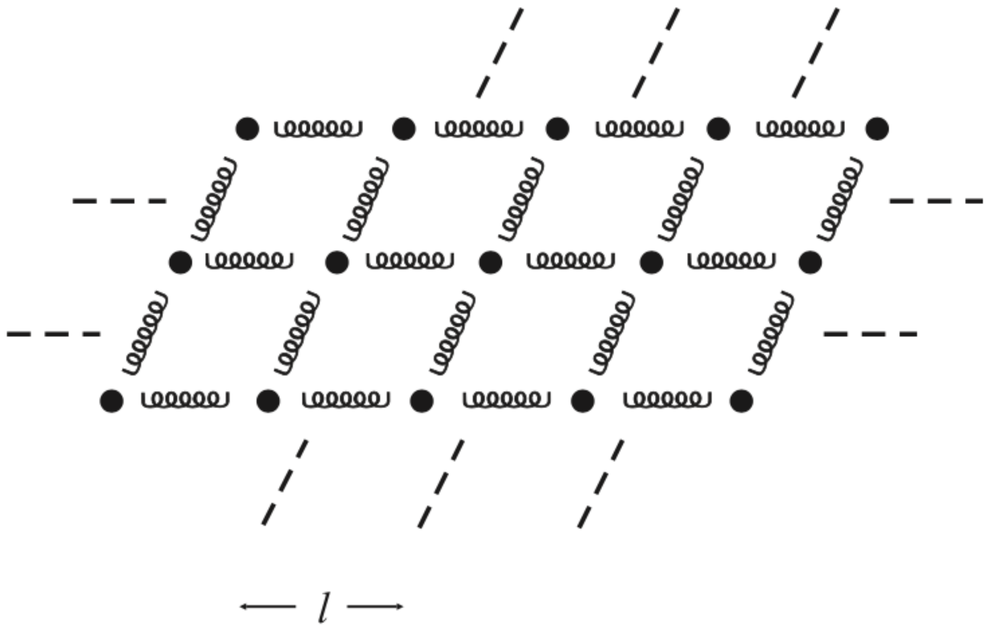

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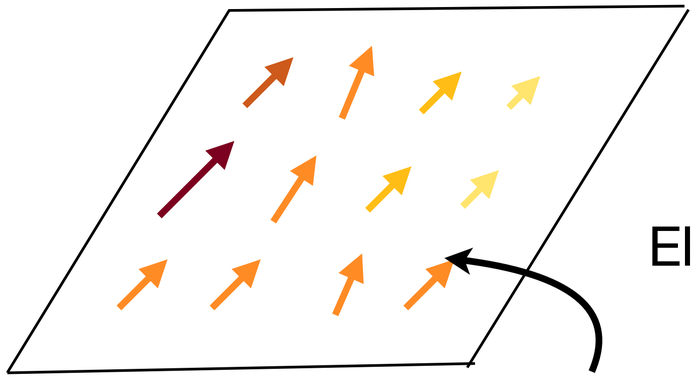


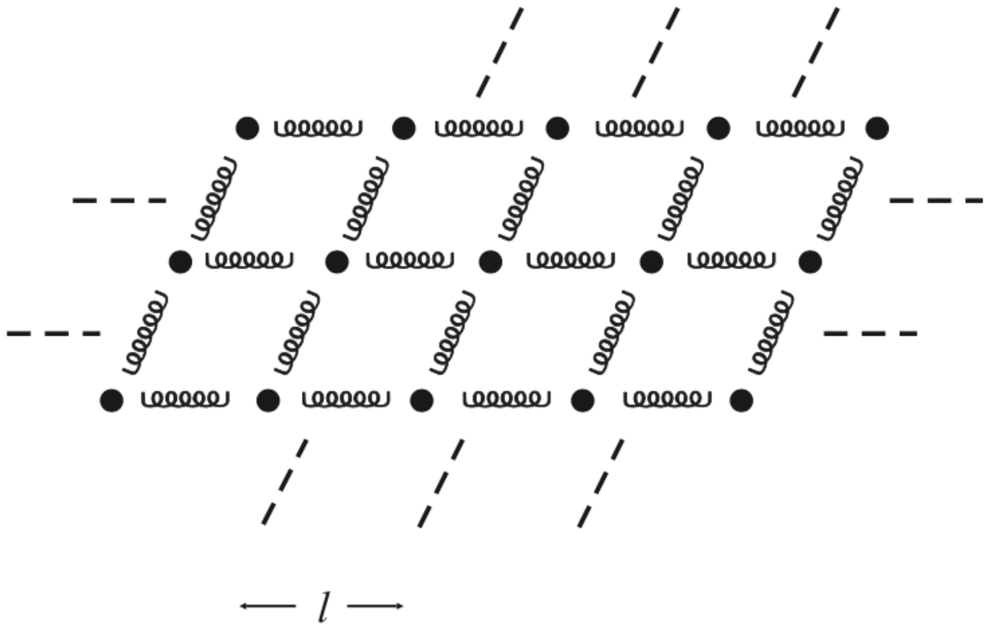
Electromagnetism

$$A(t,\vec{x}) = (A^0, A^1, A^2, A^3)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{bmatrix}$$

Classical Fields



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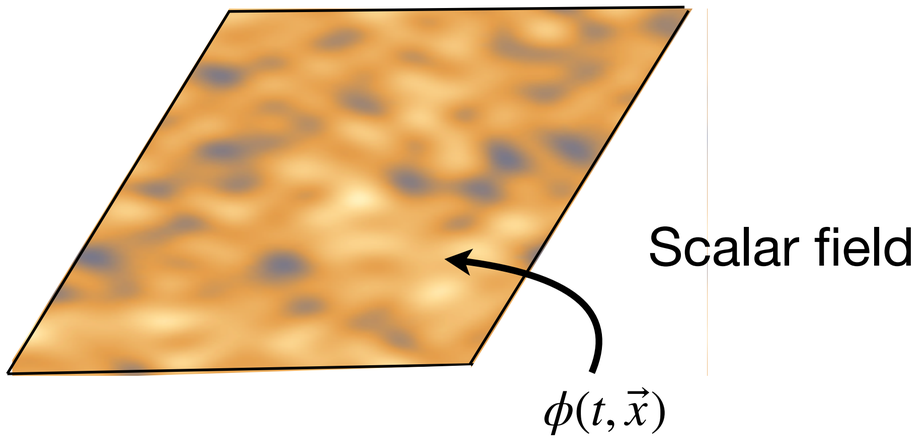
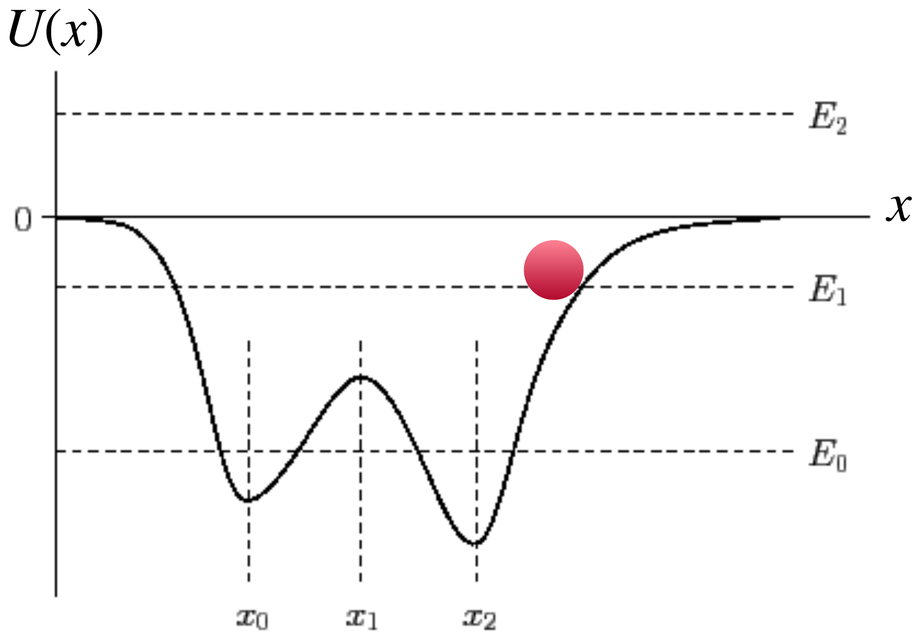


Consider a new scalar field

Classical limit = large occupation number

Particle in a potential

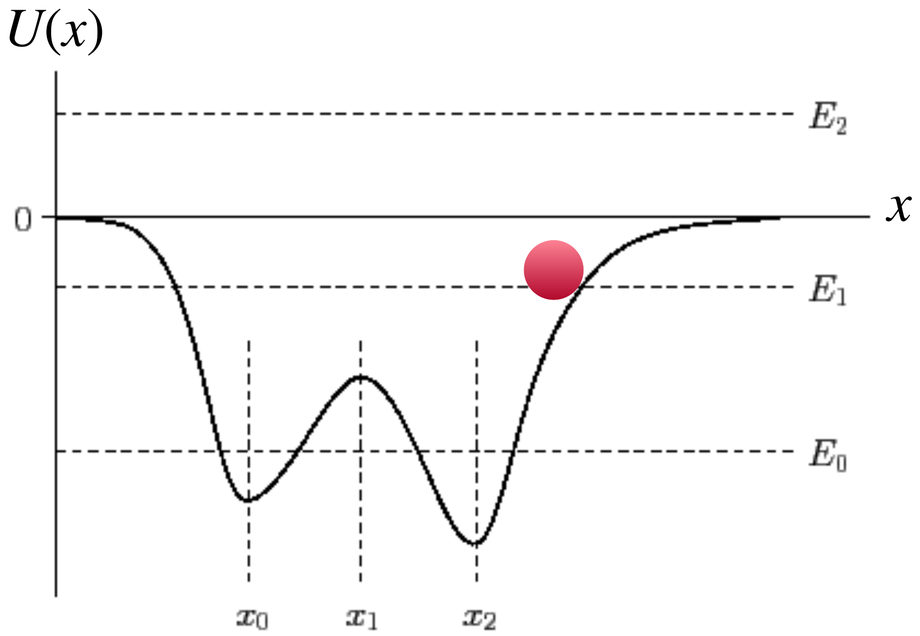


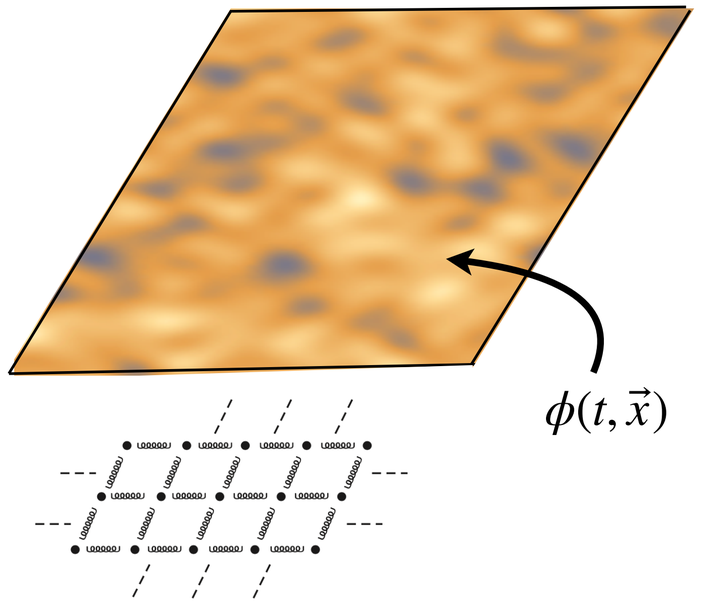
$$E = T + U(x) = \frac{1}{2}m\dot{x}^2 + U(x)$$

One degree of freedom

$$x$$

Particle in a potential





$$E = T + U(x) = \frac{1}{2}m\dot{x}^2 + U(x)$$

$$\rho \sim (\partial_t \phi)^2 + (\partial_i \phi)^2 + U(\phi)\,, \qquad E = \int d^3x\, \rho$$

One degree of freedom

One degree of freedom at each point in space

$$x \longrightarrow \phi(\vec{x})$$

Symmetry preserving potential

E.g. $U(\phi) = f_a^2\phi^2 + \lambda\phi^4$

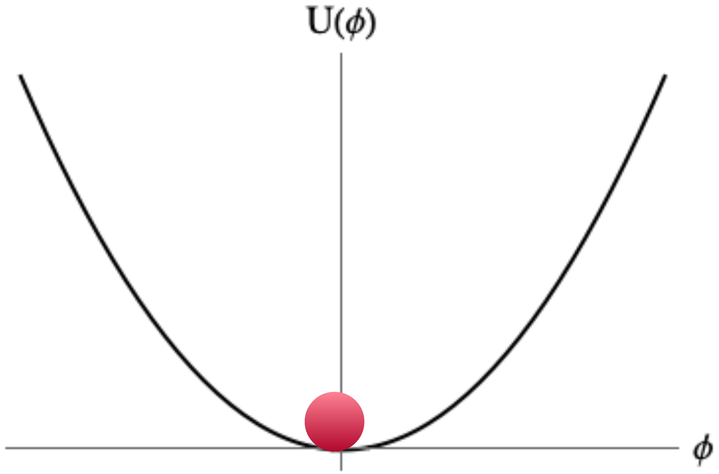
Symmetry:

$Z_2 : \ \phi \to -\phi$

$U(-\phi) = U(\phi)$

Preserved by the vacuum

$\phi_{\text{min}} = 0 = -\phi_{\text{min}}$



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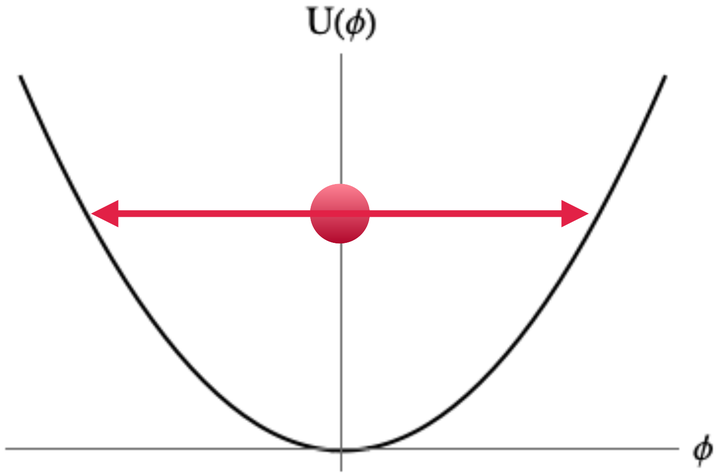

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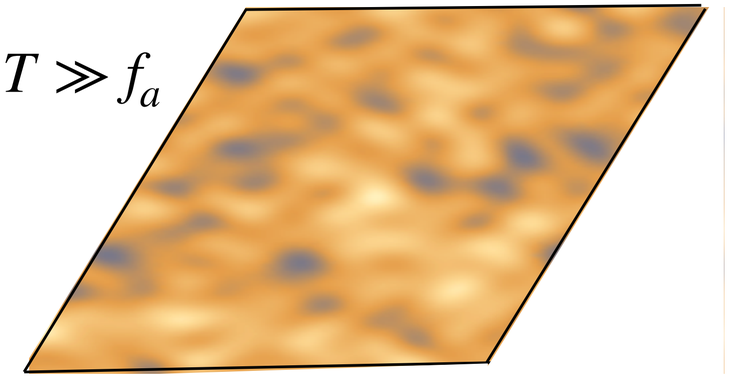

High $T \gg f_a$

Preserved by the vacuum

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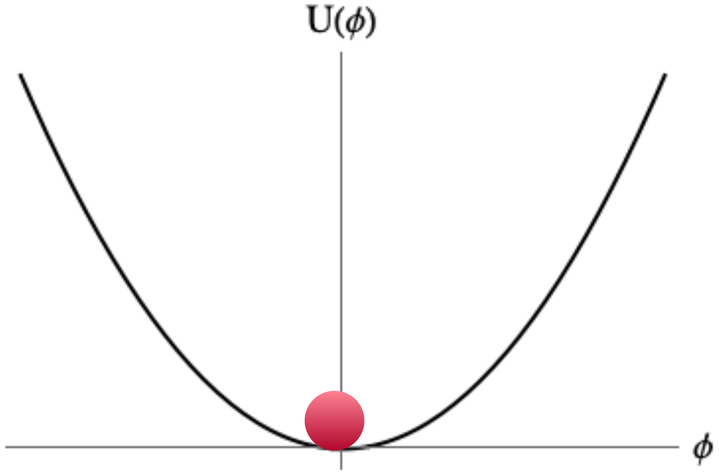

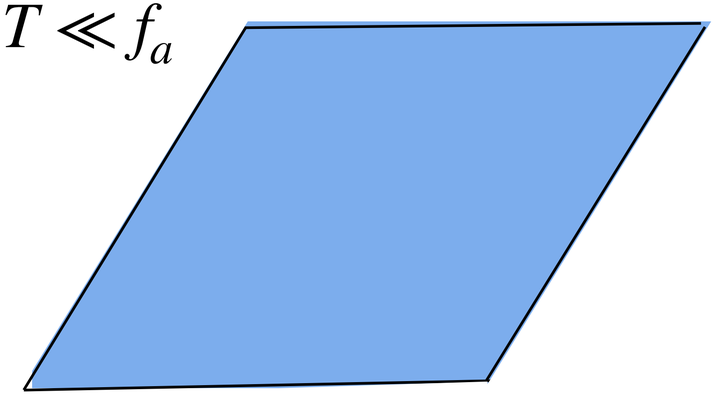

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Spontaneous symmetry breaking

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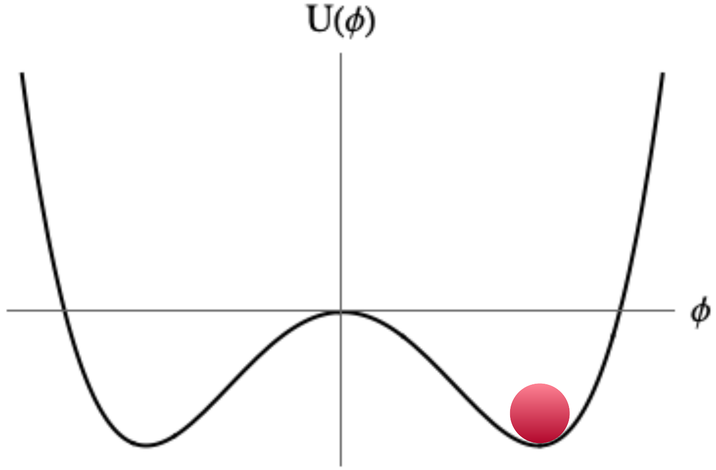

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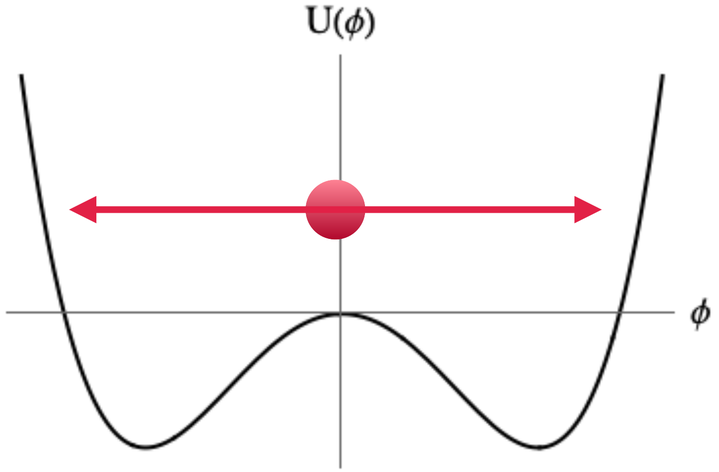

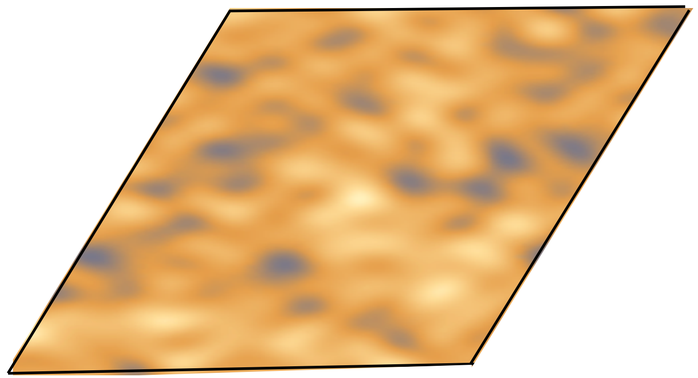

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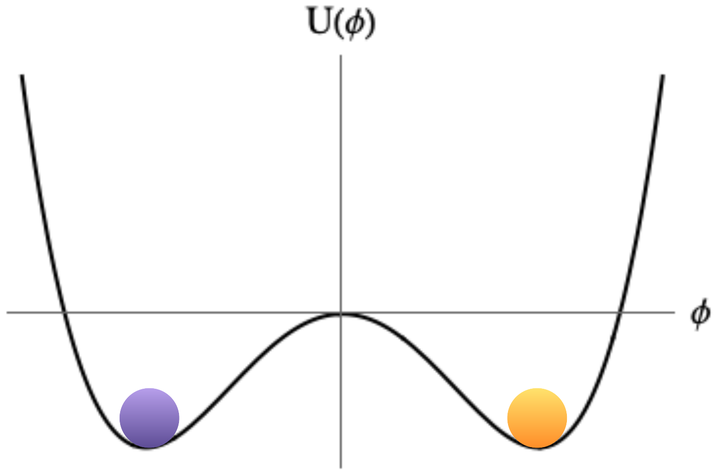

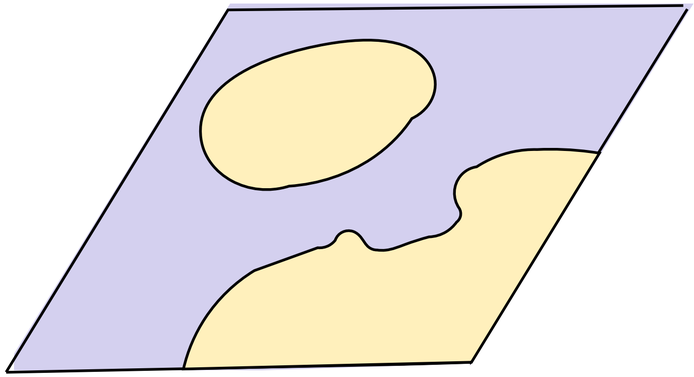

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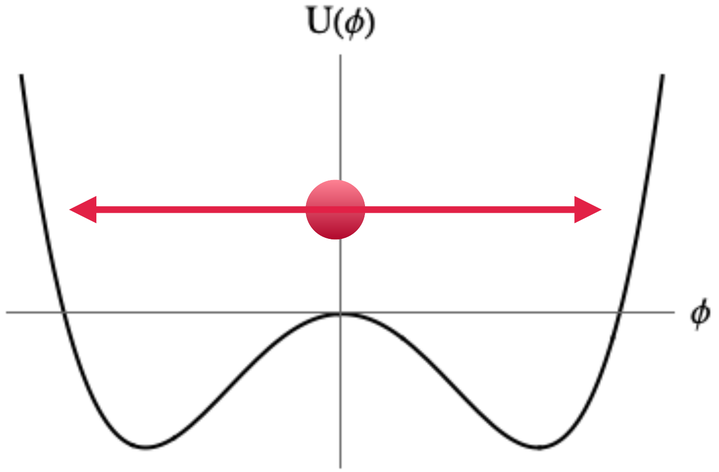

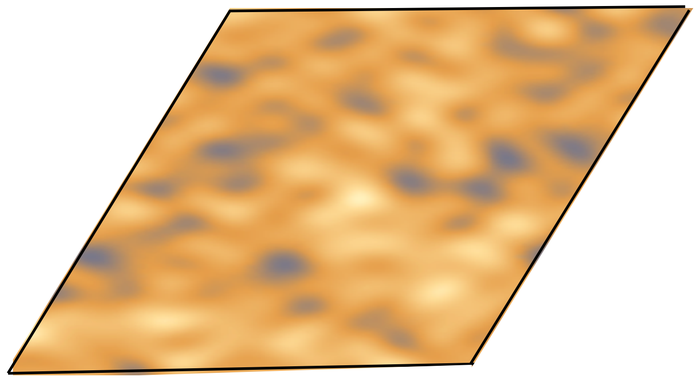

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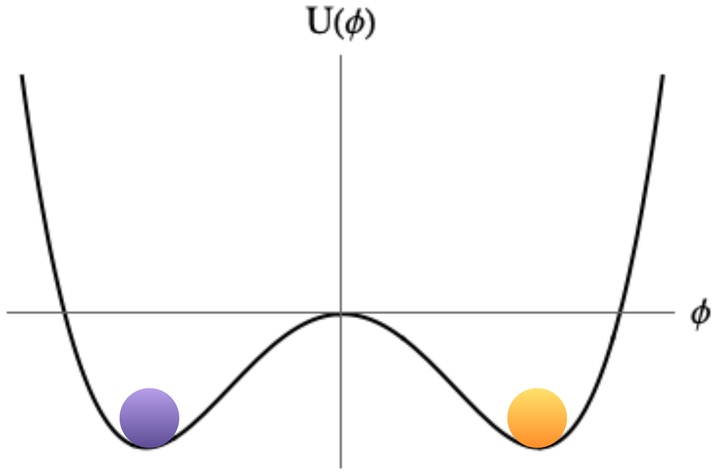
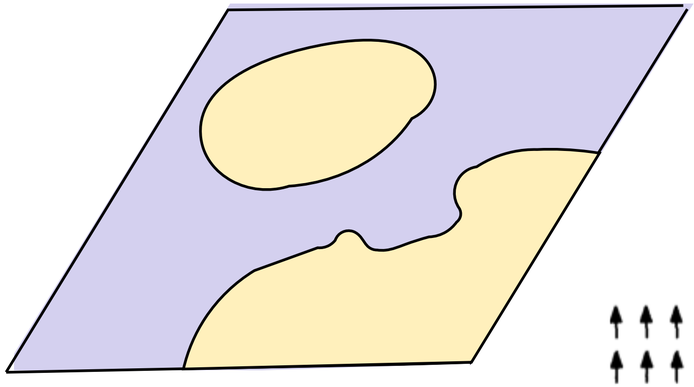
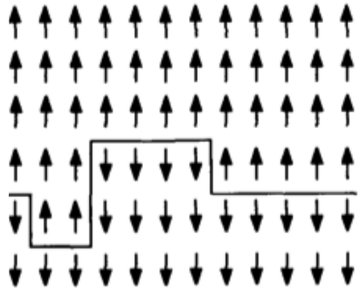




Low $T \ll f_a$

Similar to:



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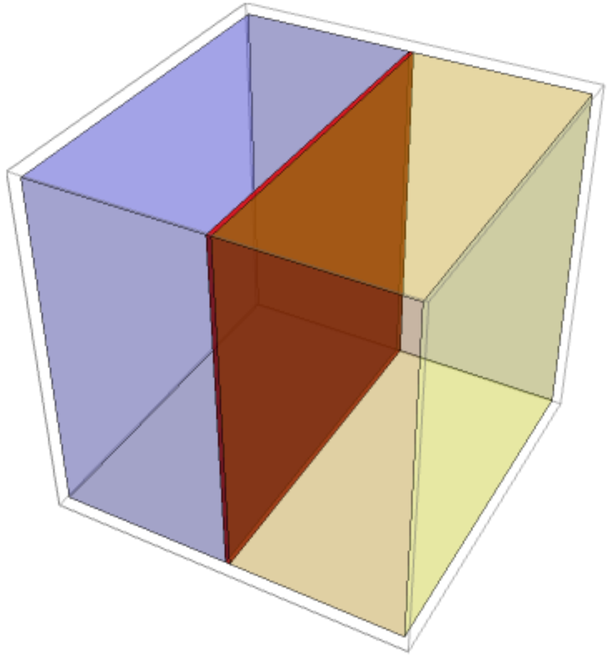
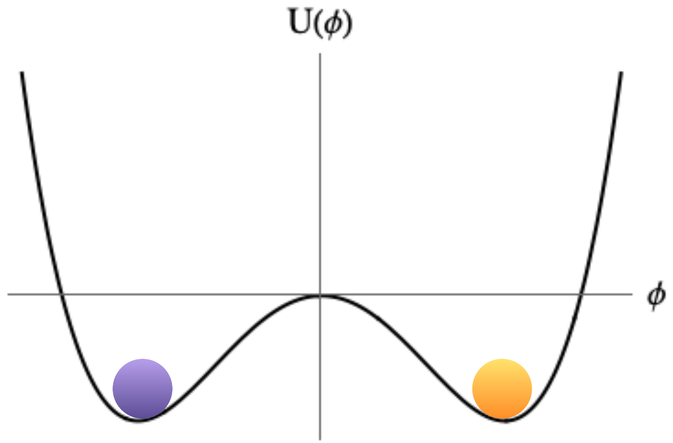
Domain walls





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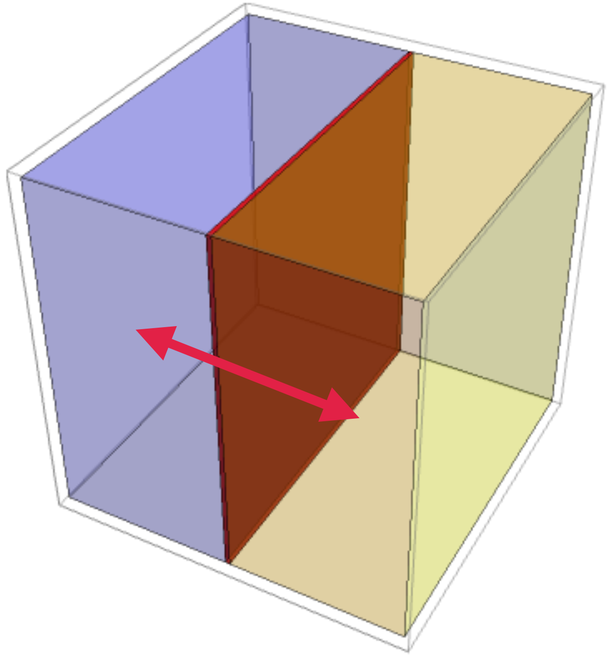
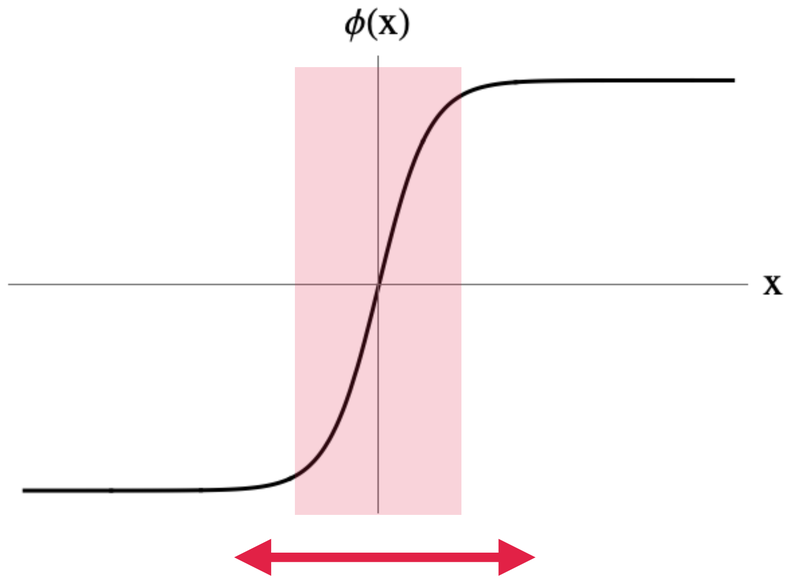
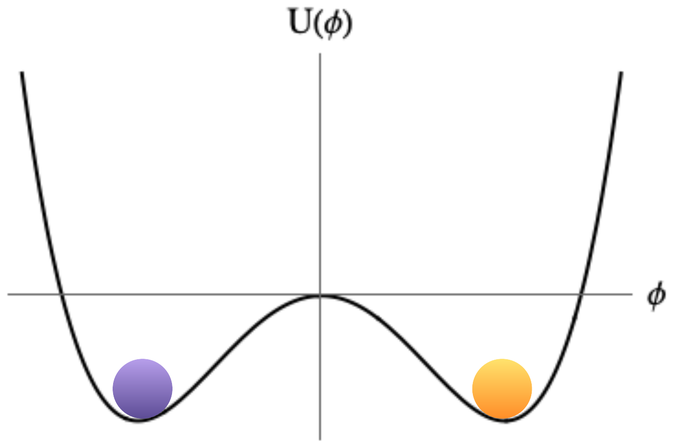
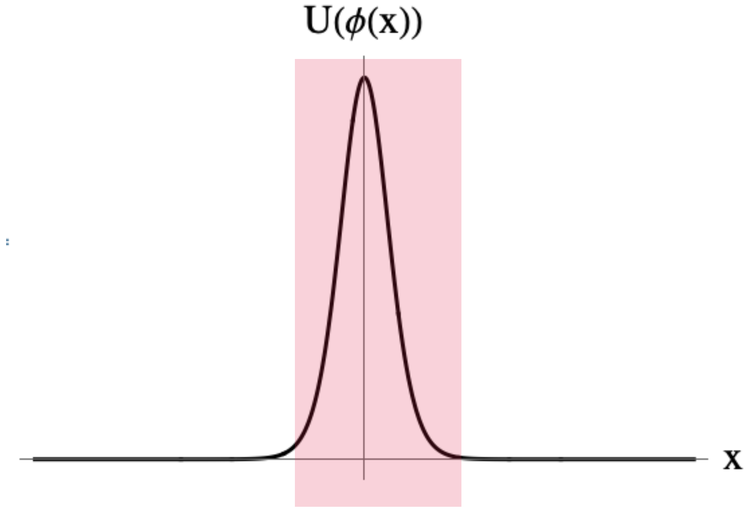
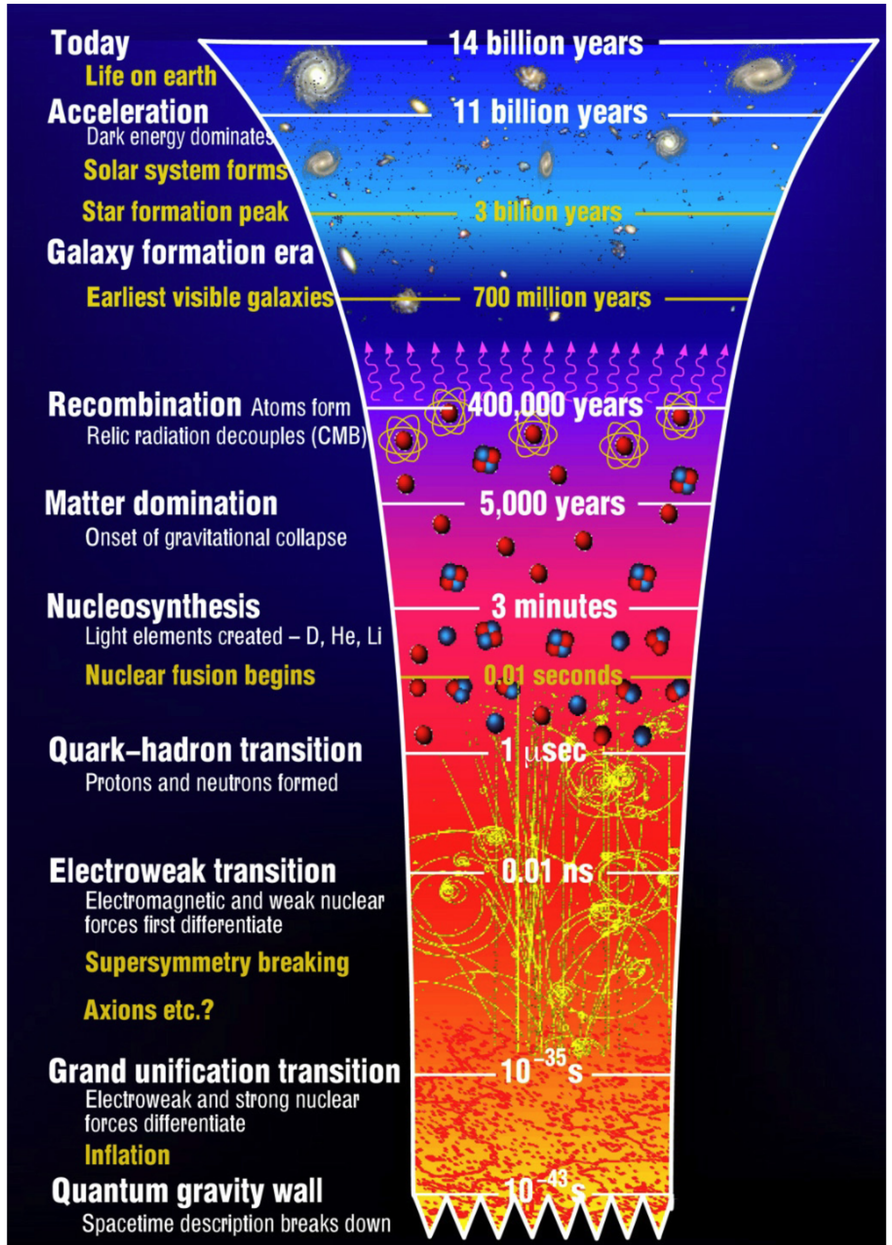








The early Universe



Temperature

10^{-4} eV

1 eV

1 MeV

1 GeV

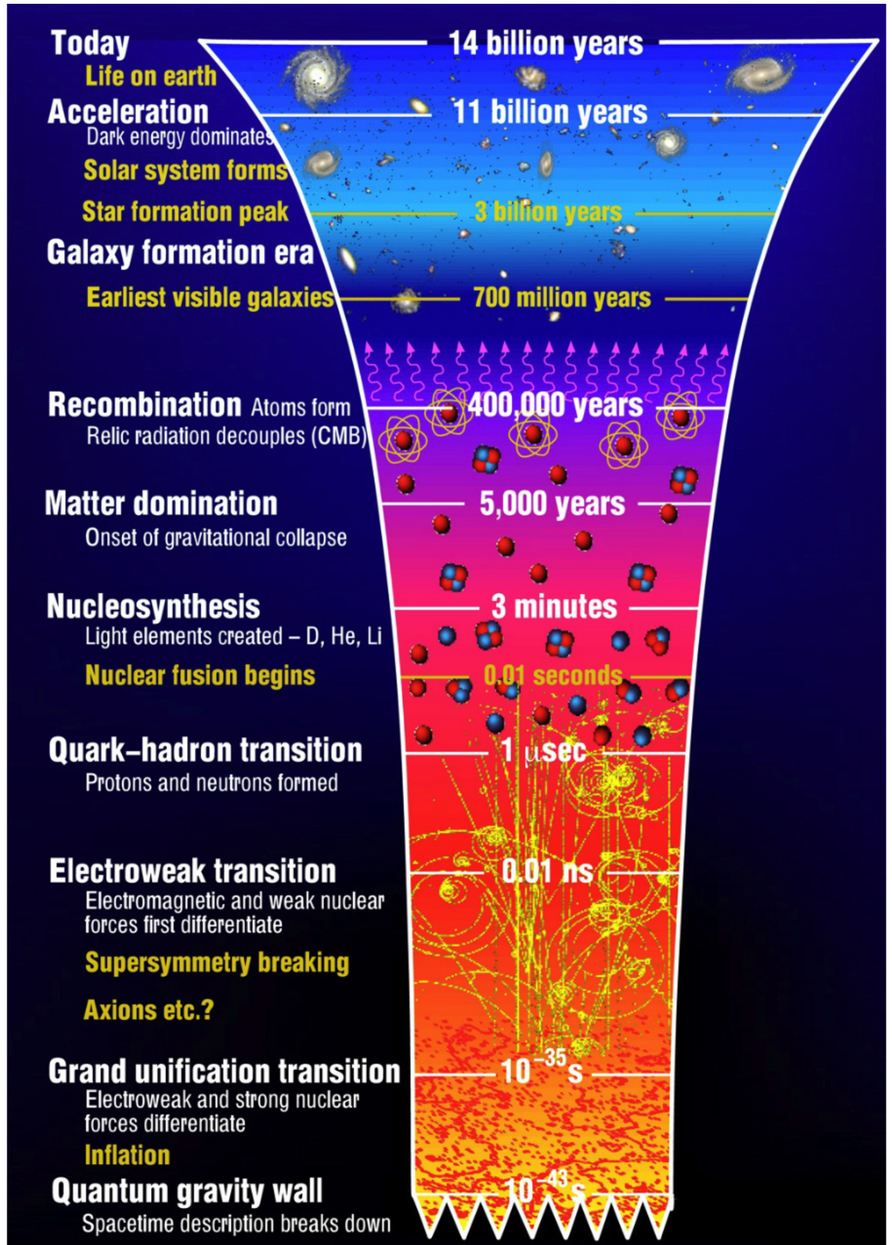
100 GeV

(LHC energies)

10^{16} GeV

$$\left(\quad (\mathrm{eV})^{-1} \sim 10^{-7}\,\mathrm{m} \qquad (\mathrm{MeV})^{-1} \sim 10^{-13}\,\mathrm{m} \qquad \left(10^{16}\,\mathrm{GeV}\right)^{-1} \sim 10^{-32}\,\mathrm{m} \quad \right)$$

The early Universe



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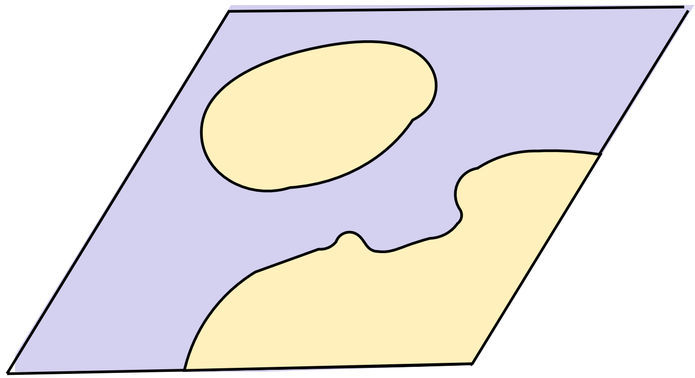
10^{16} GeV f_a 10^{-30} m



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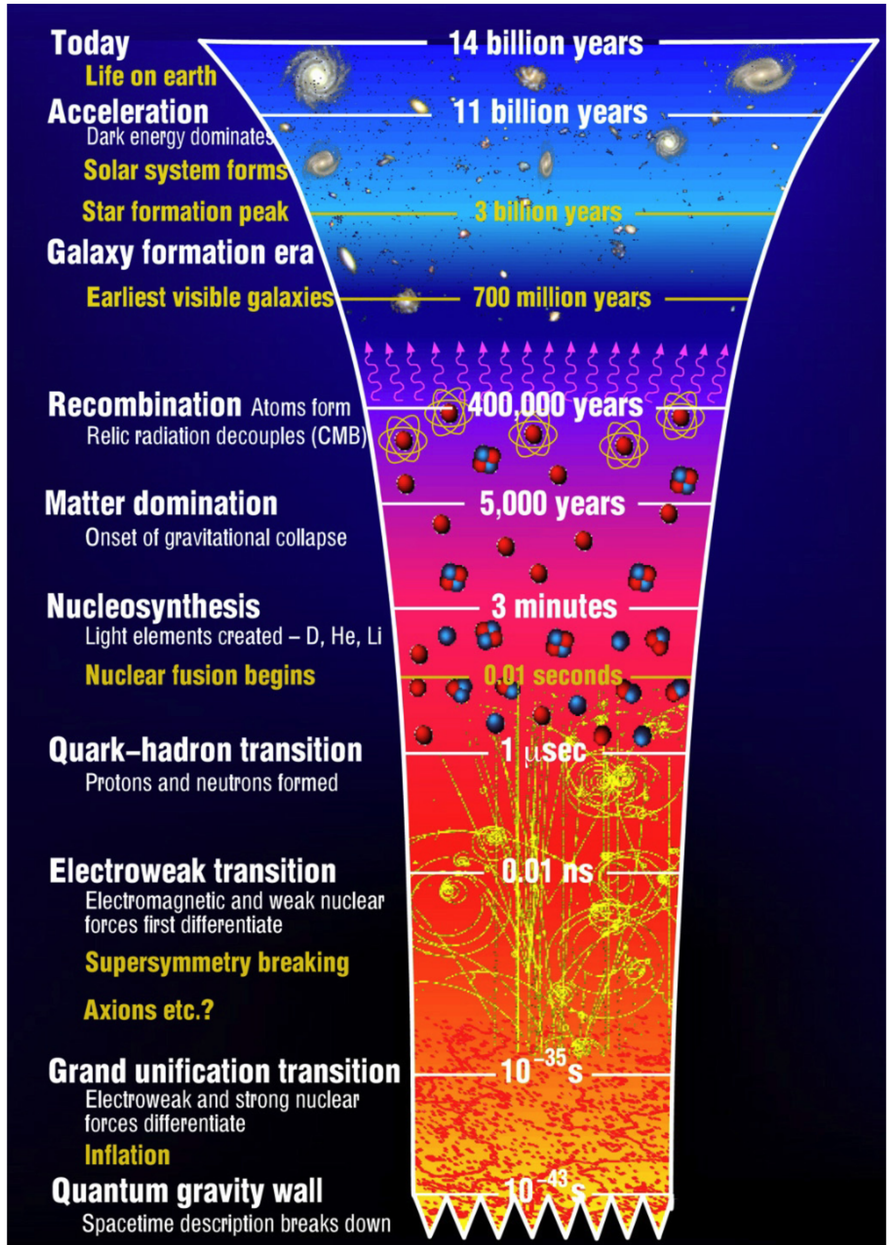
The early Universe



Temperature

Hubble horizon:

$$H^{-1} \simeq \frac{M_{\rm Pl}}{\sqrt{\rho}} \simeq \frac{M_{\rm Pl}}{T^2}$$

(during radiation domination)

10^{-4} eV $\quad 10^{26}$ m

1 eV

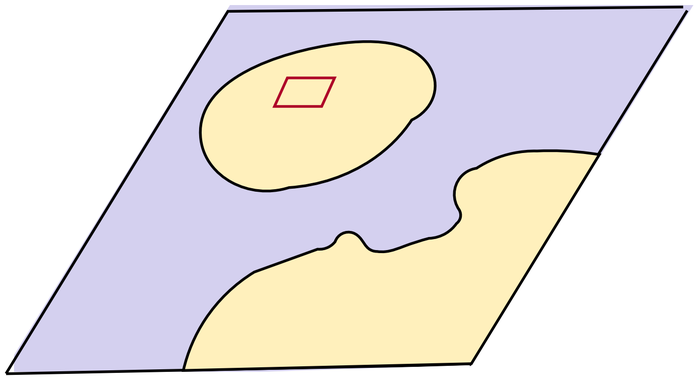
1 MeV

1 GeV $\quad 100$ m

100 GeV

(LHC energies)

10^{16} GeV $\quad f_a \quad 10^{-30}$ m



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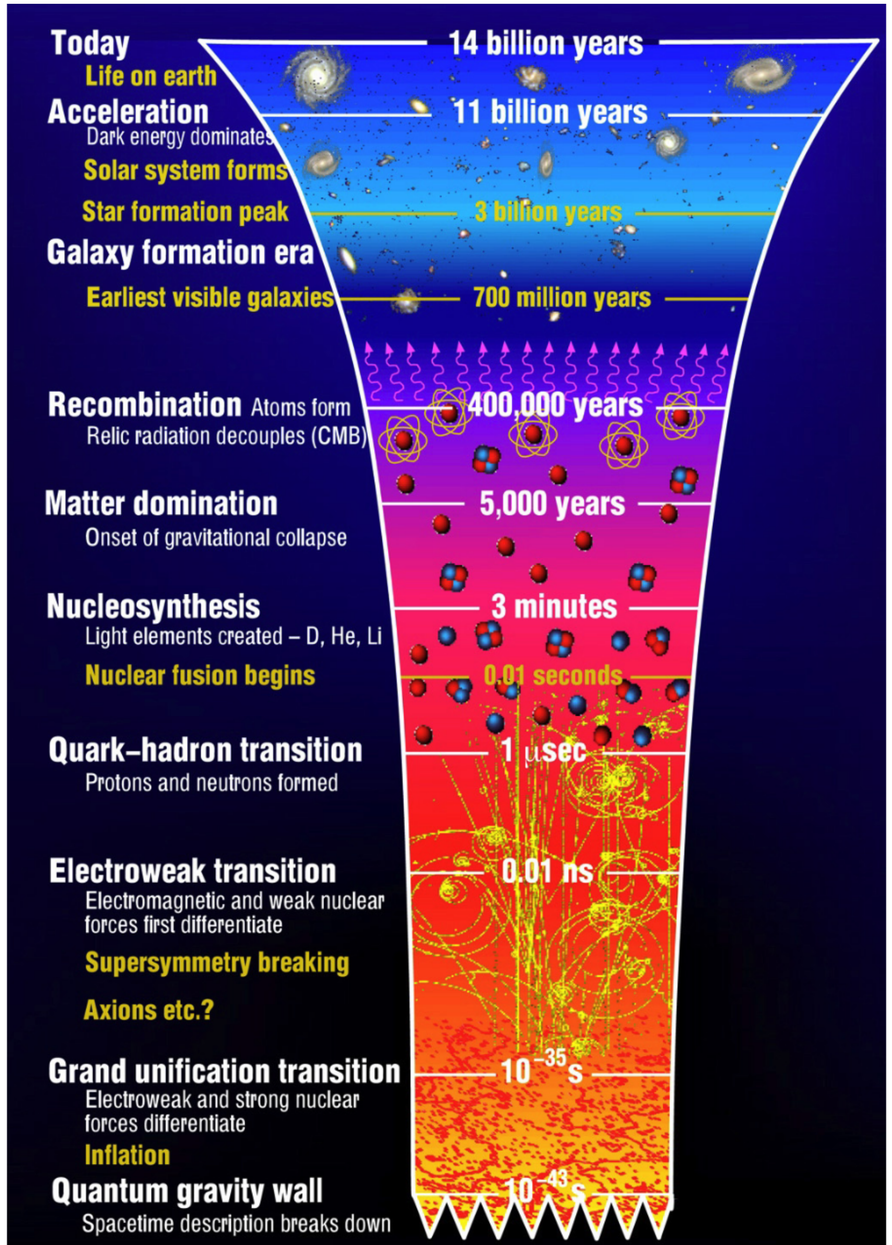
The early Universe



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(LHC energies)

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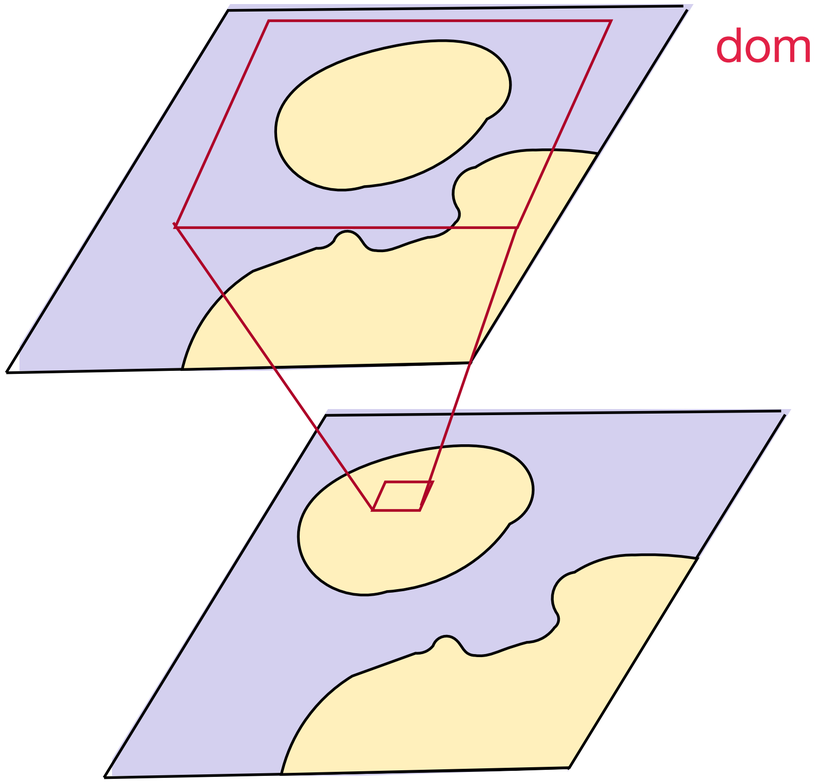

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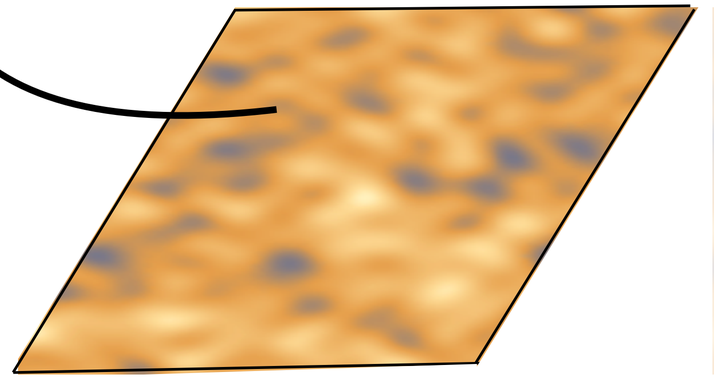
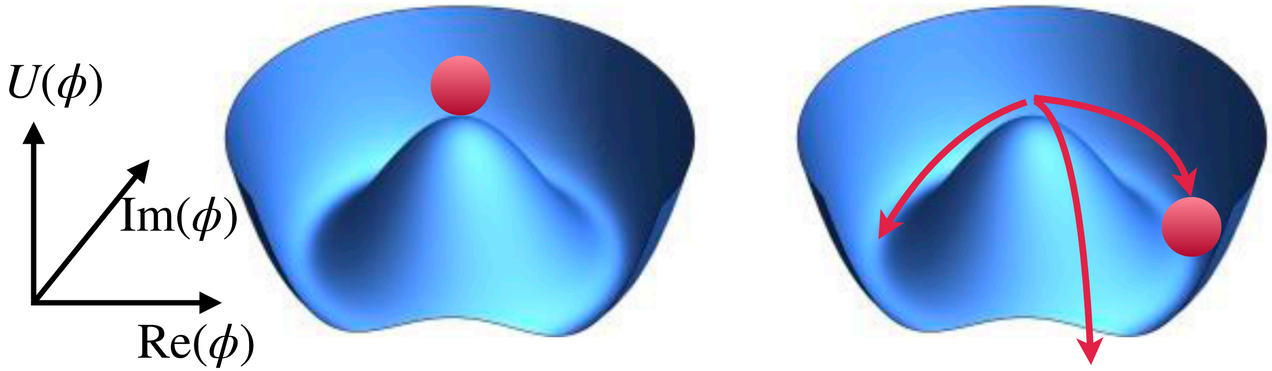
Strings

$$U(\phi) = (|\phi|^2 - f_a^2)^2$$

$\phi(t, \vec{x}) \in \mathbb{C}$





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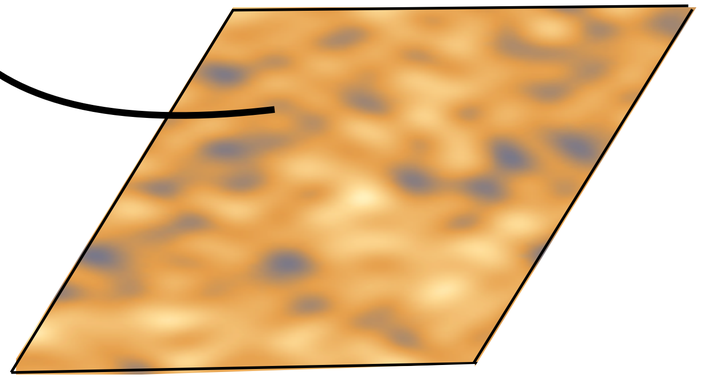
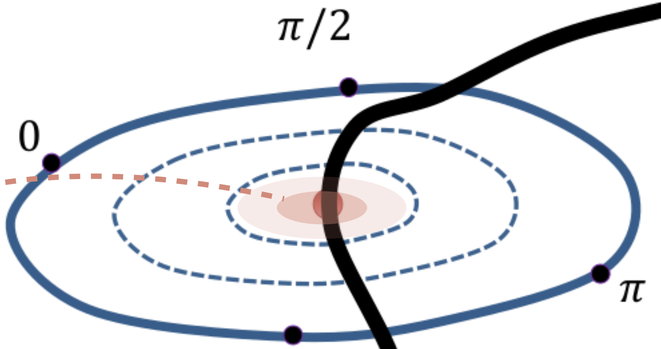
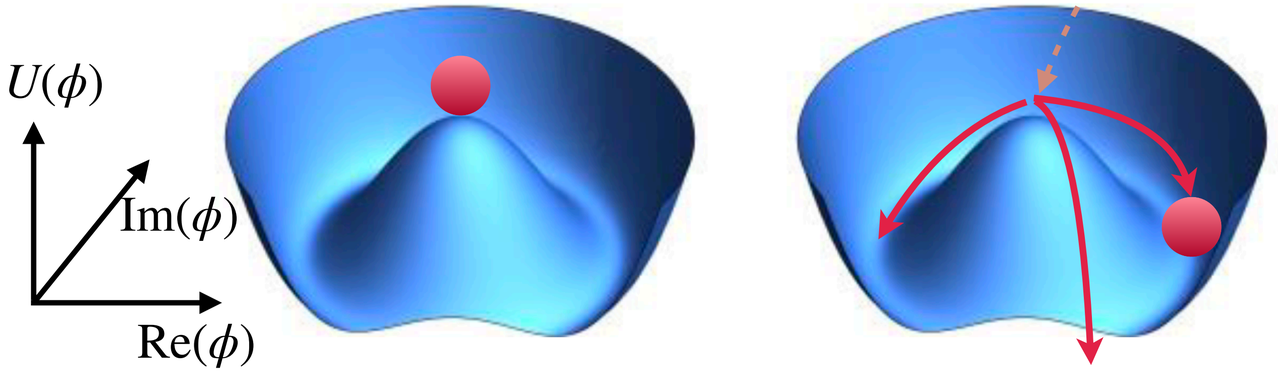


$\delta_s \simeq \frac{1}{f_a}$





$$\phi = f_a\, g(|x|) e^{i\theta}$$

$$g(0) = 0 \quad g(\infty) = 1$$

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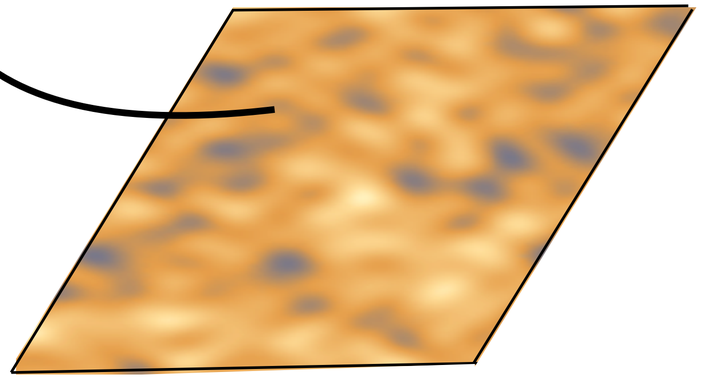

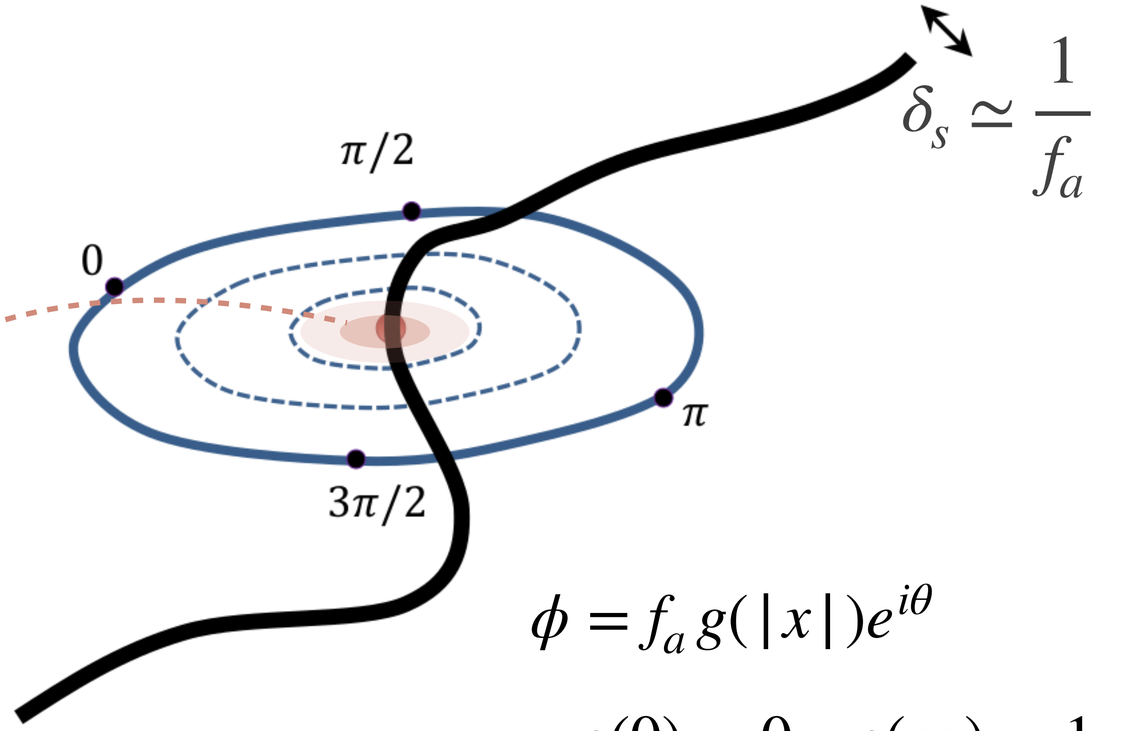

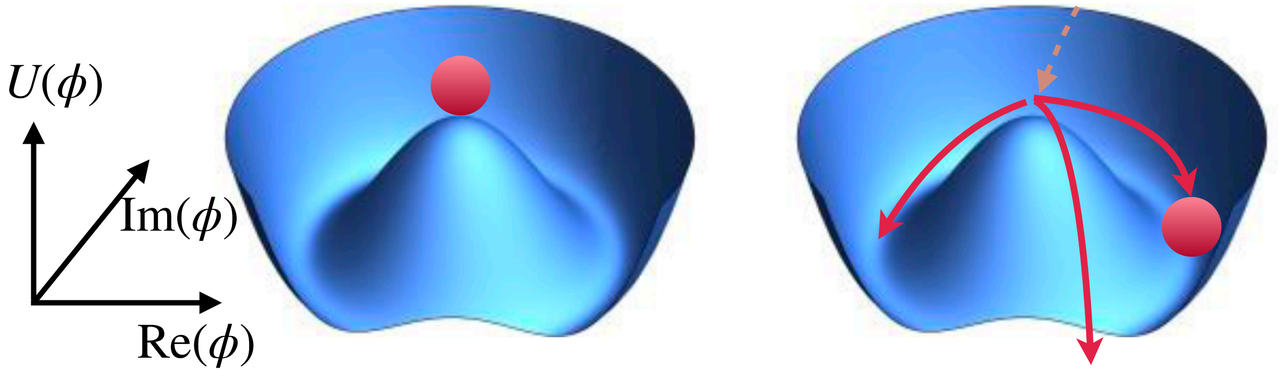

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$$g(0) = 0 \quad g(\infty) = 1$$

String tension $E/L \sim \pi f_a^2$

Cosmological evolution

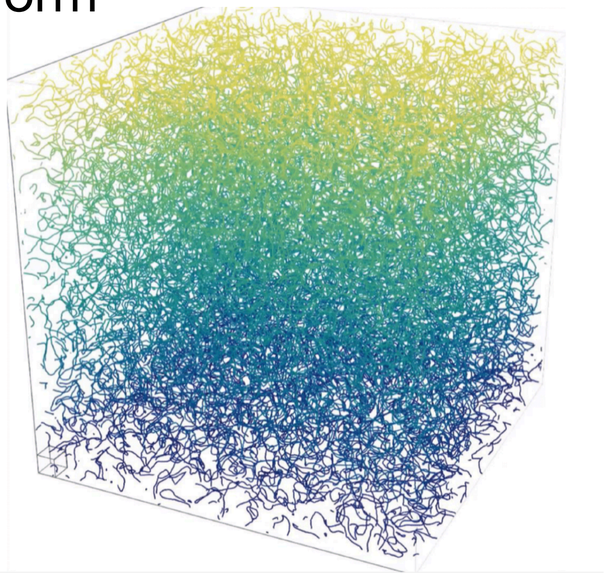
$T \gtrsim f_a$

$\sim 10^{10}\,\mathrm{GeV} \to 10^{16}\ \mathrm{GeV}$

$T \sim \mathrm{GeV}$

today

Strings form



Destroyed

Relic dark matter,
gravitational waves,
signals in the cosmic microwave background

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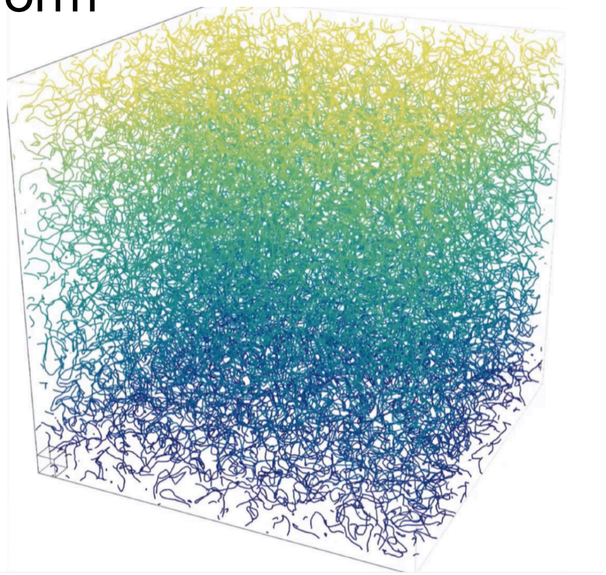


Dynamics:

- nonlinear

- large scale separation

Cosmological evolution

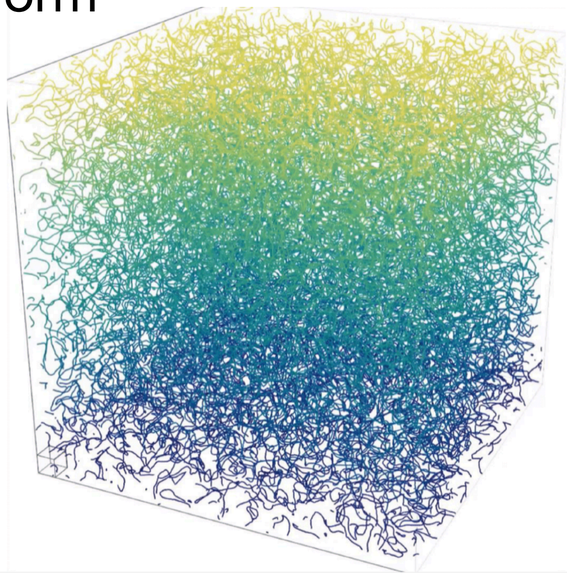
$T \gtrsim f_a$

$\sim 10^{10}\,\mathrm{GeV} \to 10^{16}\ \mathrm{GeV}$

$T \sim \mathrm{GeV}$

today

Strings form



Destroyed

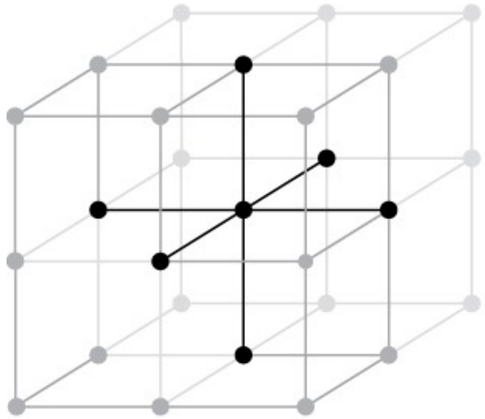
Relic dark matter,
gravitational waves,
signals in the cosmic microwave background

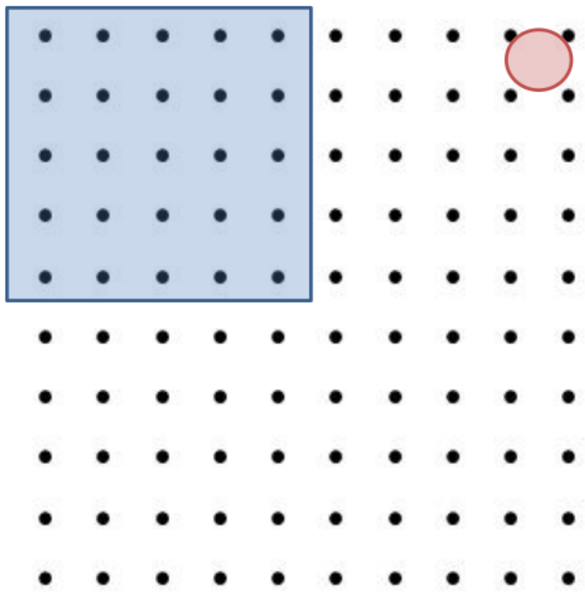
Dynamics:

- nonlinear — *analytics* ✗

- large scale separation — *numerics* ✗

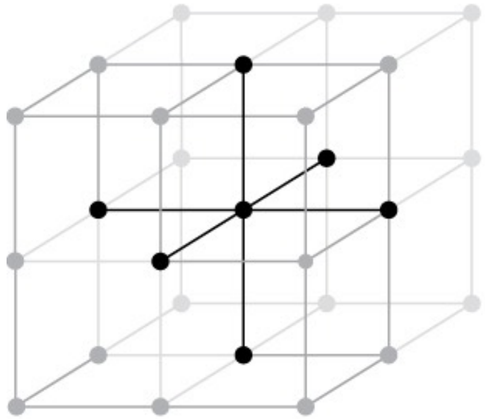
Simulations

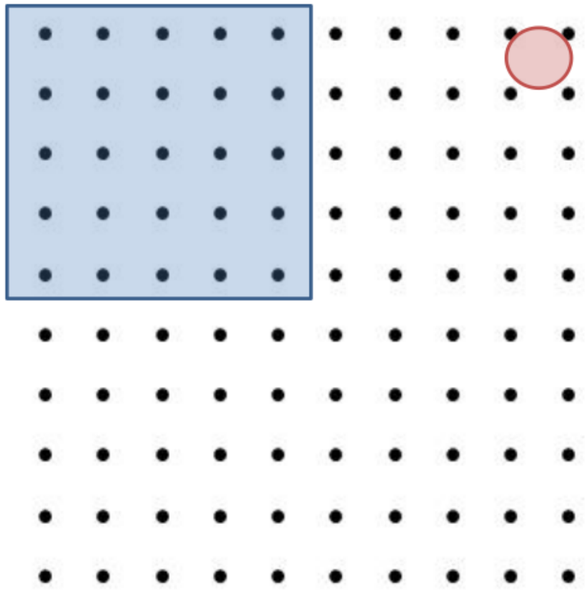




- a few lattice points per string core
- a few Hubble patches

Simulations



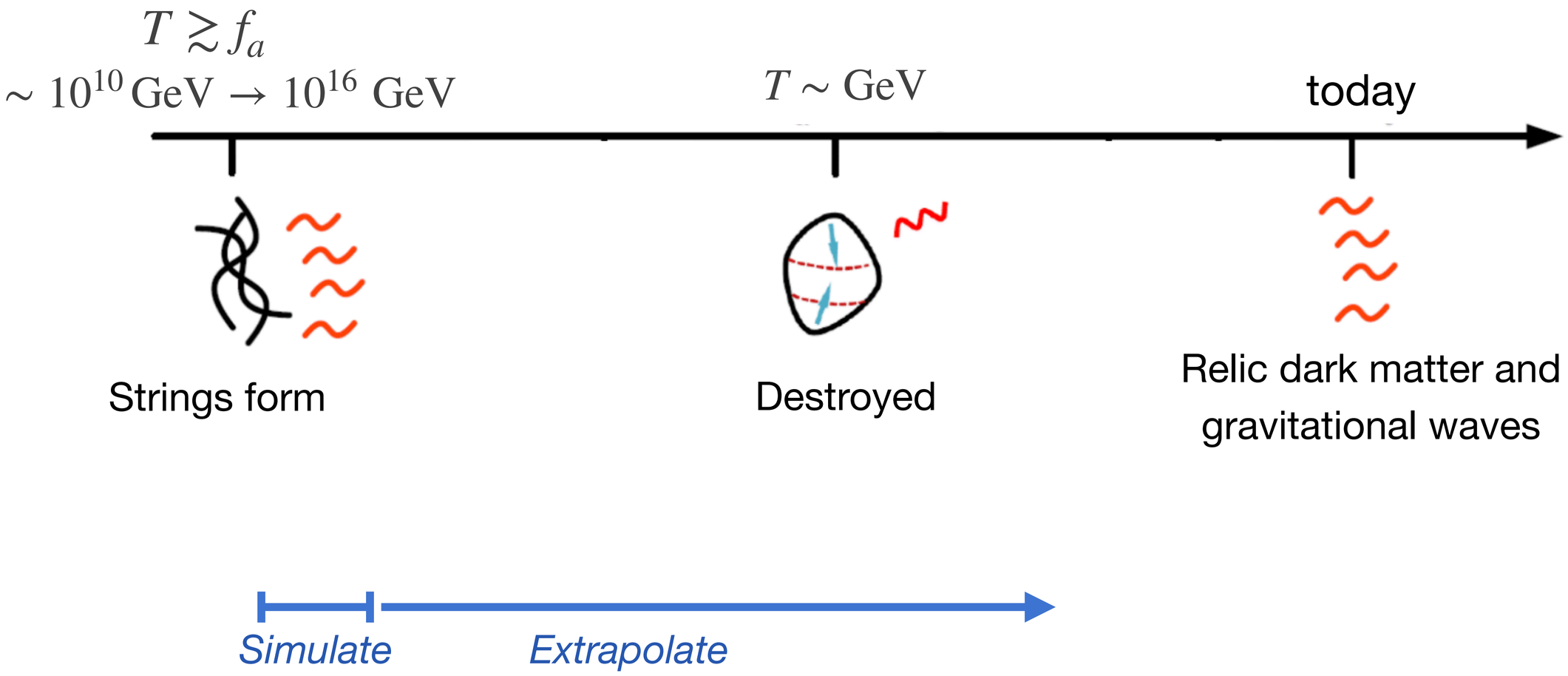


- a few lattice points per string core
- a few Hubble patches

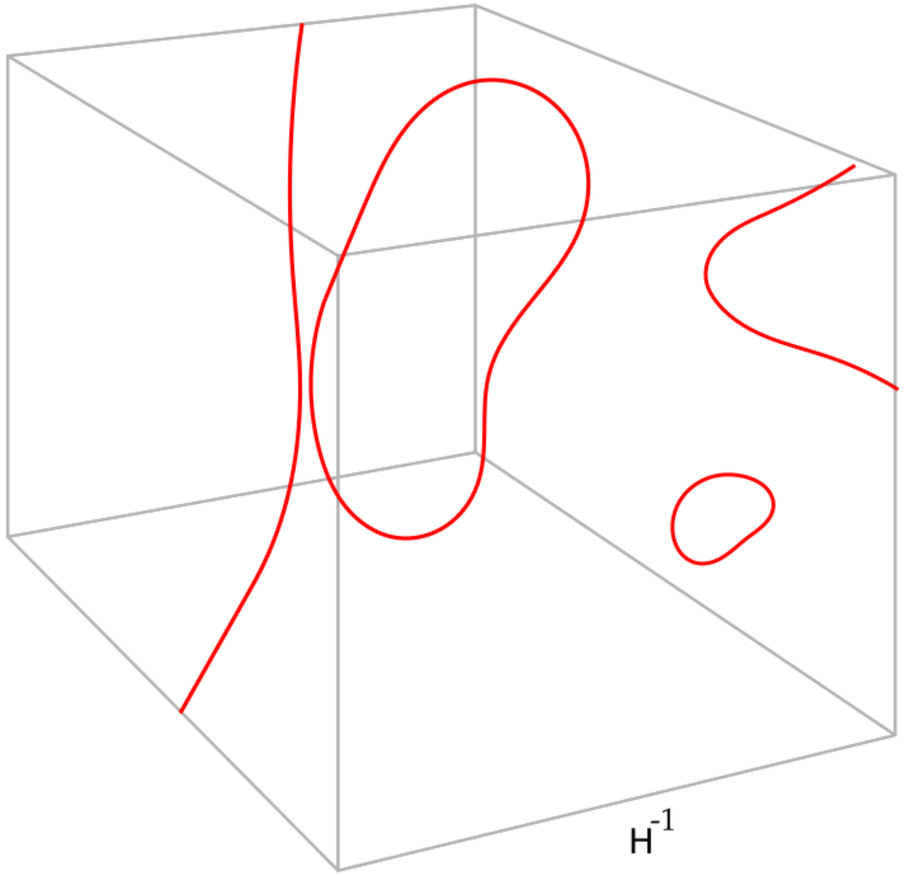
Memory constraints → max 5000^3 grid points

Simulations $\frac{f_a}{H} \lesssim \frac{\square}{\bigcirc} \lesssim 1000$

Physical $\frac{f_a}{H} \sim 10^{30}$


$T \gtrsim f_a$
$\sim 10^{10}\,\mathrm{GeV} \to 10^{16}\,\mathrm{GeV}$
$T \sim \mathrm{GeV}$
today
Strings form
Destroyed
Relic dark matter and gravitational waves
Simulate
Extrapolate

Scaling solution



~ one Hubble length of string per Hubble patch

$T \gtrsim f_a$

$\sim 10^{10}\,\mathrm{GeV} \to 10^{16}\ \mathrm{GeV}$

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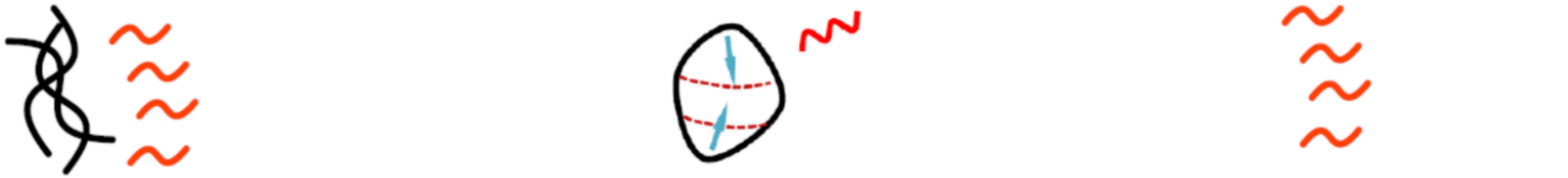
today



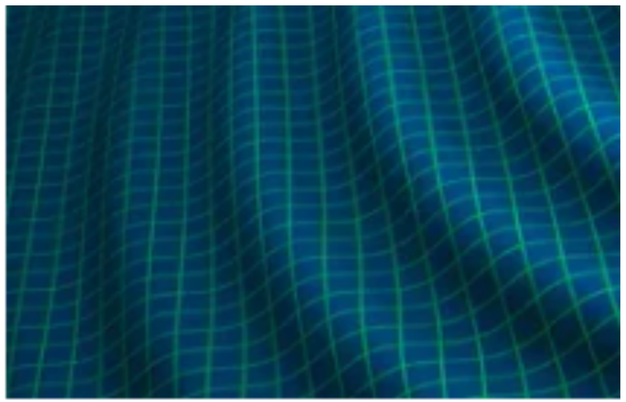
Strings form

Destroyed

Energy emitted per Hubble time and Hubble volume $\simeq \pi f_a^2 \times H^{-1}$

Dark matter

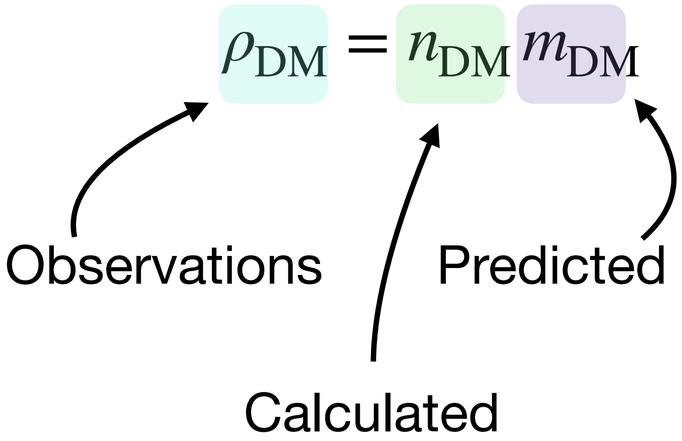
Gravitational waves



Dark matter abundance

$$\rho_{\mathrm{DM}} = n_{\mathrm{DM}}\, m_{\mathrm{DM}}$$

Dark matter abundance



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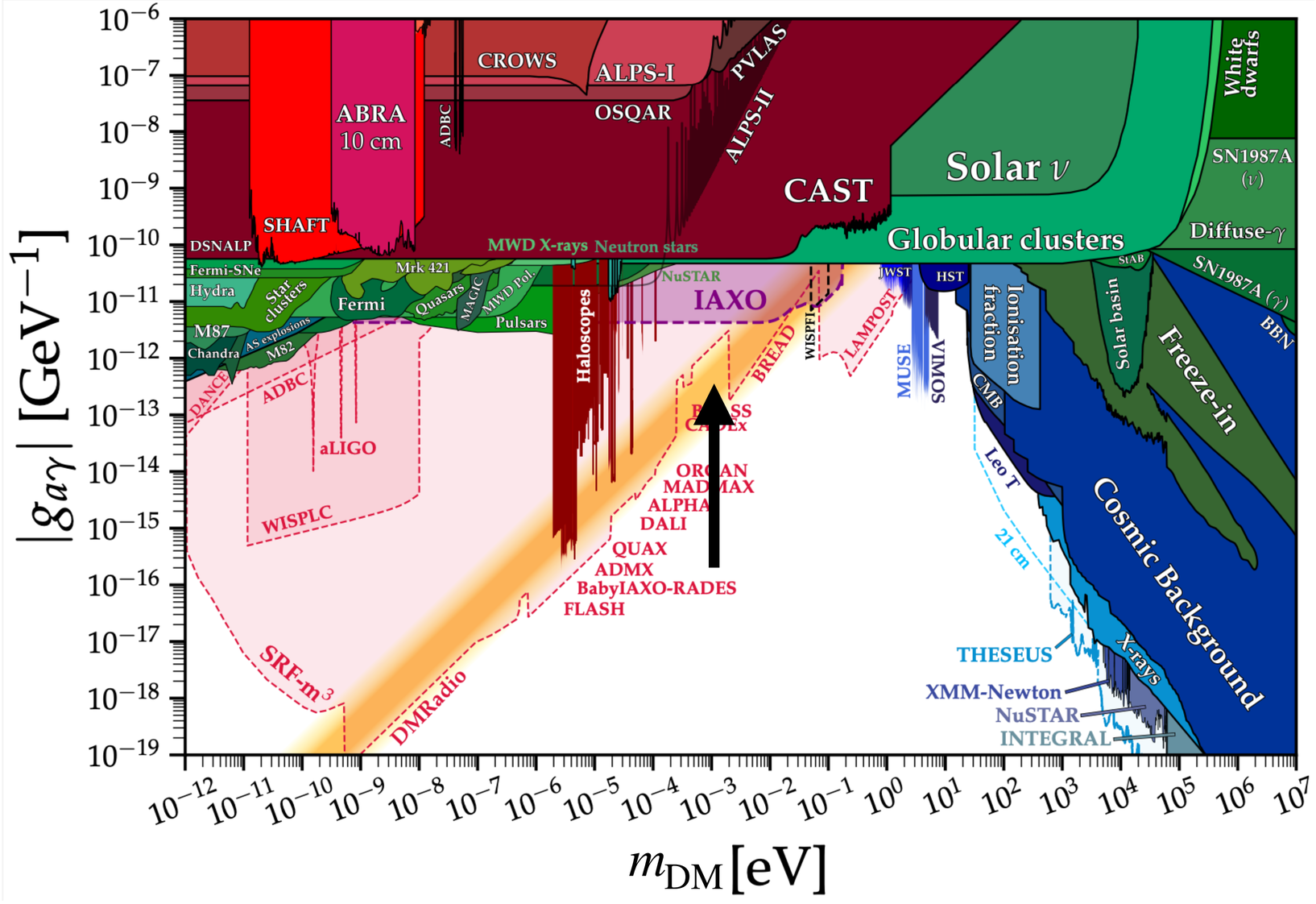
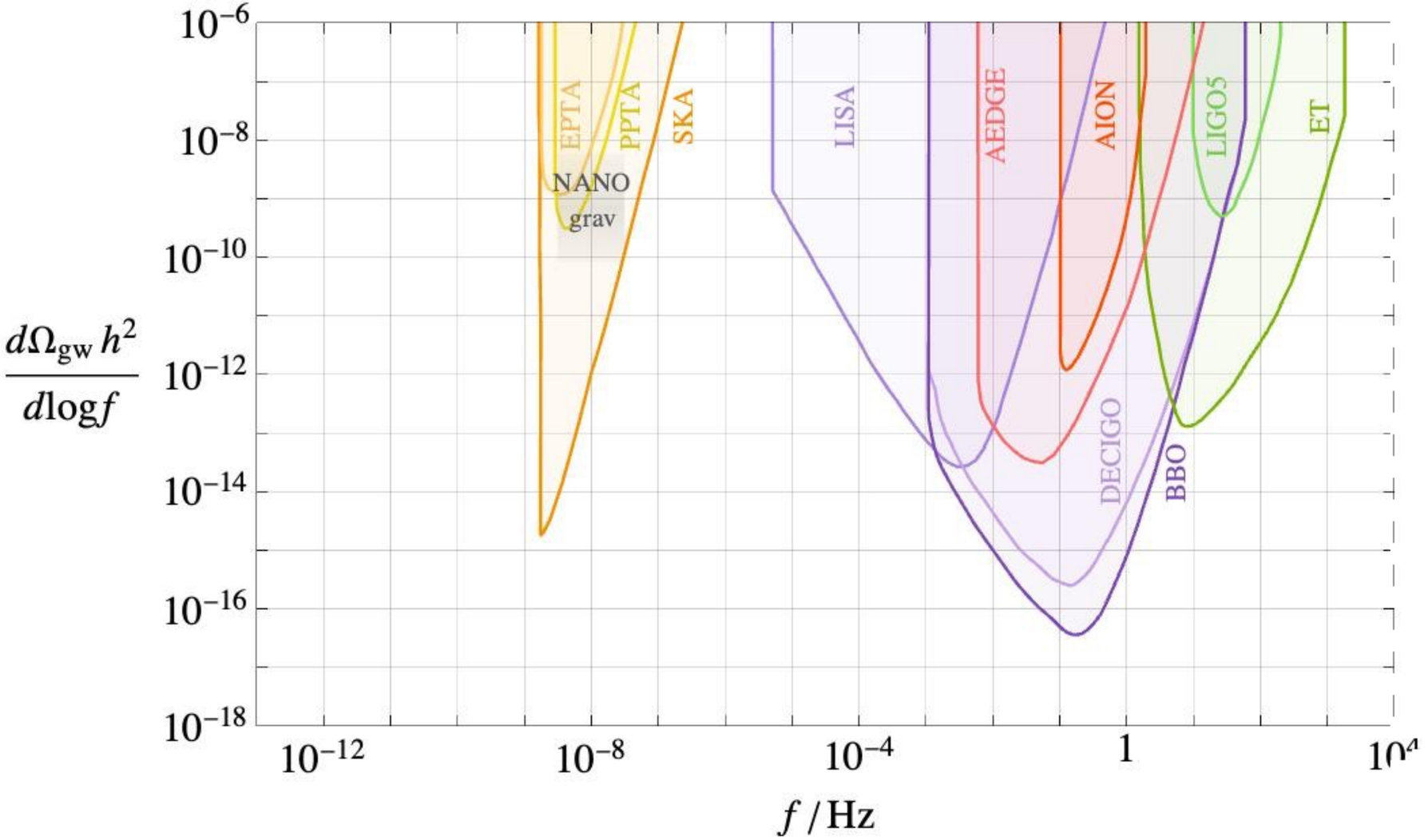
Observations — Predicted — Calculated



Gravitational waves



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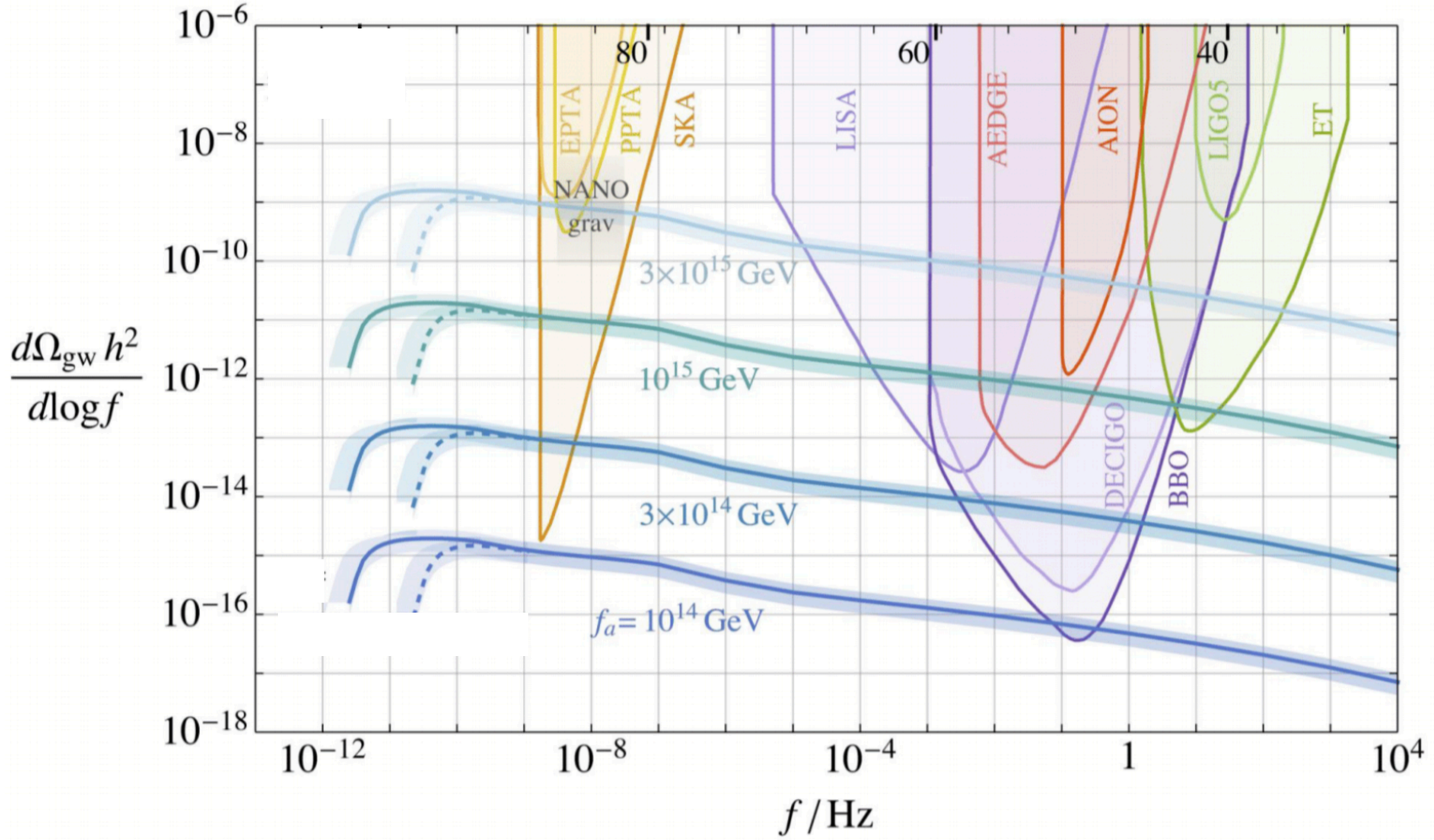

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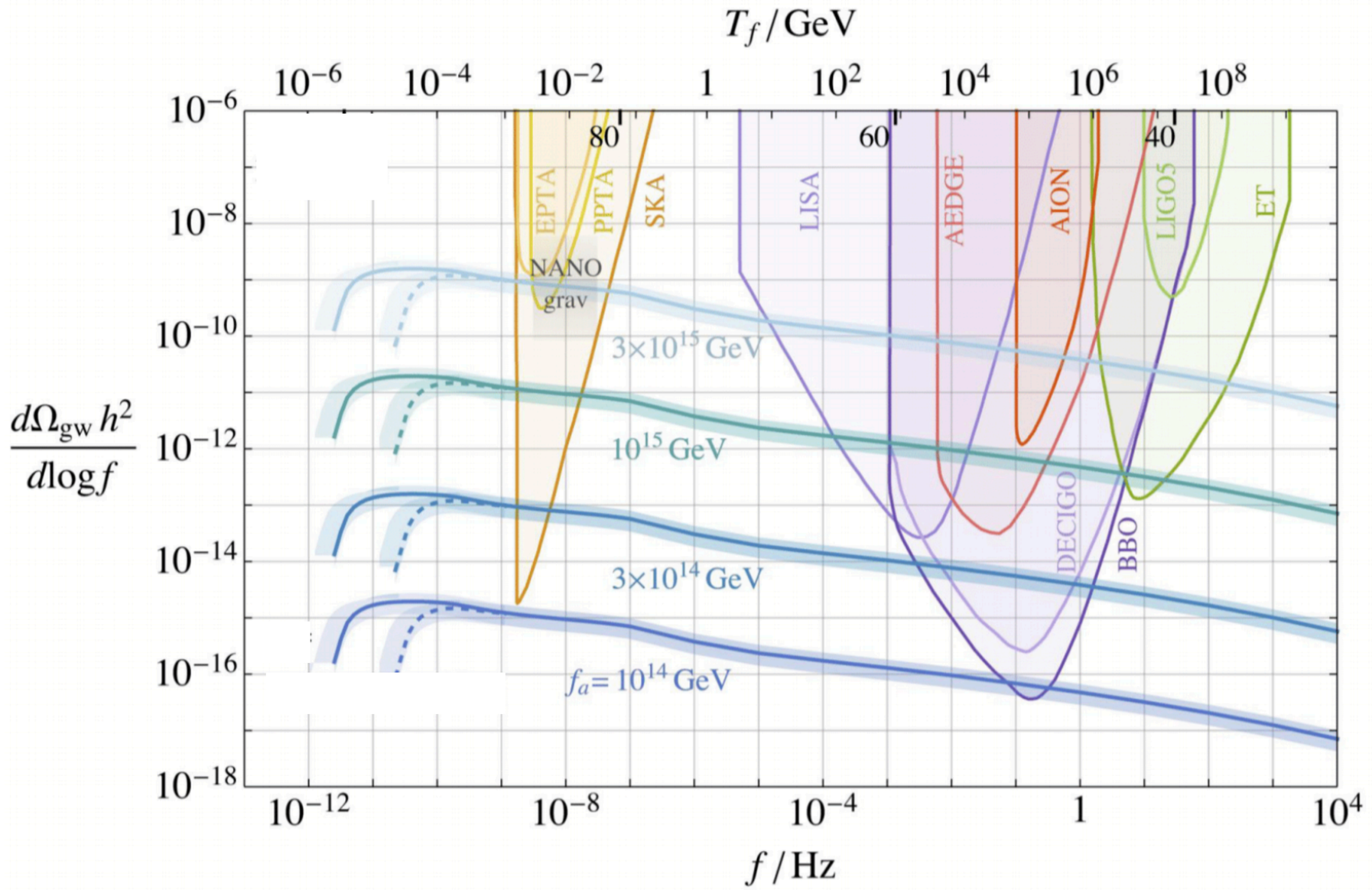
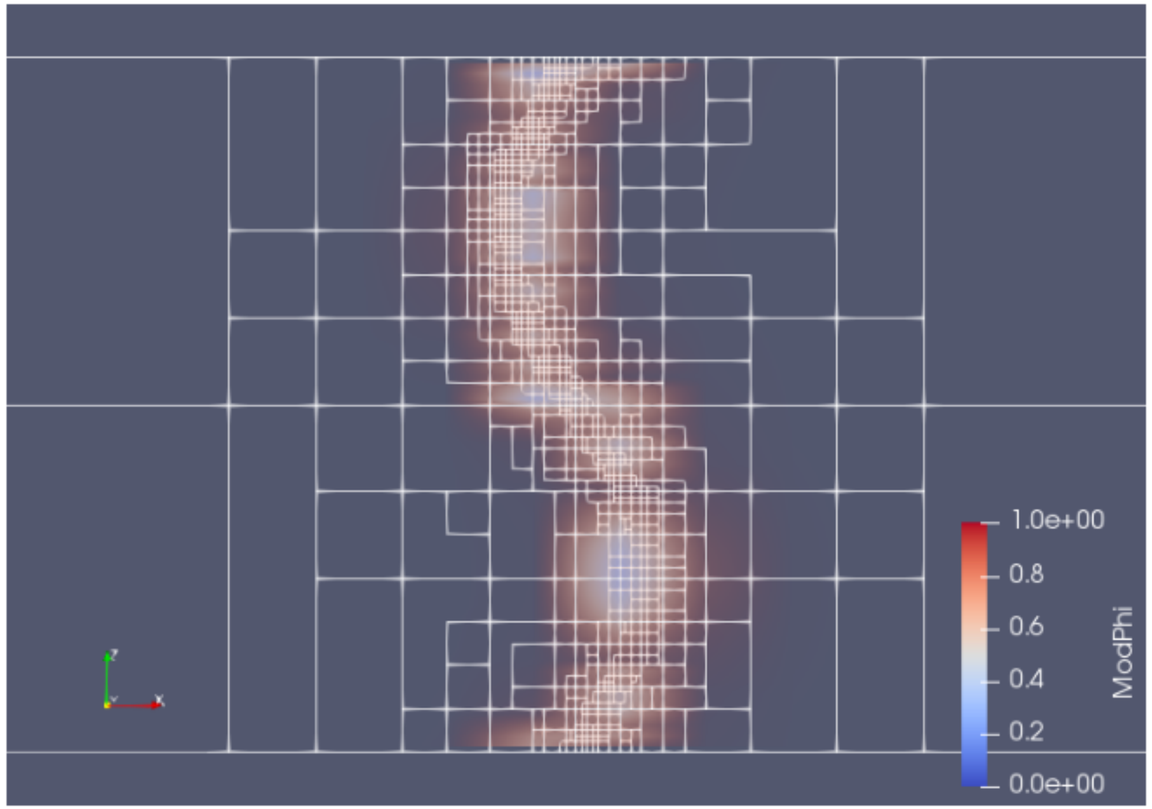
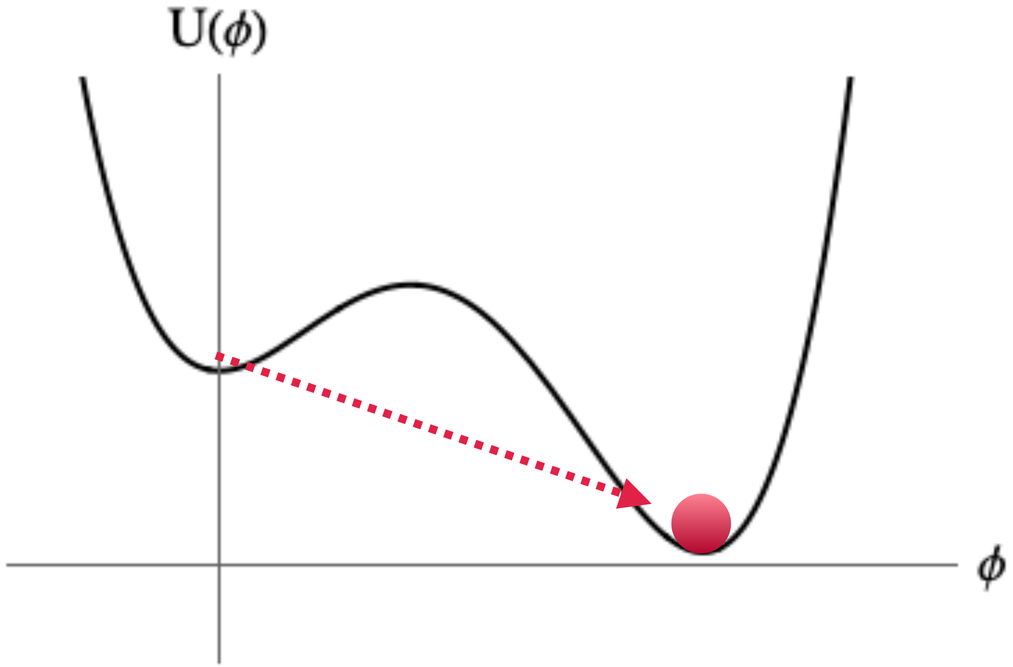


Work in progress

Adaptive meshing



1st order phase transitions



Summary

- Spontaneous symmetry breaking $\xrightarrow{\text{(often)}}$ “Topological defects”
- Persist from the early universe
- Access to ultra-high energy scales $\sim 10^{16}\,\mathrm{GeV}$
- Also the very early Universe: $T \sim 10^{8}\,\mathrm{GeV} \implies t_{\mathrm{universe}} \sim 10^{-22}\,\mathrm{second}$
- Ongoing experimental and theoretical effort

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Thanks

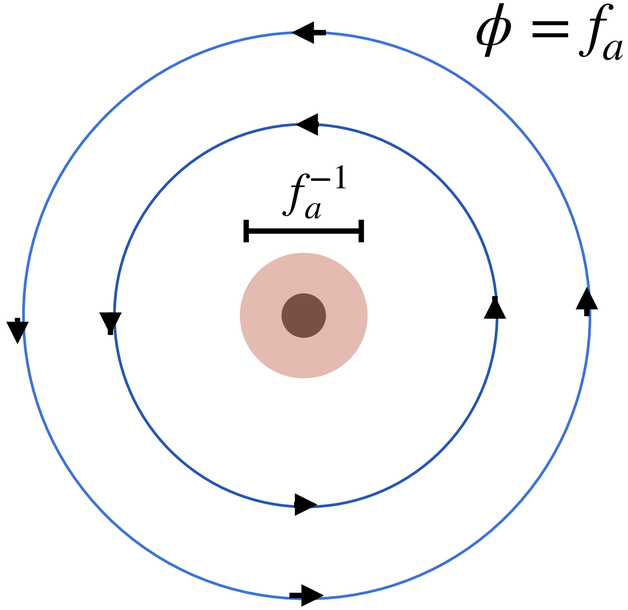
Backup

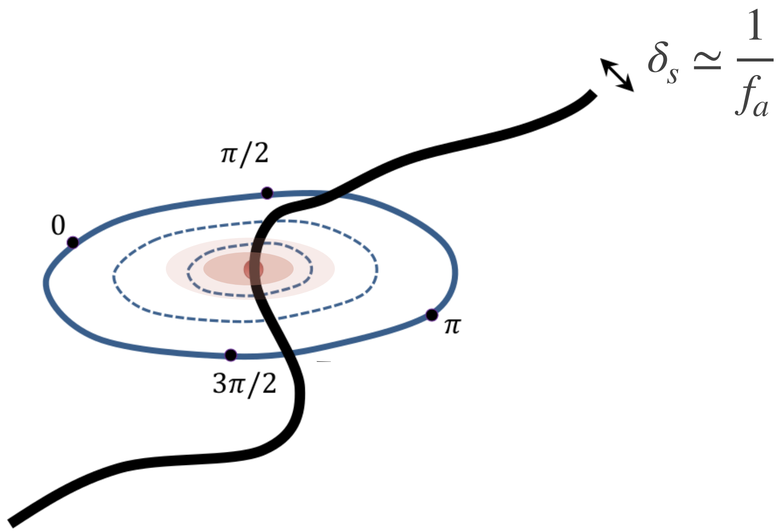
$$U(\phi) = (|\phi|^2 - f_a^2)^2$$

String tension $\mu = E/L$

$$\mathscr{H} \sim (\partial_i \phi)^2 + U(\phi)$$

$$\phi = f_a\, g(|x|) e^{i\theta}$$





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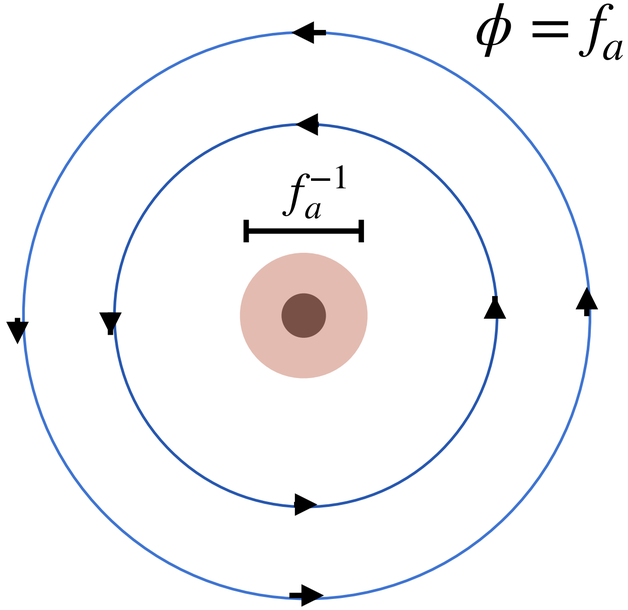

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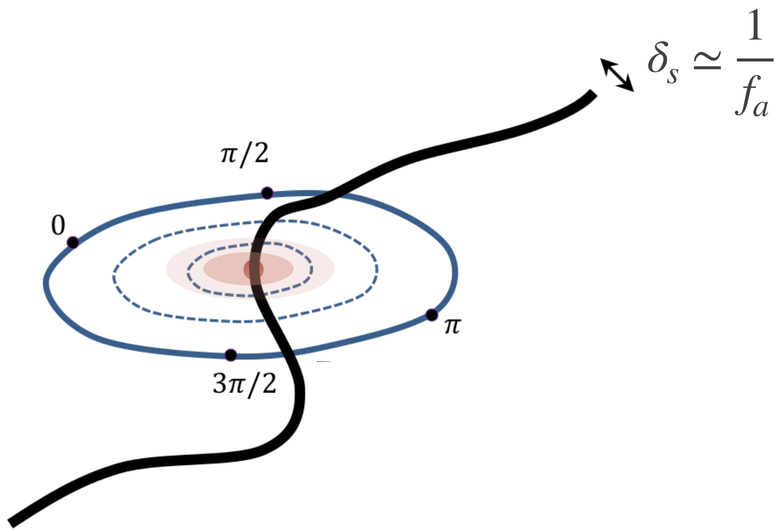
Core : πf_a^2



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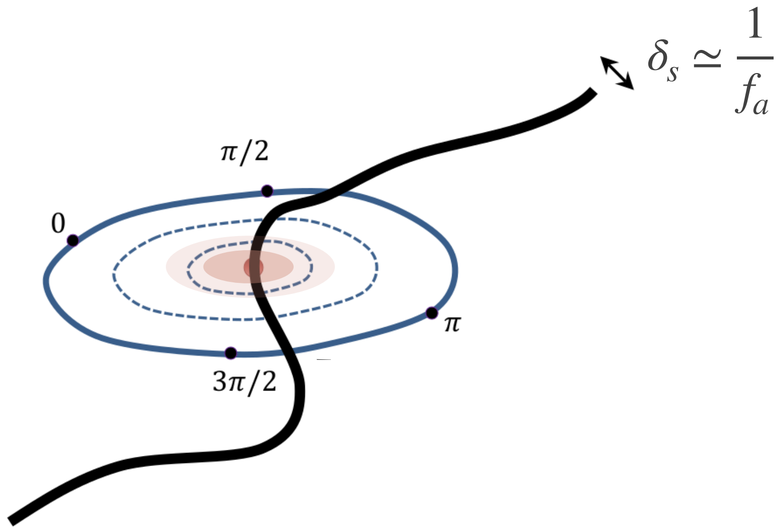

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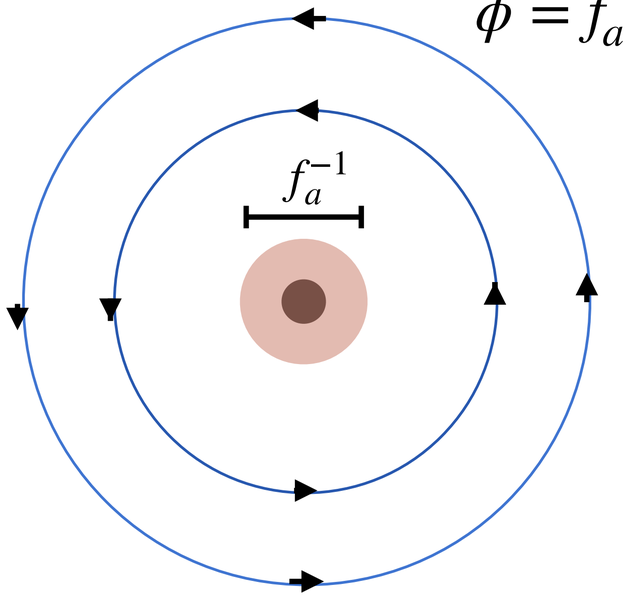

Core : πf_a^2

Gradient : $$\int_{f_a^{-1}}^{r_{\max}} dr\, 2\pi r \left| \frac{\partial_\theta \phi}{r} \right|^2$$

$$\sim \pi f_a^2 \log\left(\frac{r_{\max}}{f_a^{-1}} \right)$$

$$U(\phi) = (|\phi|^2 - f_a^2)^2$$

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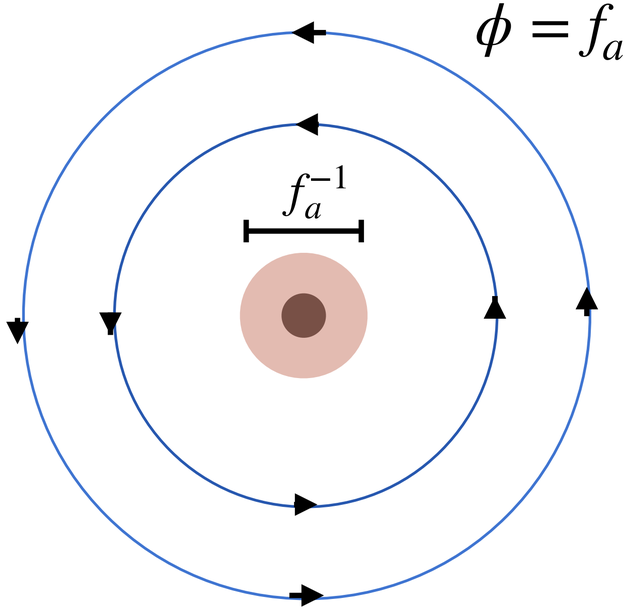

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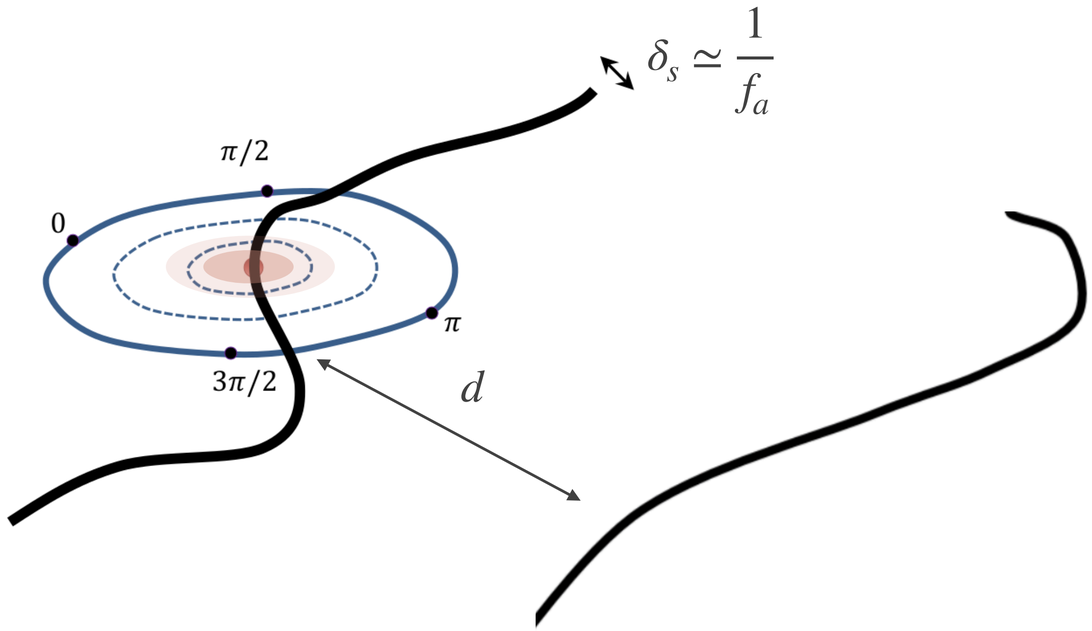

$$\mu \simeq \pi f_a^2 \log\left(\frac{d}{f_a^{-1}}\right)$$

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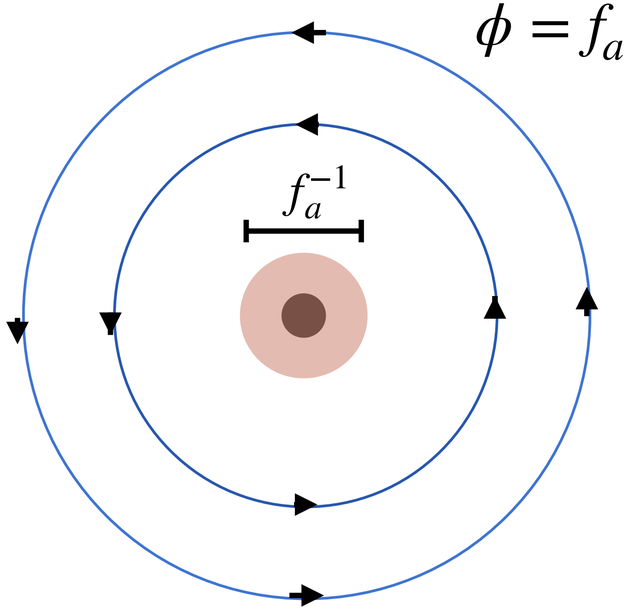

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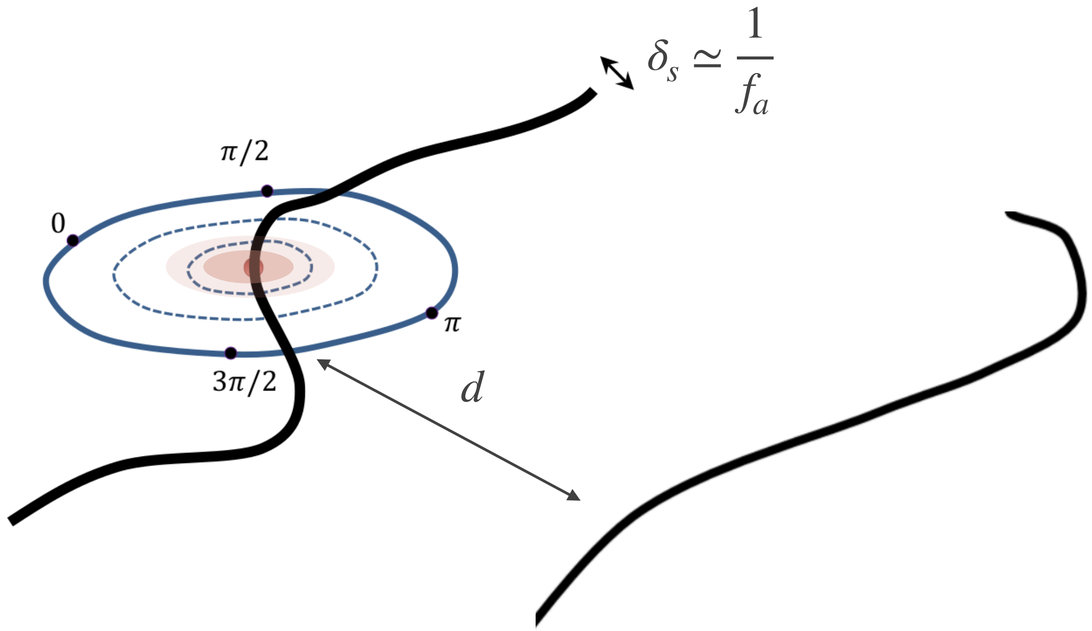


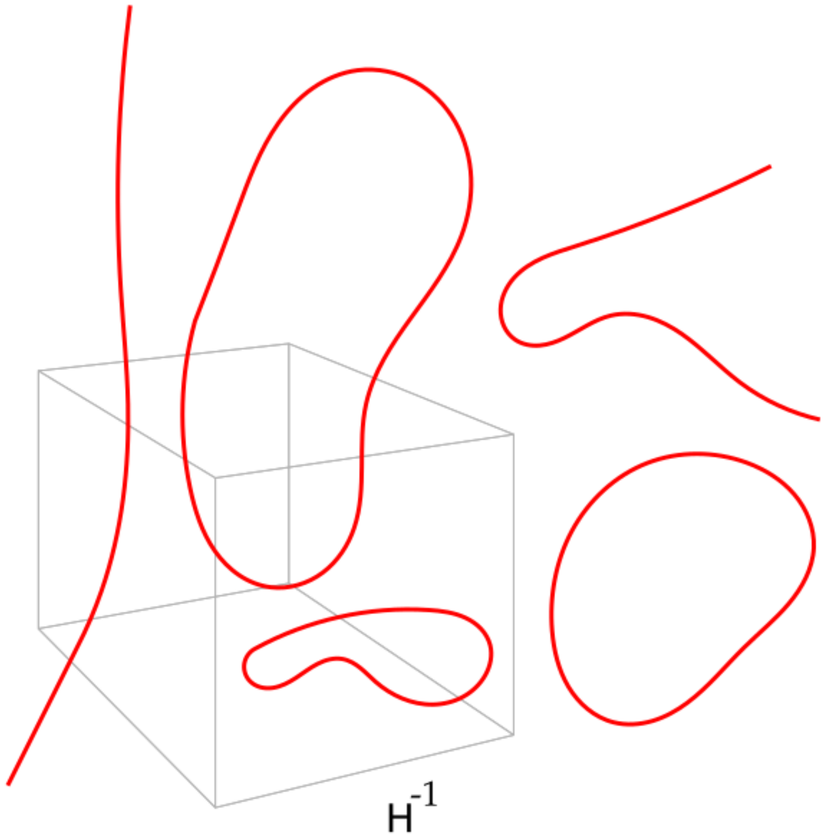
$$\mu \simeq \pi f_a^2 \log\left(\frac{d}{f_a^{-1}}\right)$$

Early universe:

$$\mu \simeq \pi f_a^2 \log\left(\frac{f_a}{H}\right)$$

Grows with time

Scaling solution



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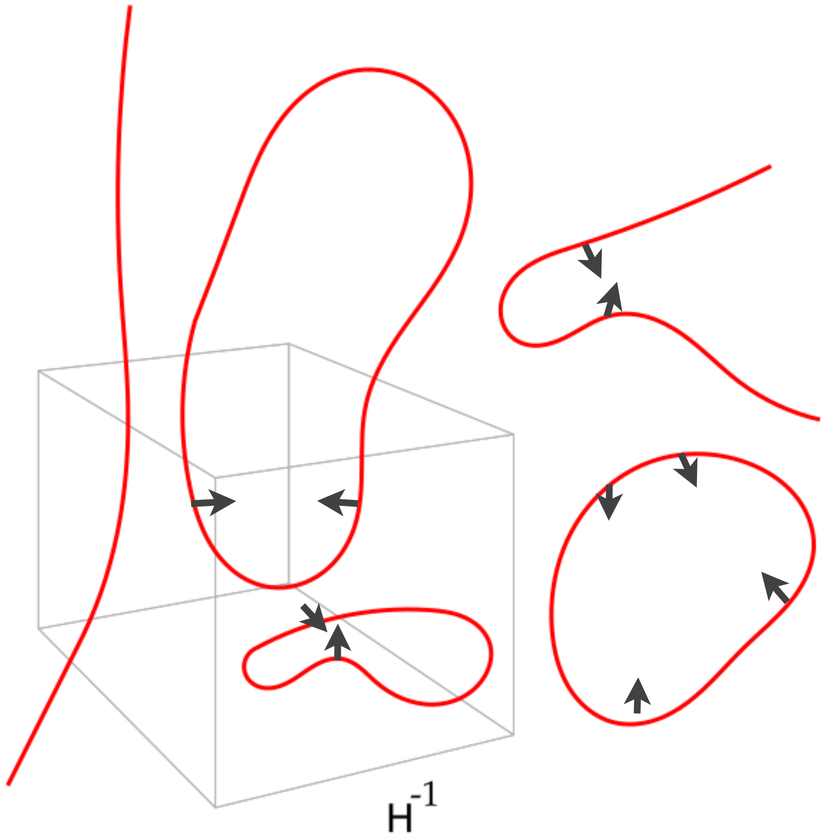

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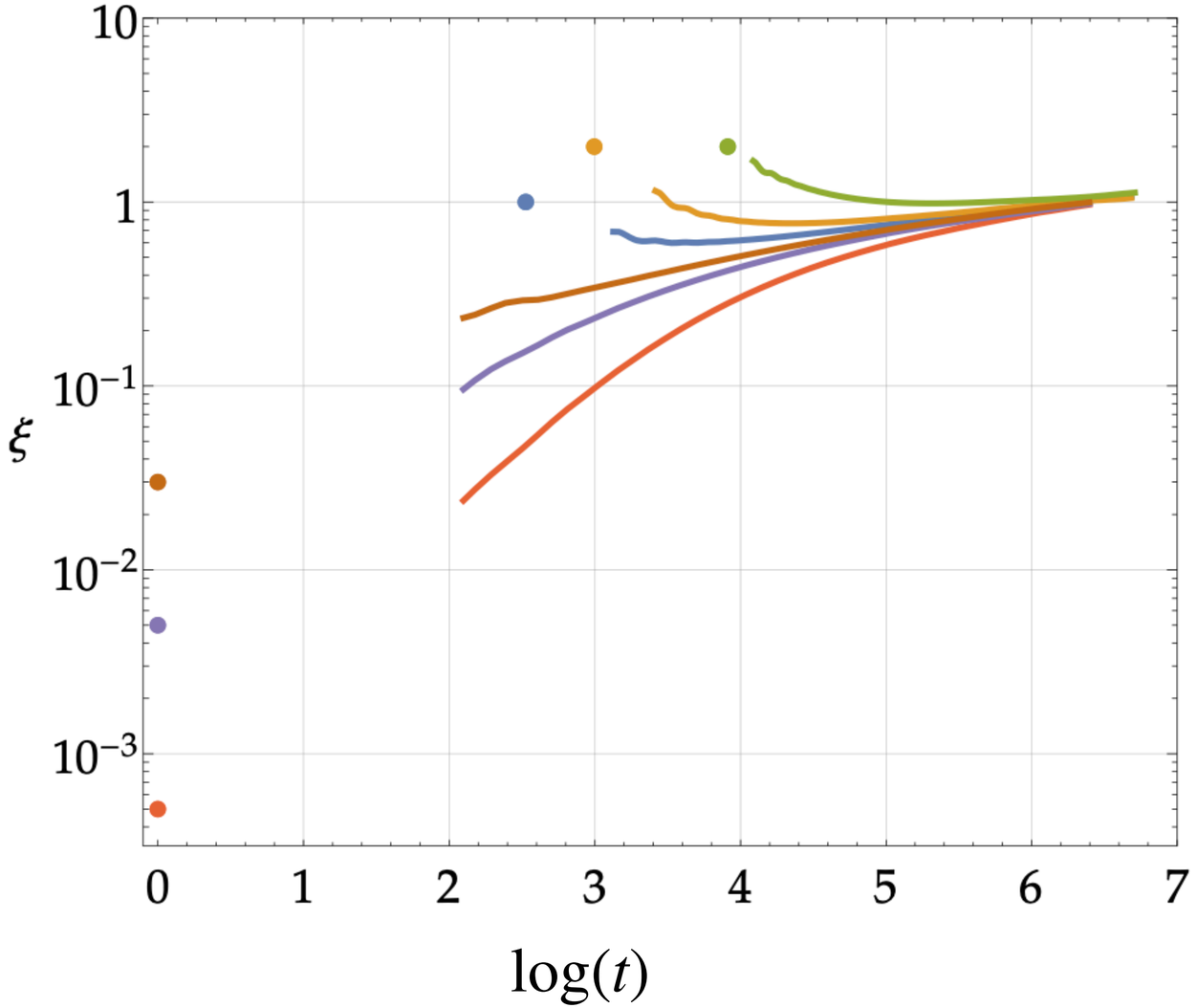

Small violations of scaling