# A New Twist on Topology:
# The Rise of "Moiré Materials"

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Oxford Physics

Saturday Morning of Theoretical Physics, February 8, 2025

# The story so far…

Topology has profoundly changed our view of phases of matter

Topology has led us to ideas such as "anyons", which might even be *useful* for quantum computing*

* not investment advice

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I'll explain recent progress in terms of 3 key ingredients

**topology** **correlations** **tunability**

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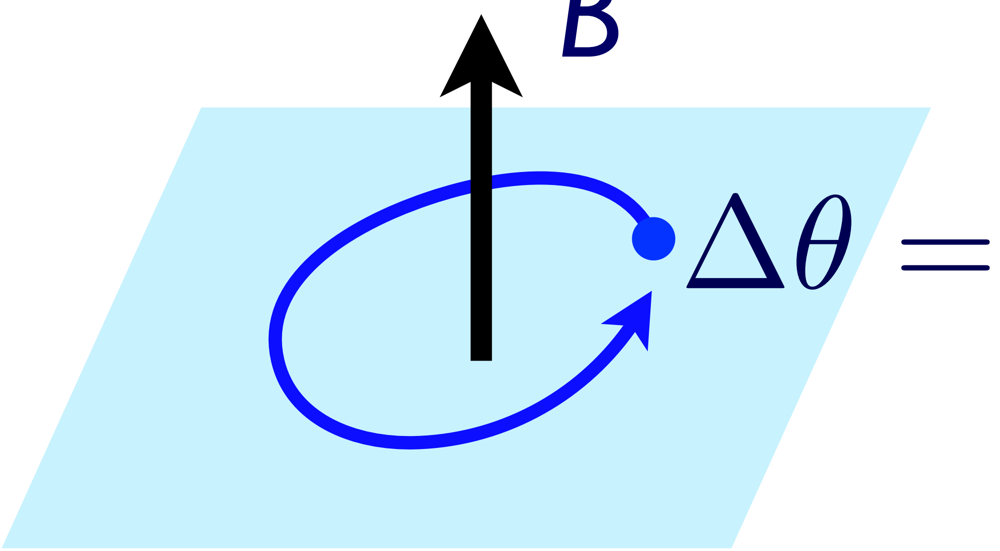
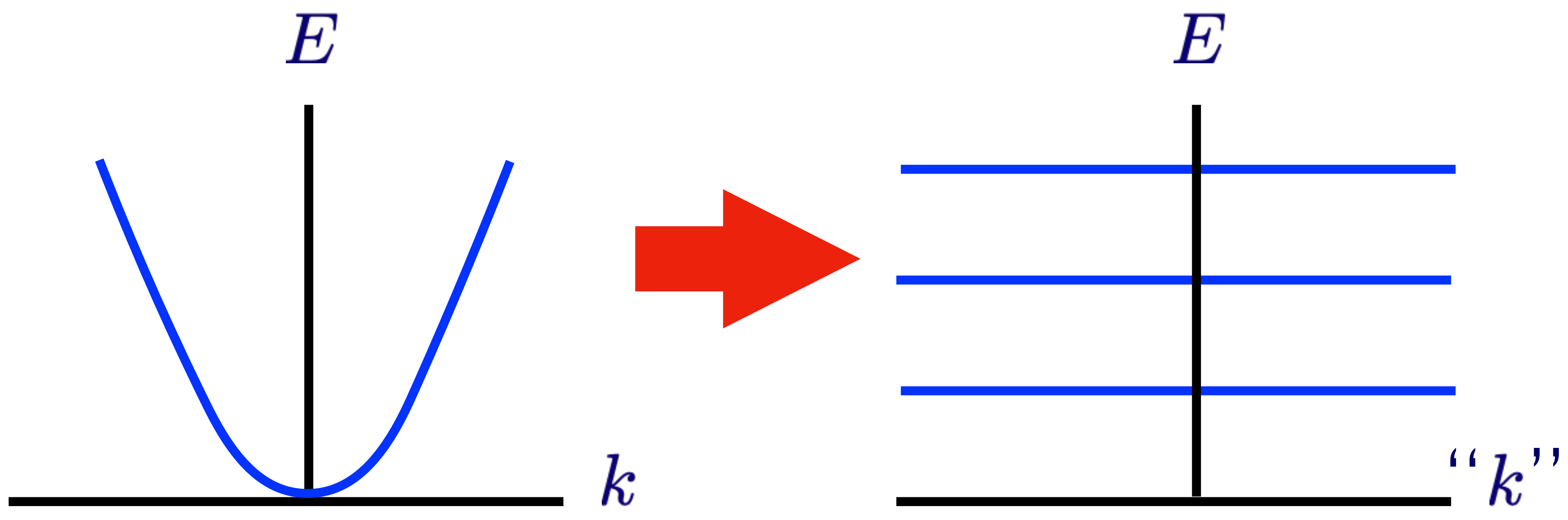
- How does the "original" quantum Hall effect embody these 3 ingredients?
- Why are they challenging to achieve in other systems?
- How did physicists manage to do this?

# 2D Electrons in a Magnetic Field

- Circular orbit + Aharonov-Bohm phase — kinetic energy frozen into flat "Landau levels"




$$\Delta\theta = \frac{e}{\hbar c} \times B \times \text{ area}$$

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also defines *magnetic length* $\quad \ell_B = \sqrt{\frac{\hbar c}{eB}}$

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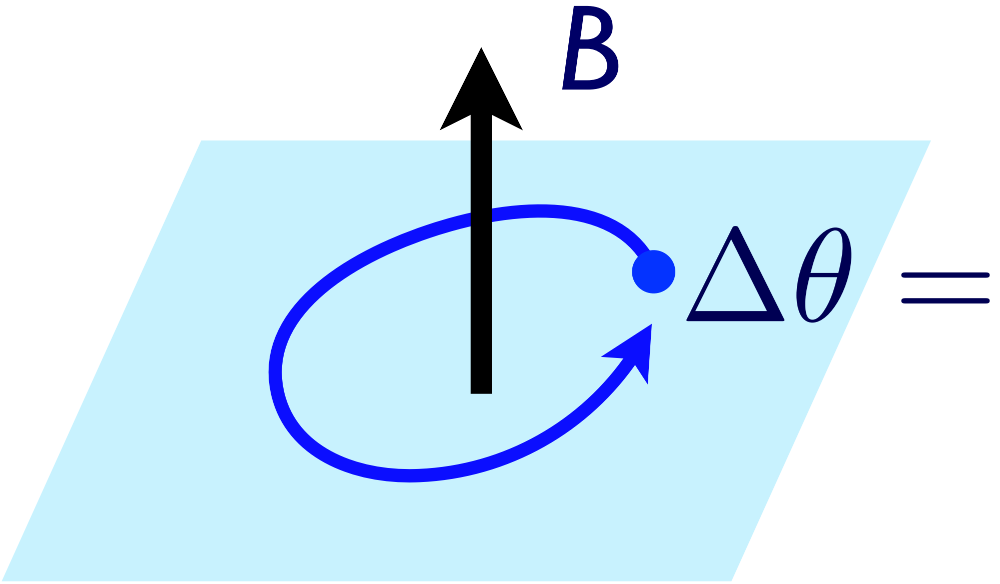


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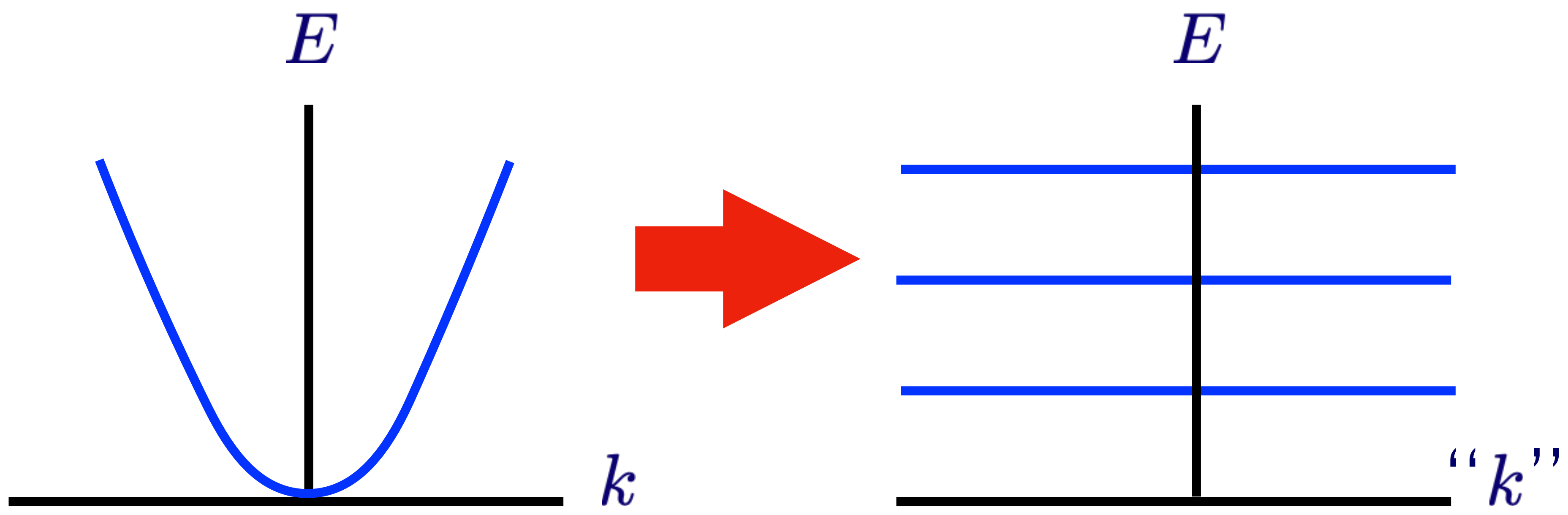


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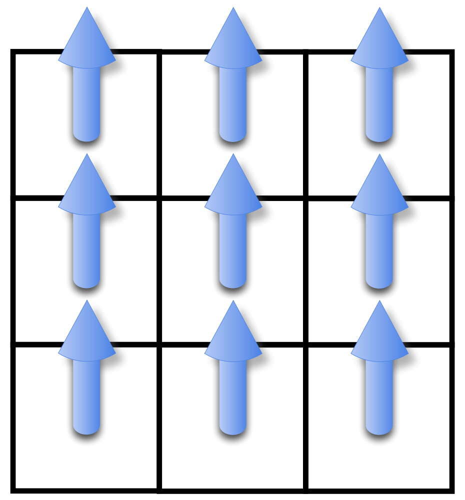
Sample with magnetic field $B$ and area $A$ has $N_\Phi = BA/\Phi_0$ "unit cells"

Each Landau level: 1 electronic state per "unit cell" $\Longrightarrow$ extensive degeneracy



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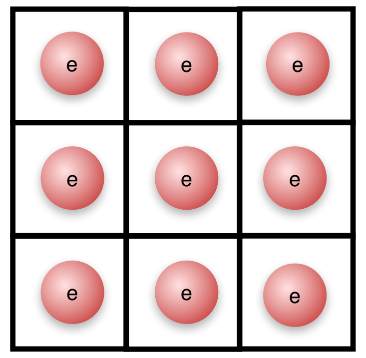

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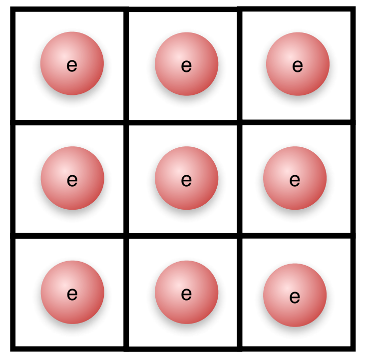
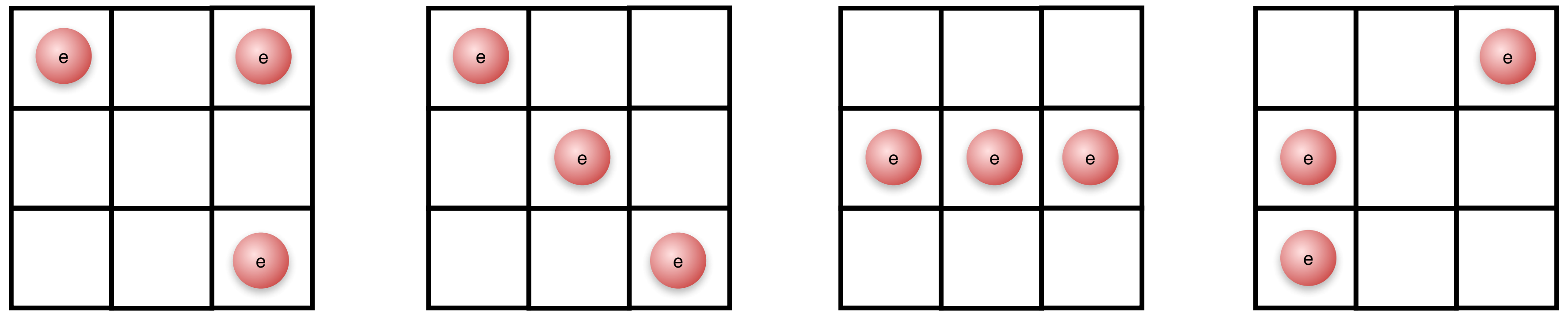


**fractional** $\nu$: interactions “pick” ground state (degenerate perturbation theory!)



+ 80 more…

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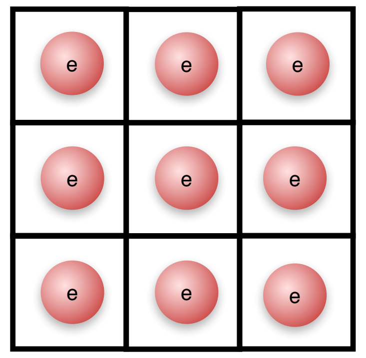

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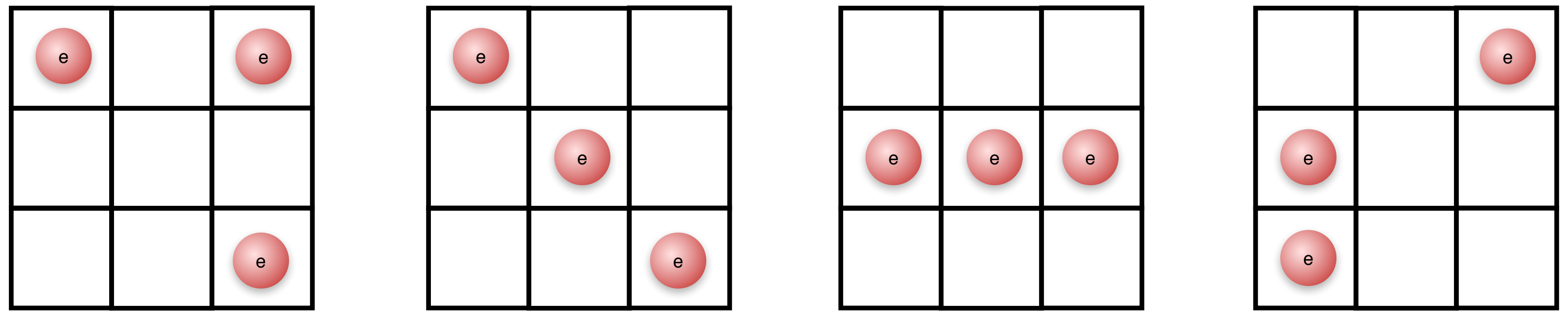

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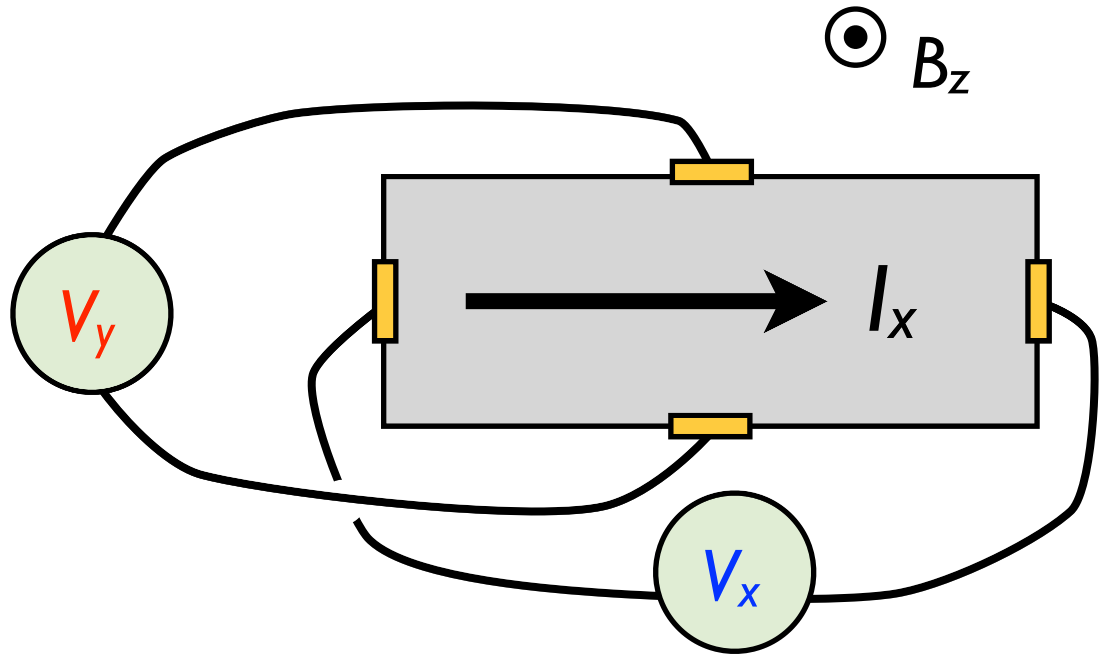
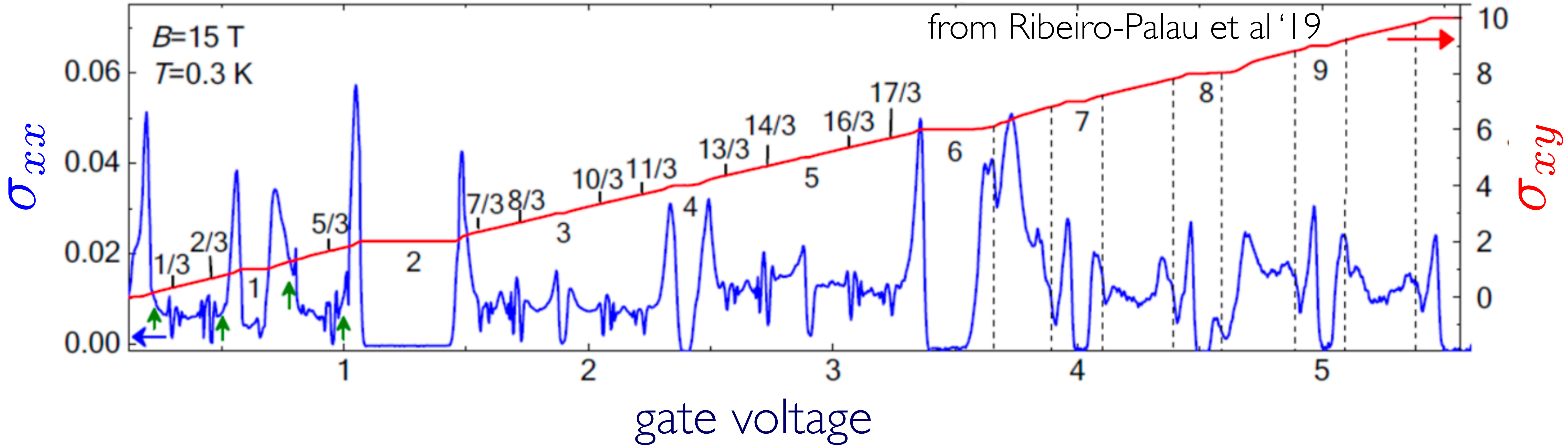
- **Quantum Hall Effect:** insulators w/ quantized response for integer or fractional $\nu$

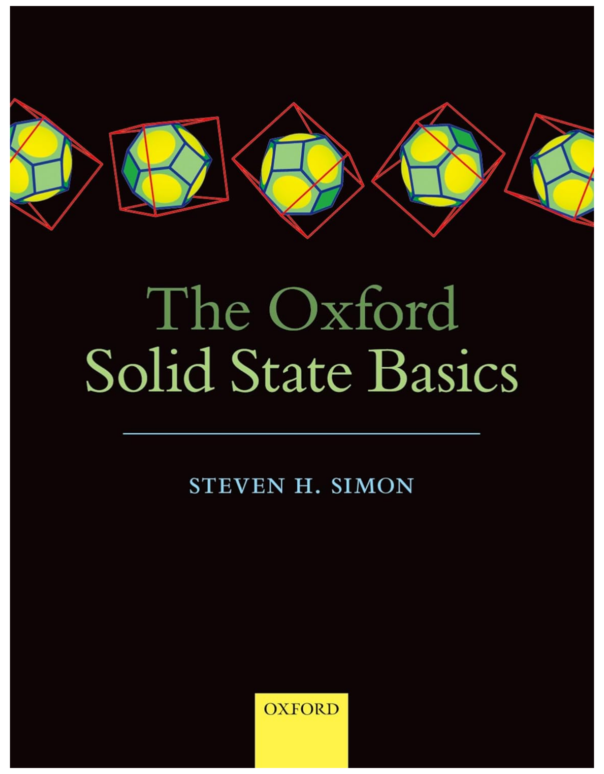



(cf Shivaji Sondhi’s Talk)



# Ingredient #1: Topology

# Back to B6!




Electrons in solids: Schrödinger equation in periodic potential

$$\left(\frac{\mathbf{p}^2}{2M} + V(\mathbf{r})\right)\psi(r) = E\psi(\mathbf{r}) \qquad V(\mathbf{r}+\mathbf{R}) = V(\mathbf{r}) \qquad \mathbf{R} \in \text{lattice}$$

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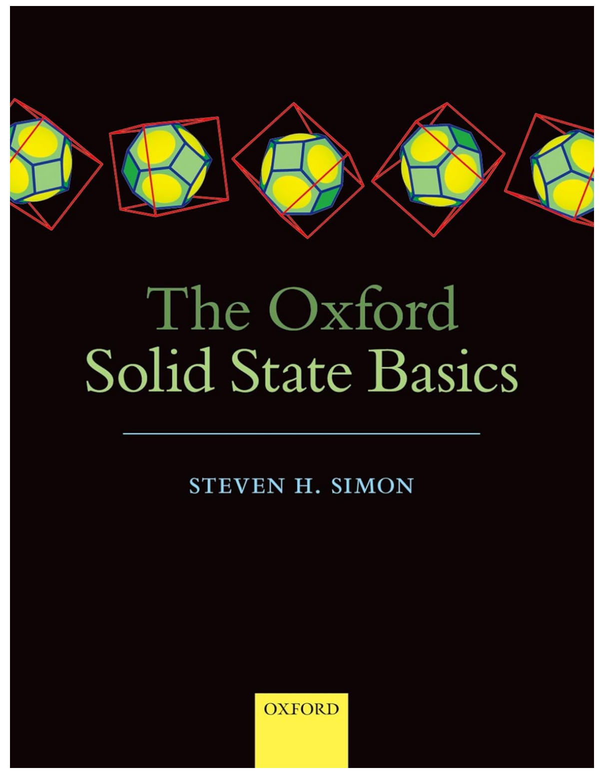


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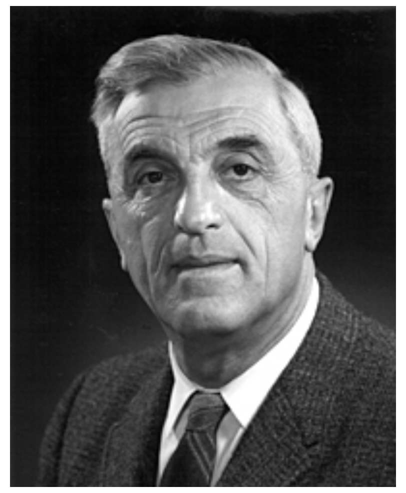

Wikipedia

**Bloch**: eigenstates = (plane wave) × (periodic function)

$$H\psi_{n\mathbf{k}}(\mathbf{r}) = E_{n\mathbf{k}}\psi_{n\mathbf{k}}(\mathbf{r})$$

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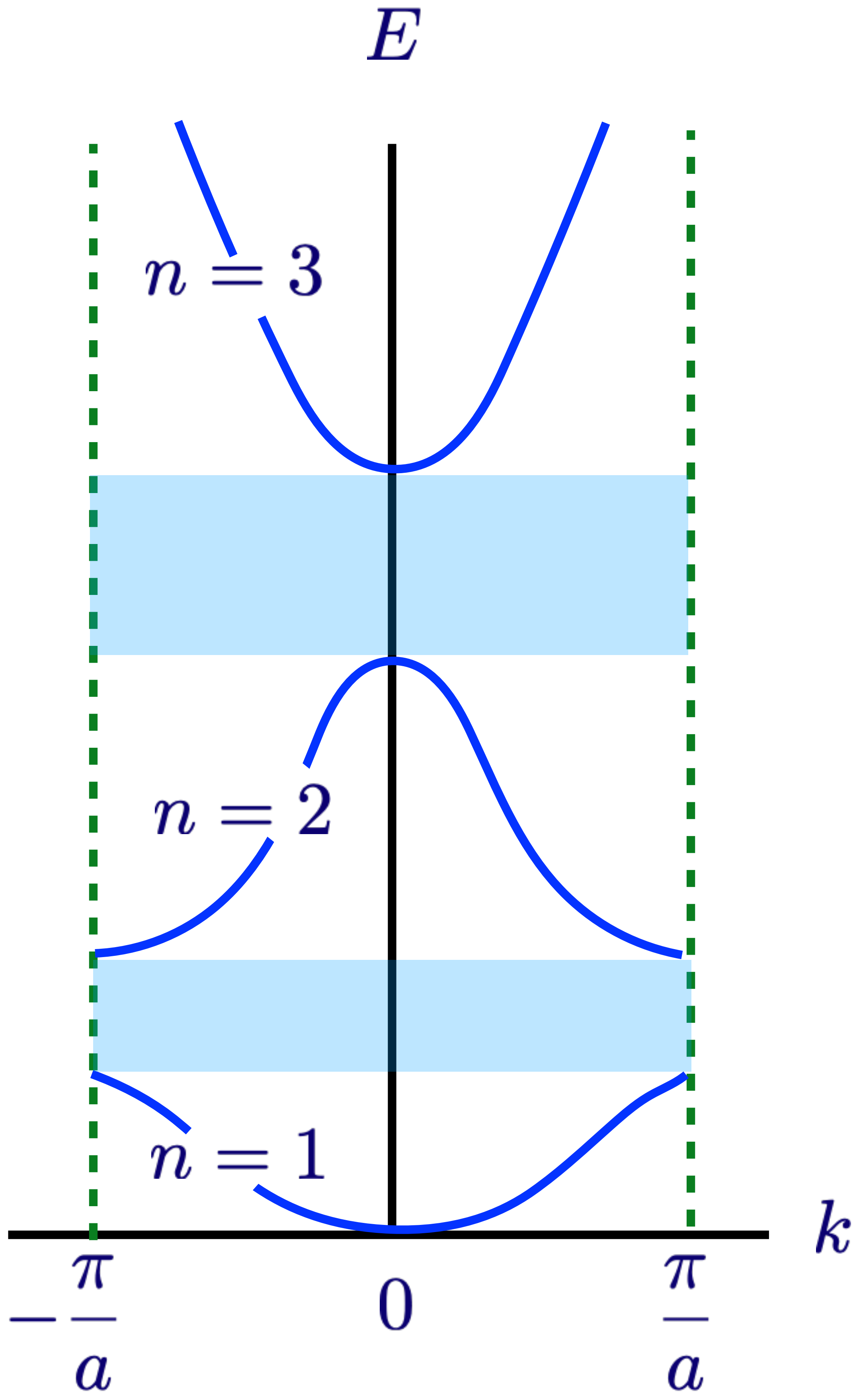

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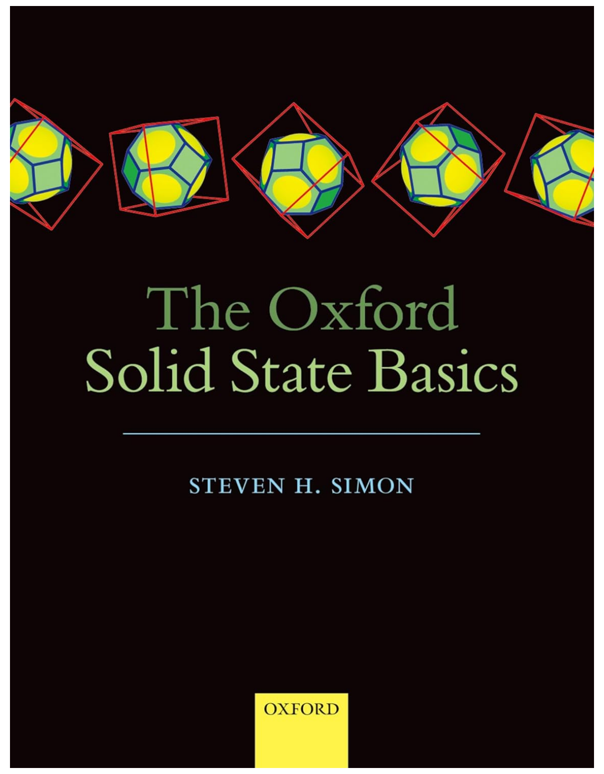


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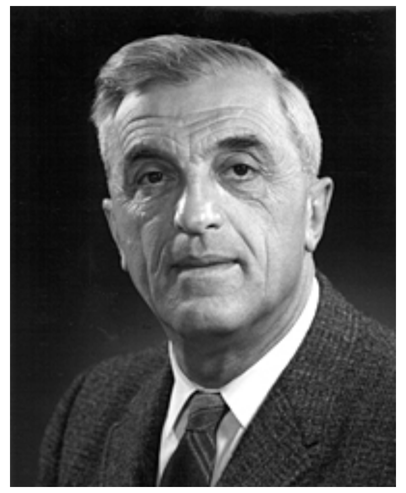

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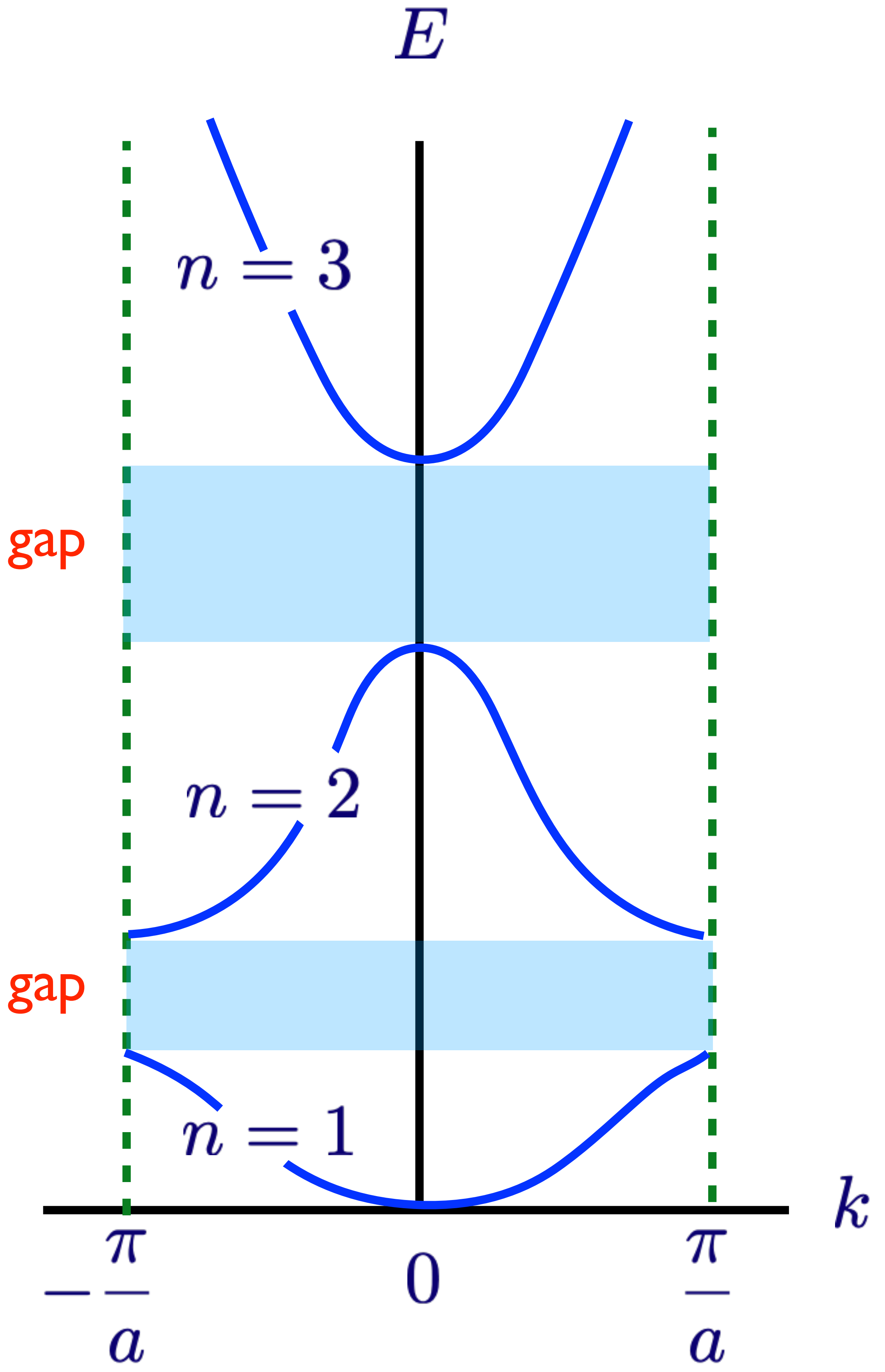
  - 1 band per orbital in the unit cell



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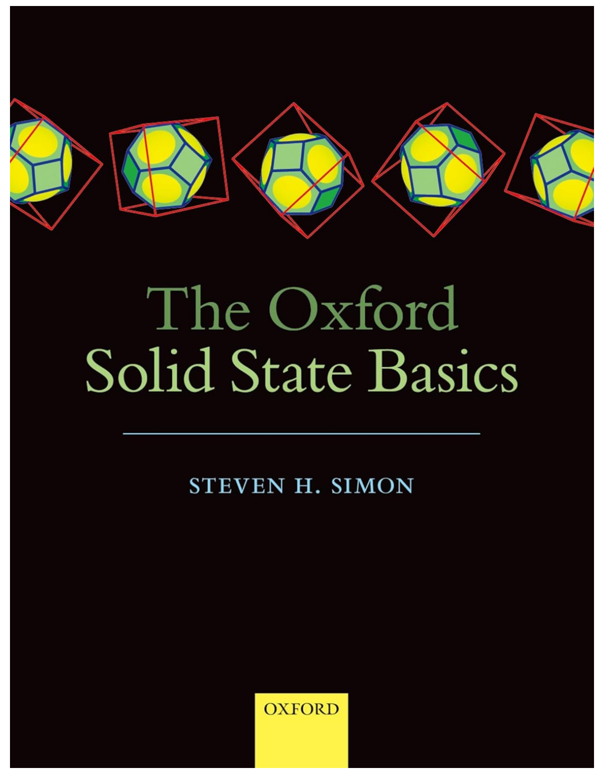


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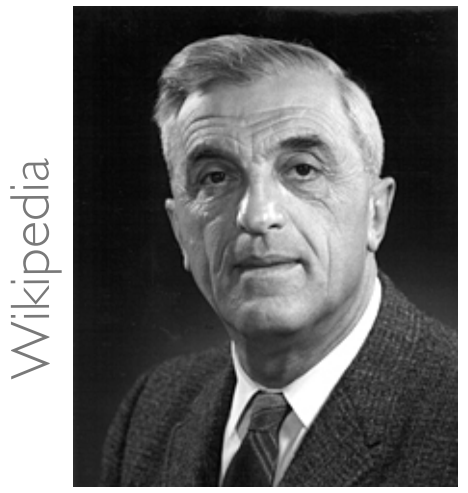


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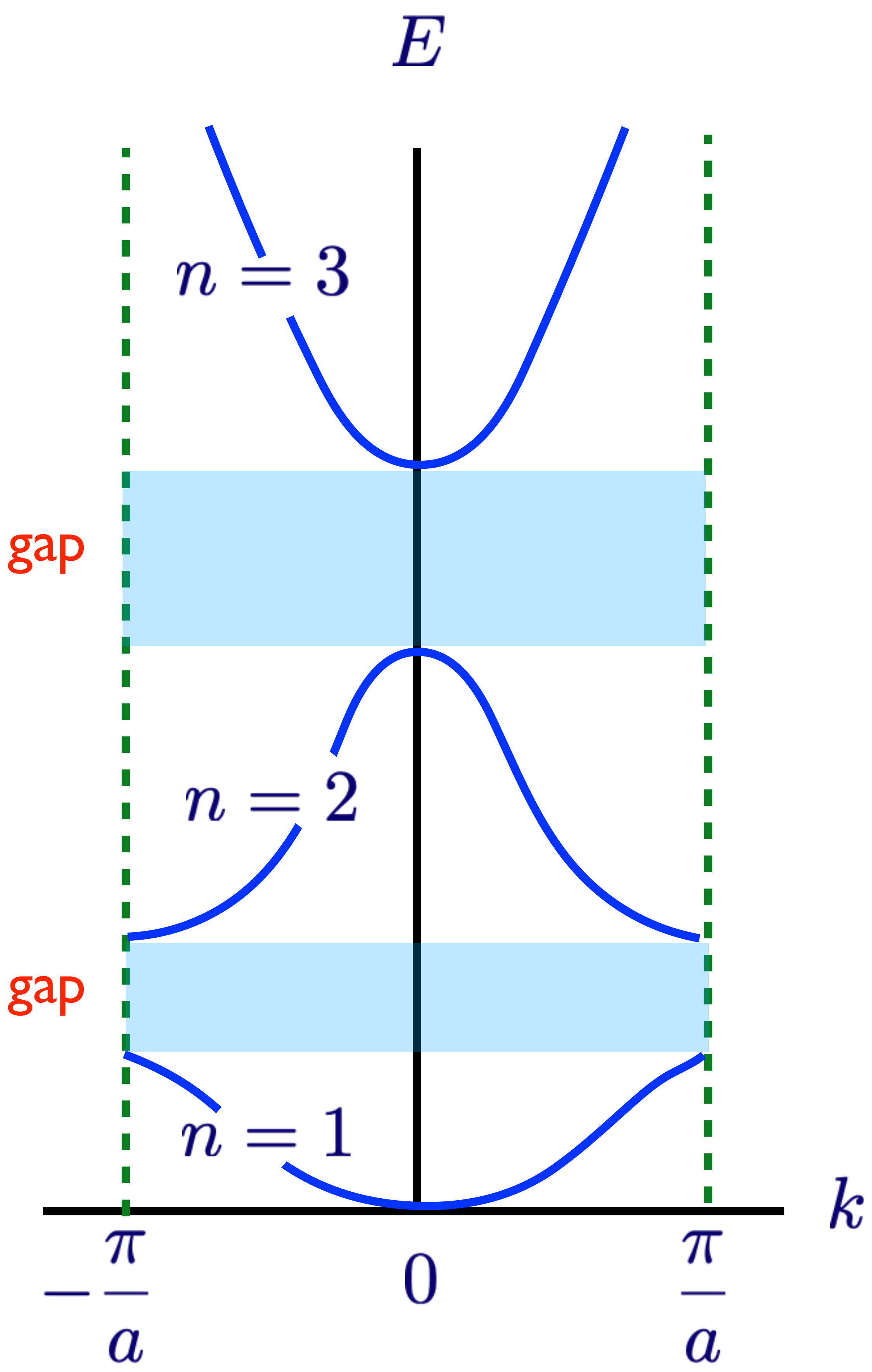



- energy levels : **bands** w/ discrete label $n$ + "gaps"
  - 1 band per orbital in the unit cell
- **crystal momentum** $k$ is <u>periodic</u>
  - e.g. in 1D, $k \equiv k + 2\pi/a$

# Topology in Bloch Bands

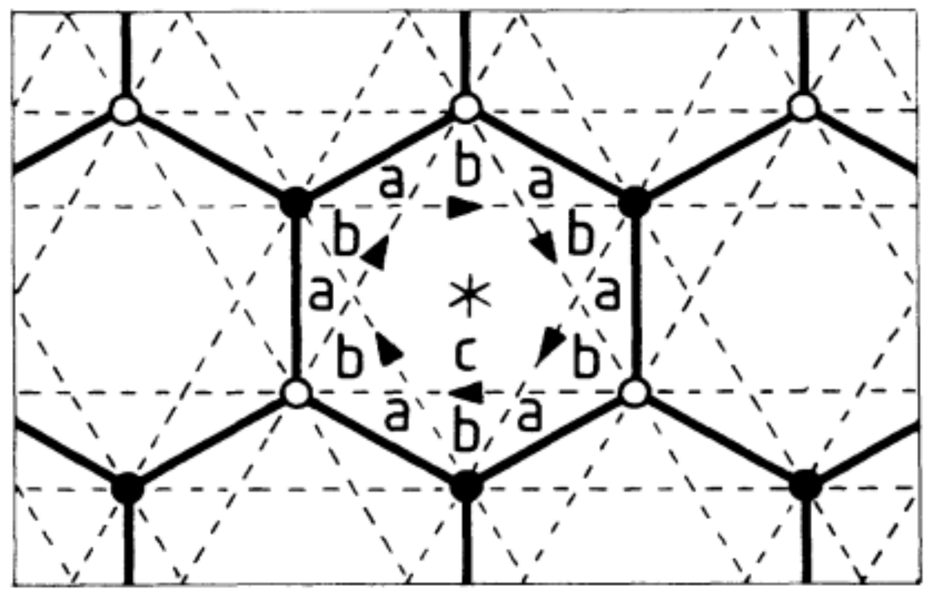
As we learned in Shivaji's talk, Bloch bands can also have topology ("Chern insulators")

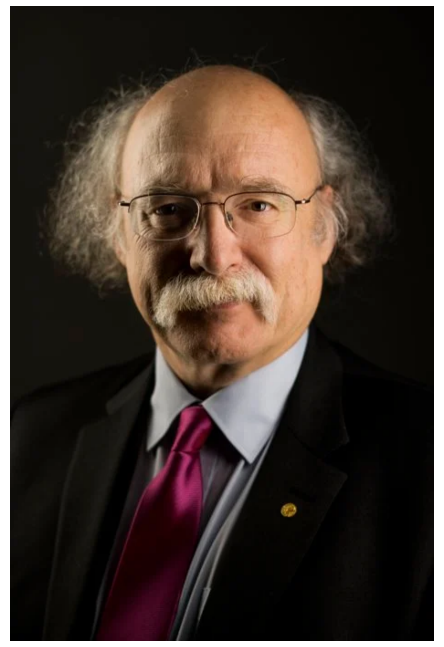
**Chern bands** — $B=0$ lattice versions of Landau levels

topological index: integer "Chern number" C

filled Chern band: quantized Hall response

$$\sigma_{xy} = C \times \frac{e^2}{h}$$



Nobel Foundation

[Haldane PRL '88; Thouless et al PRL '82]

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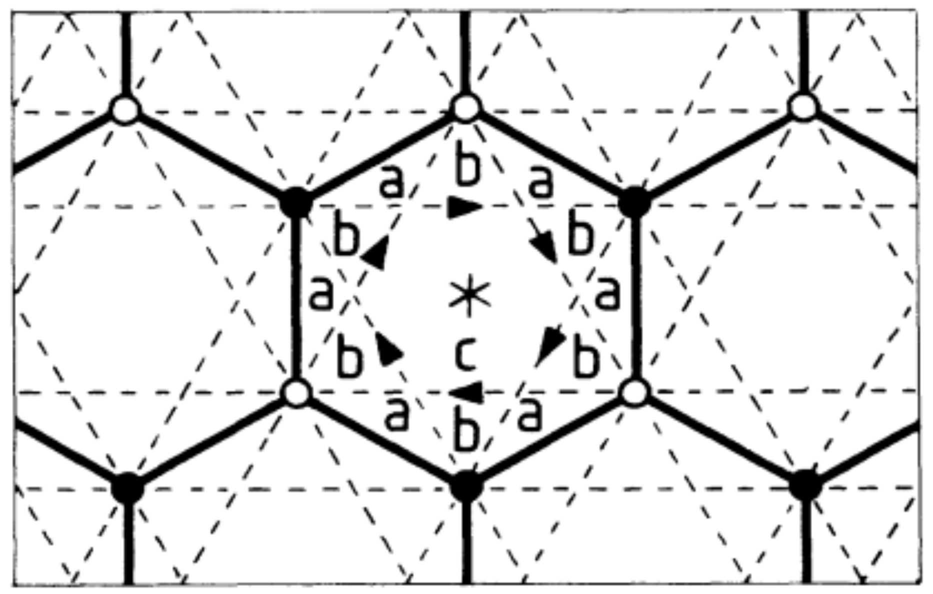

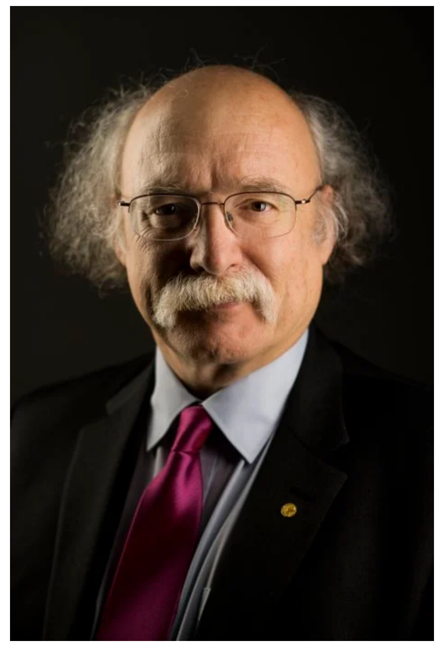

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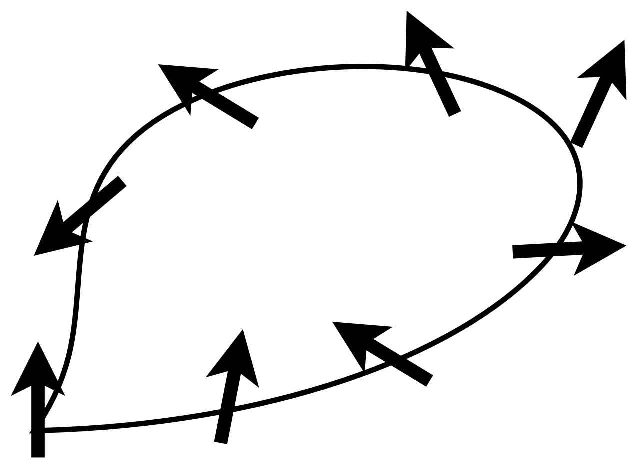
## Where does the topology come from?

**Berry's Phase** ~ "winding" of electron wavefunction as it moves

- cf Aharonov-Bohm phase in magnetic field
- often arises in systems w/ strong spin-orbit coupling



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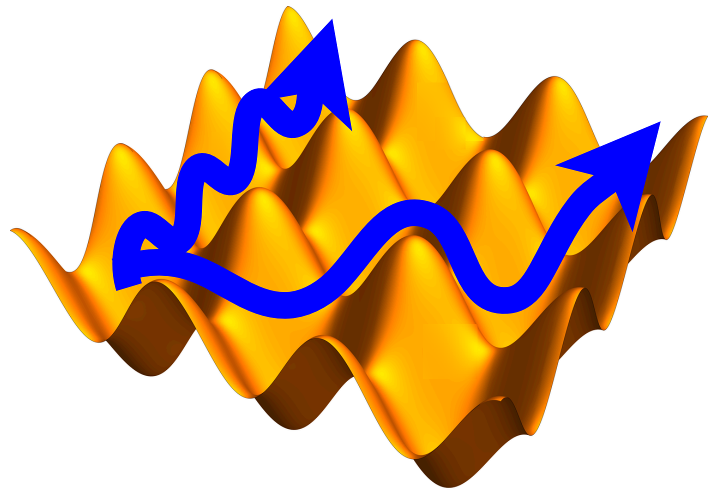
# Ingredient #2: Correlations

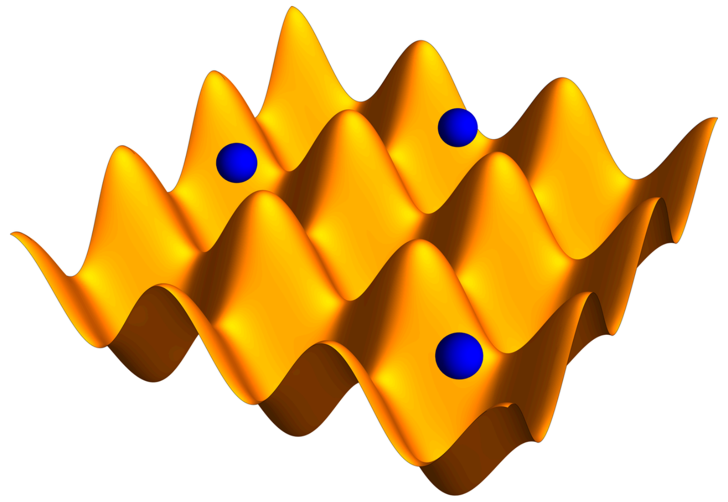
# What do we mean by "correlations"?

competition between kinetic energy & interactions decides the fate of matter

more itinerant
(wave-like)



more localized
(particle-like)



interaction
strength

metals
superconductors

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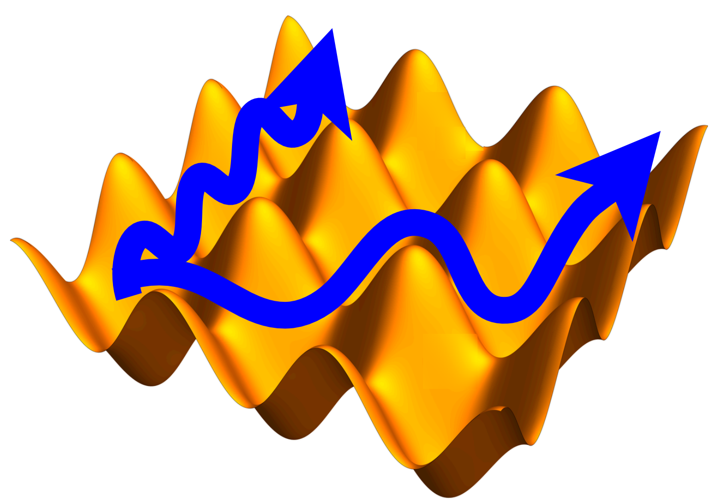

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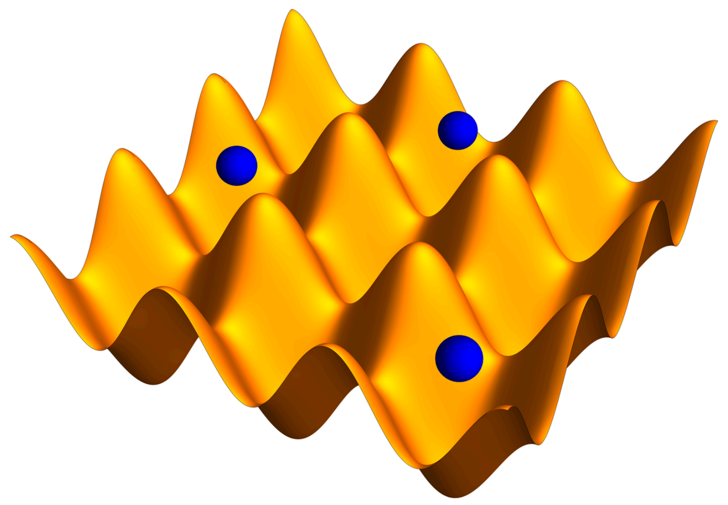

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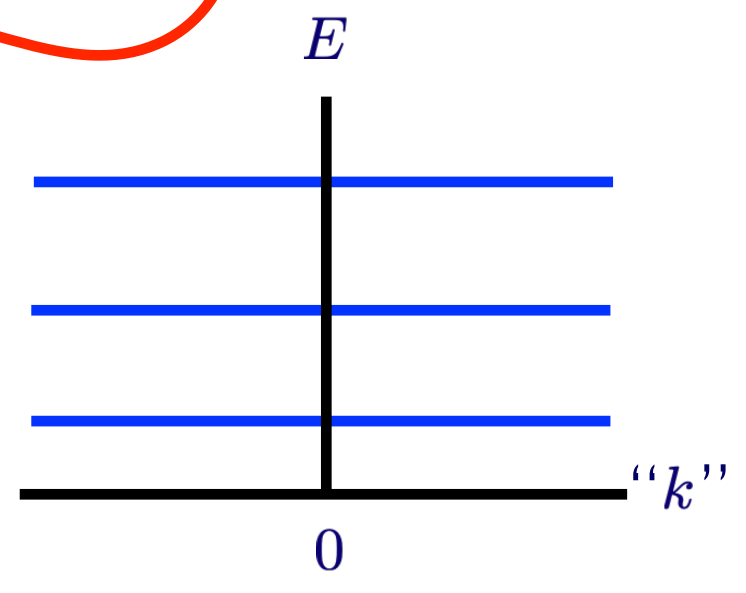
Landau levels: zero kinetic energy
— intrinsically “strong coupling”



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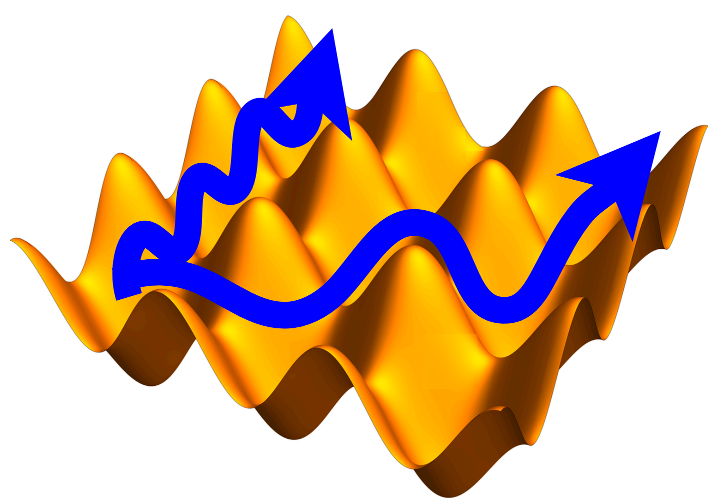
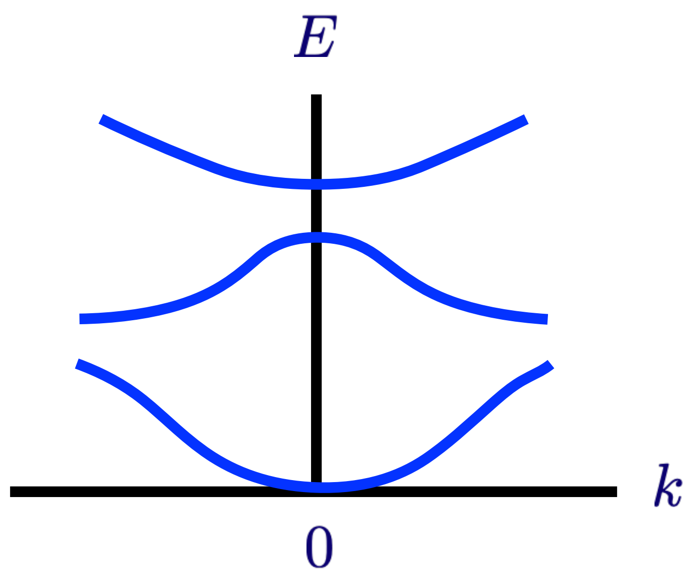


Bloch bands usually have non-zero kinetic energy: "intermediate coupling"

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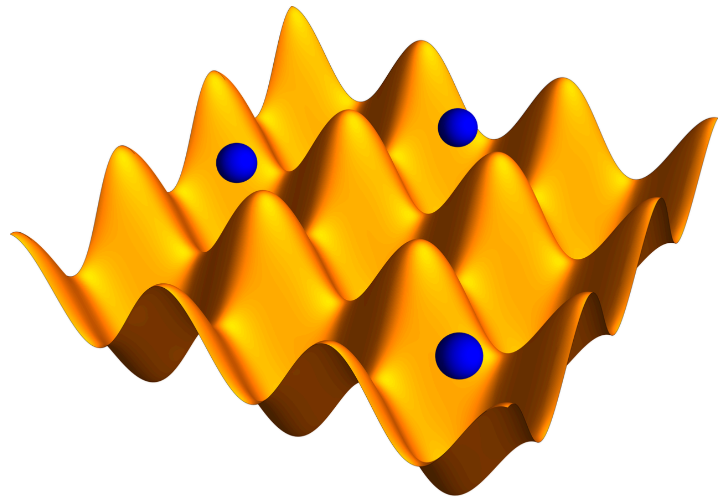

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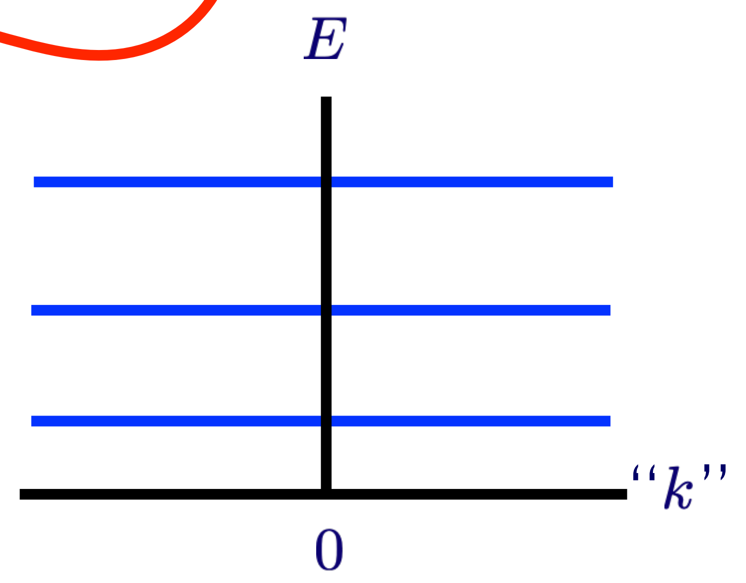
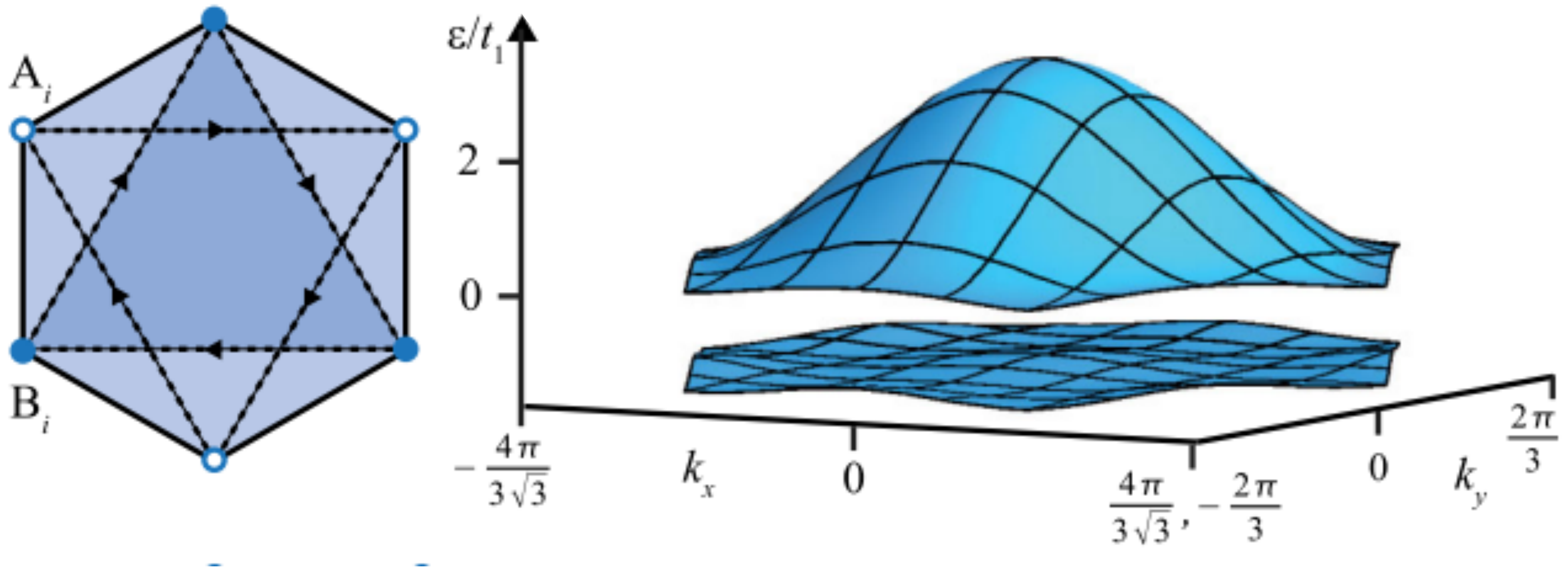


# "Fractional Chern Insulators"

"Fractional Chern insulators": lattice analog of fractional QHE
[Tang et al PRL'11; **Neupert et al PRL '11**; Sun et al PRL '11]




"flatten" dispersion of Chern bands to enhance correlations (easy in <u>theory</u>)

Under right conditions, interactions ≈ those in a Landau level [Qi, PRL '11; SP, Roy, Sondhi PRB '12]

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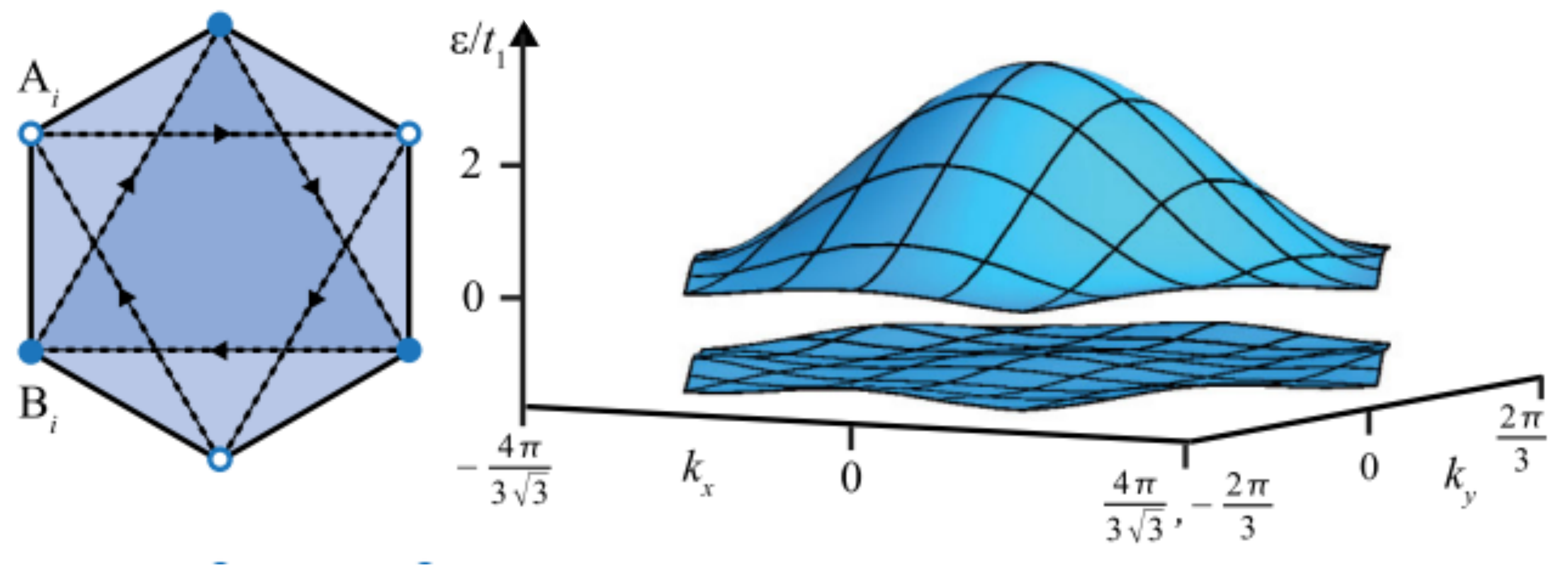


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Lots of subsequent theory… enough for a review article already in 2013

Rahul Roy
Oxford Postdoc
2009-11


C. R. Physique 14 (2013) 816–839

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Topological insulators/Isolants topologiques


Fractional quantum Hall physics in topological flat bands

*Physique de l'effet Hall quantique fractionnaire dans des bandes plates topologiques*

Siddharth A. Parameswaran[a,*], Rahul Roy[b], Shivaji L. Sondhi[c]

… but "flattening" bands is not easy in experiments!

# Ingredient #3: Tunability

# What do we need to tune?

Ideally… **lots** of things!

But if we had to choose **one**: it would be the electron density

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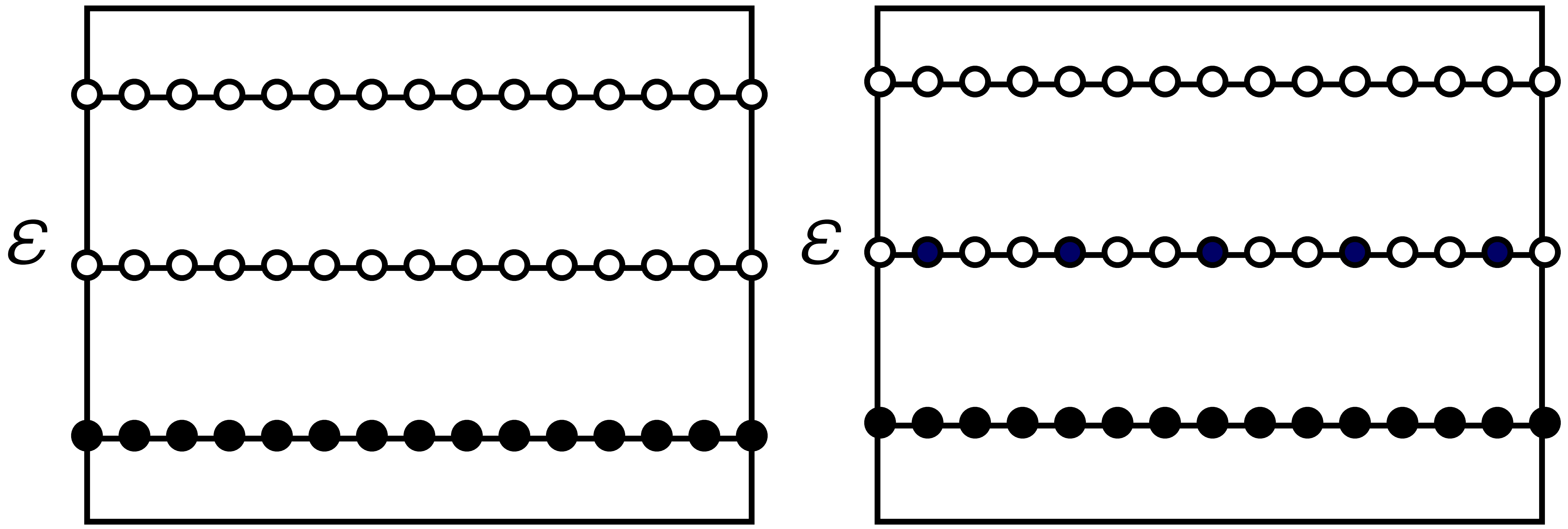
**Landau Levels**




IQHE

FQHE
(w/ interactions)

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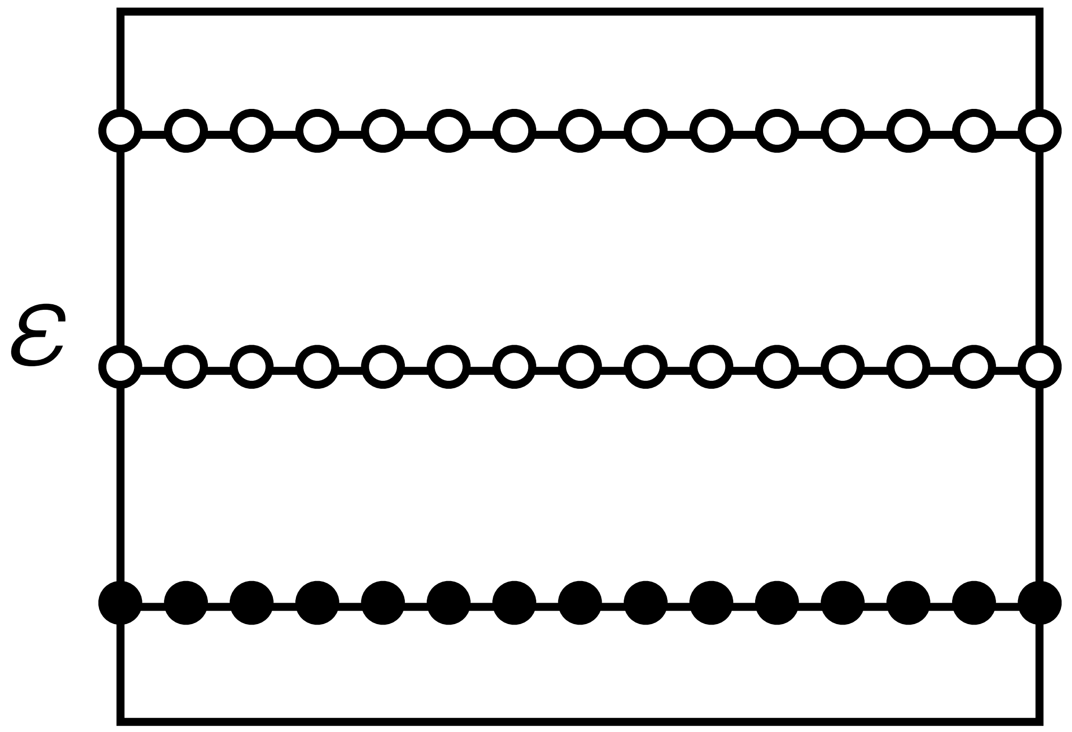


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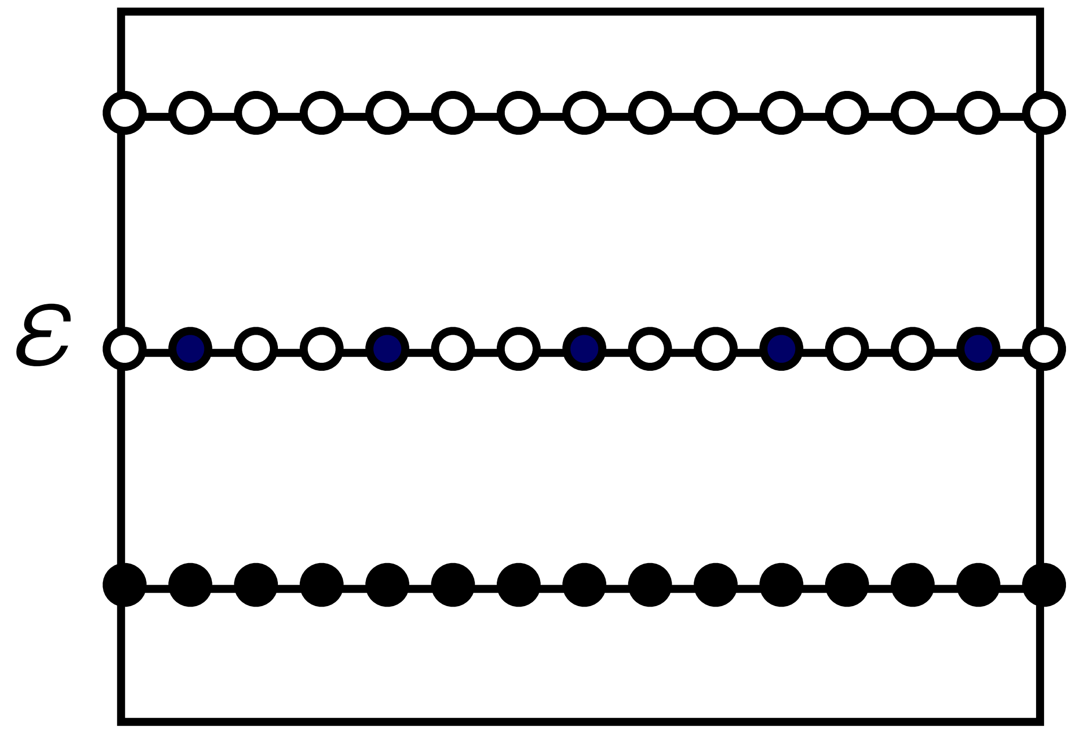


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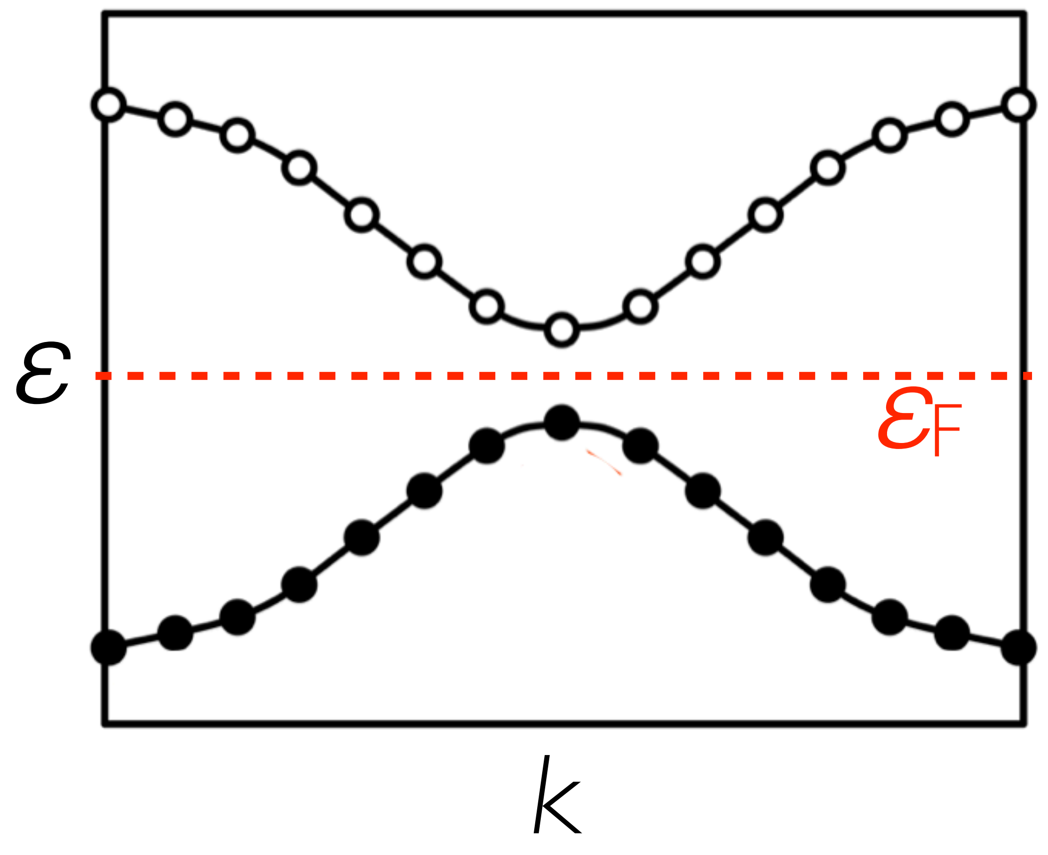
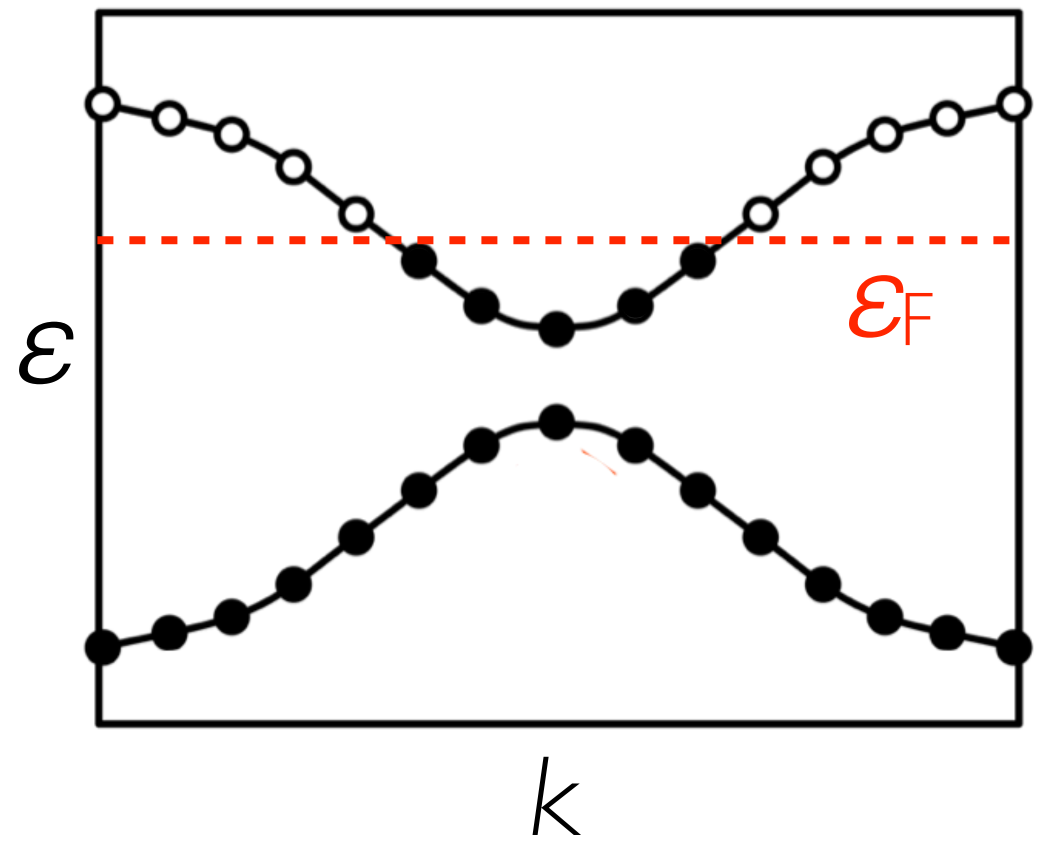
## Bloch Bands




band insulator




metal
"Mott" insulator w/
strong correlations

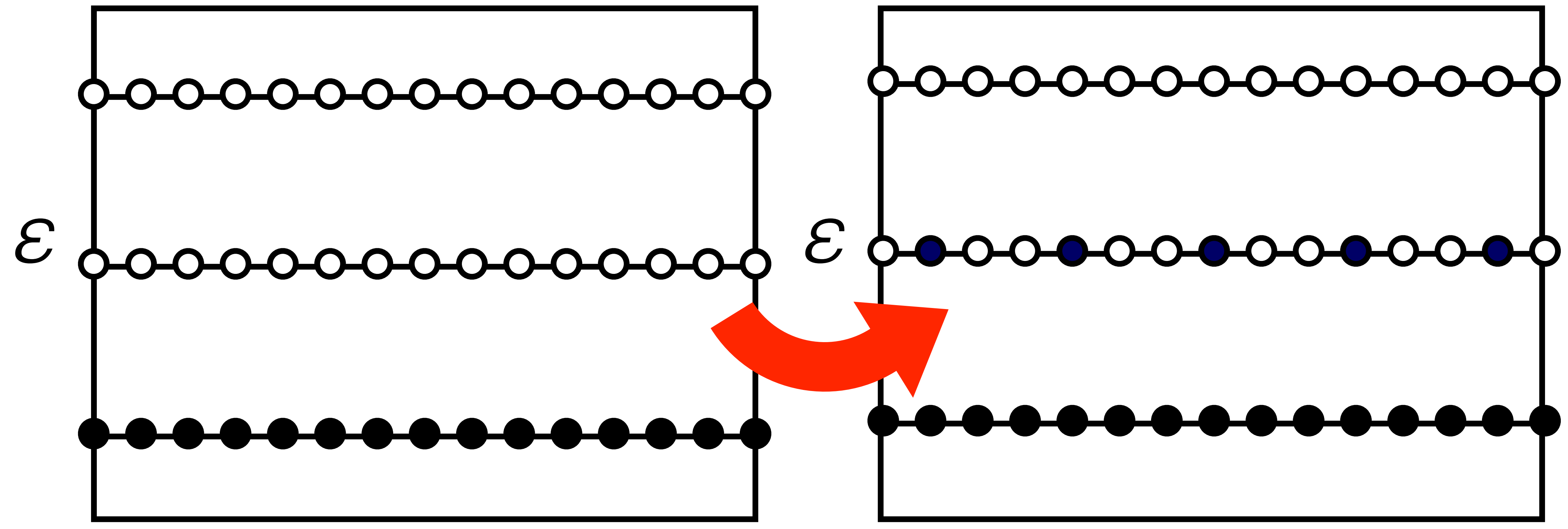
Chern/fractional Chern insulator if $C \neq 0$

# What do we need to tune?

By tuning density, **"integer" states** → **"fractional" states**

(no correlations needed) (correlations essential)

**Landau Levels**

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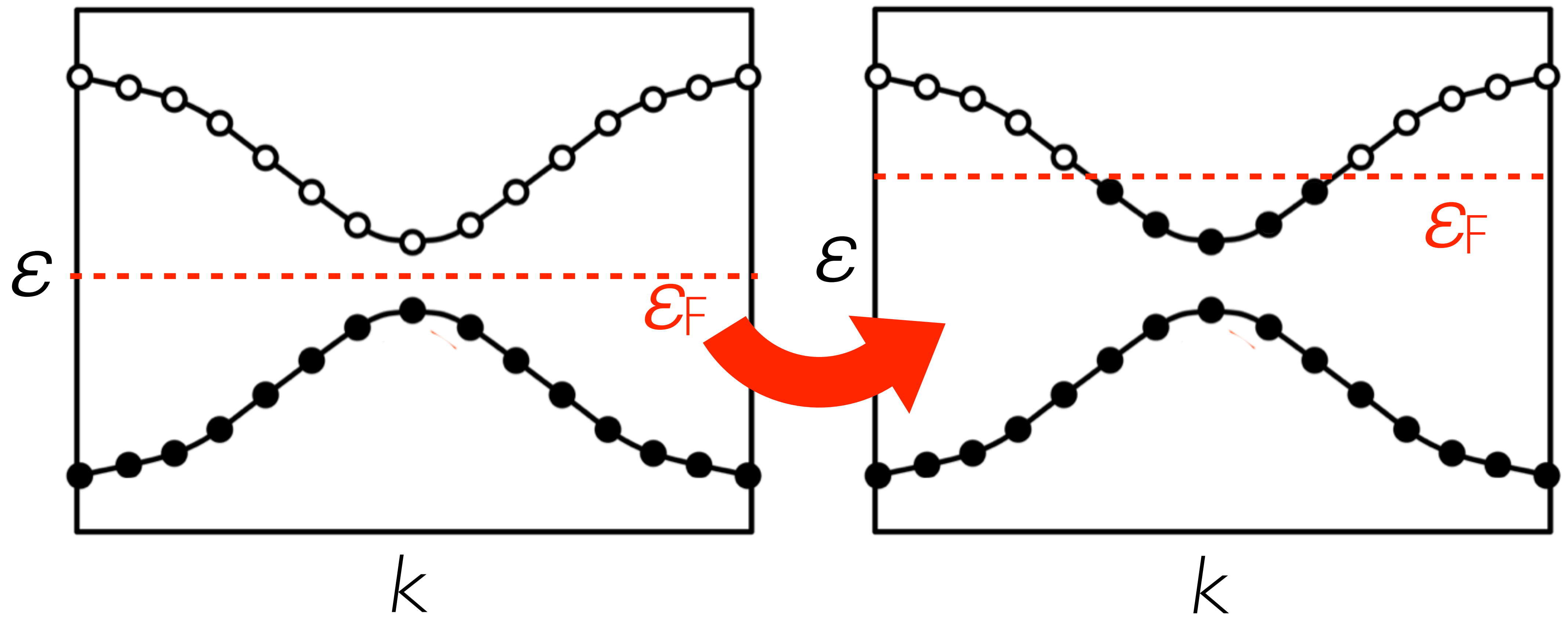


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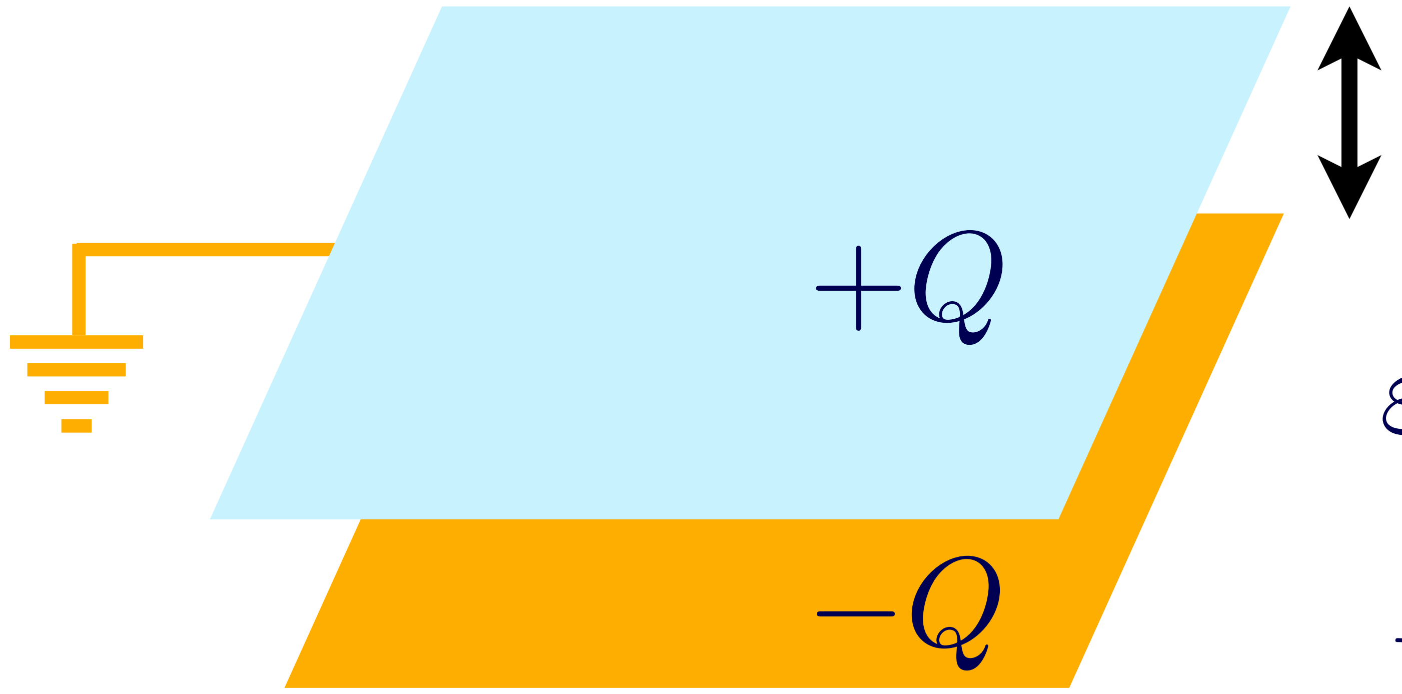


gate voltage $V$

$d \sim 10\,\mathrm{nm}$

$\varepsilon_r \sim 10$

$V \sim 1$ Volt

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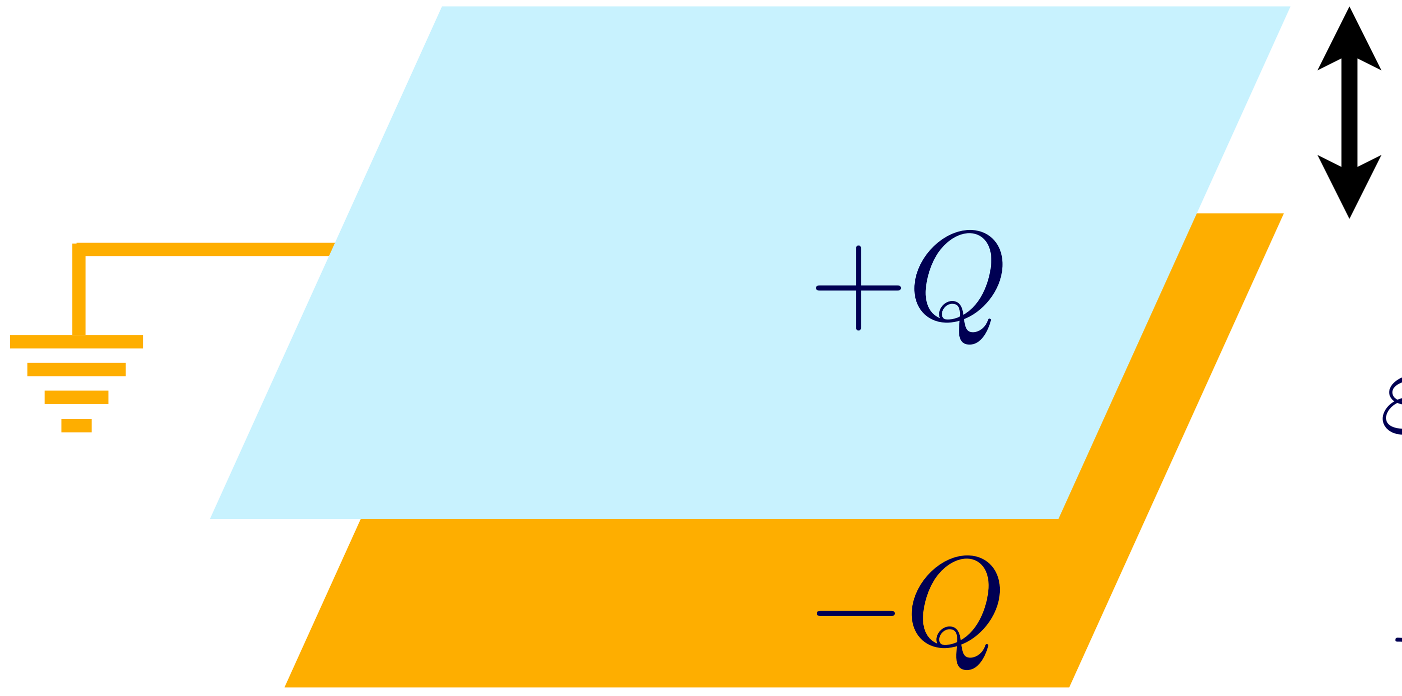


gate voltage $V$

$d \sim 10\,\mathrm{nm}$

$\varepsilon_r \sim 10$

$V \sim 1$ Volt

Charge density:

$$ne = \frac{Q}{A} = \frac{CV}{A} = \frac{\varepsilon V}{d}$$

$$n \sim \frac{1 \text{ electron}}{(10 \text{ nm})^2}$$

# How can we change the filling in experiments?

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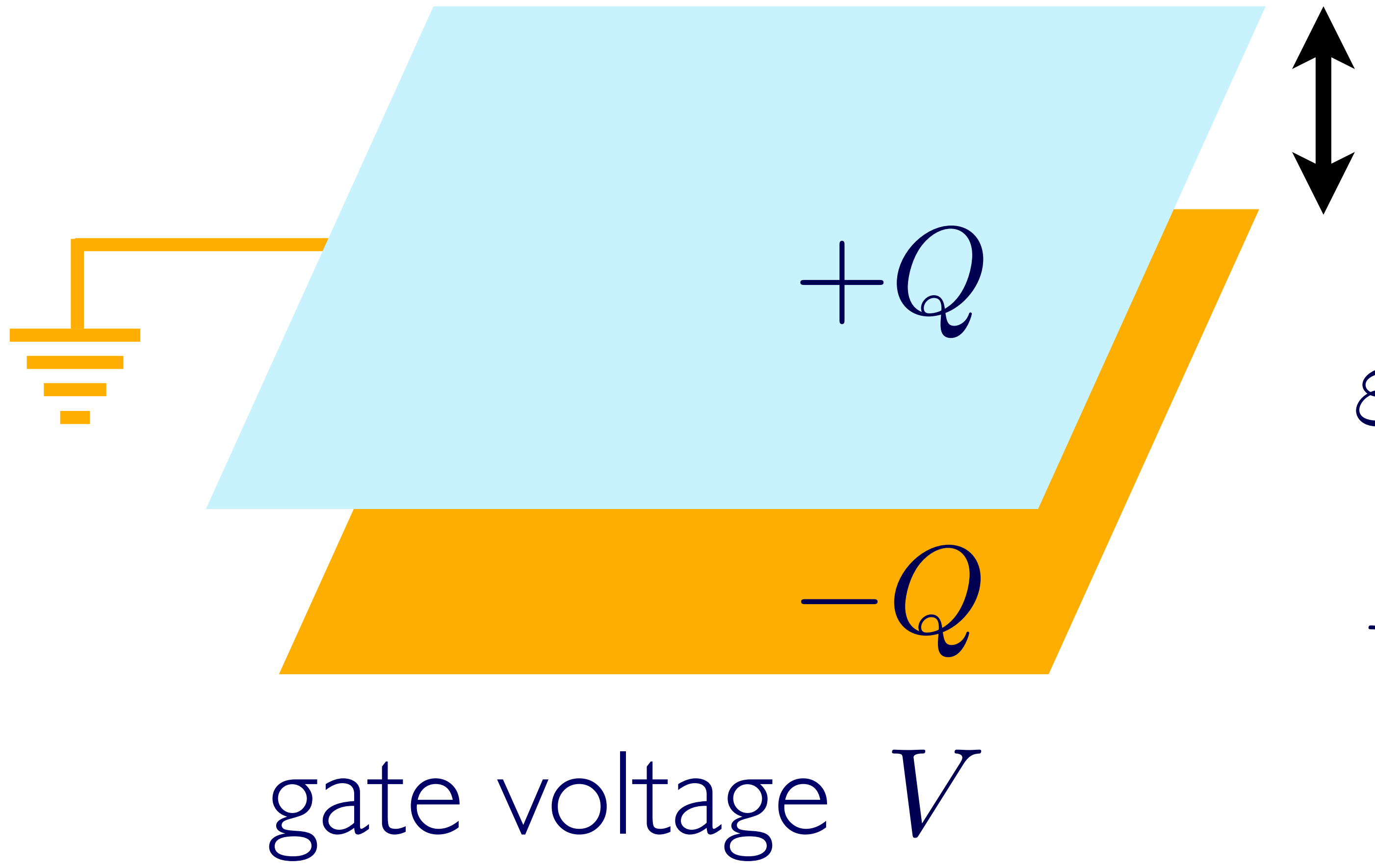


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How does this compare to the length scales we have at hand?

# Length scales in Landau Levels vs. Materials

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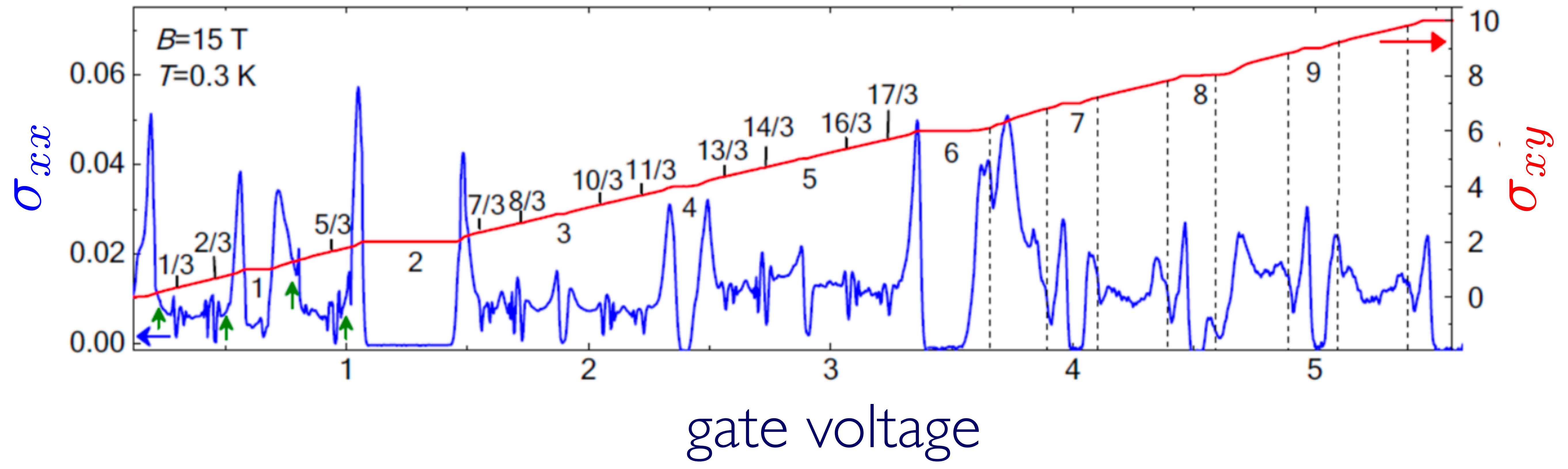

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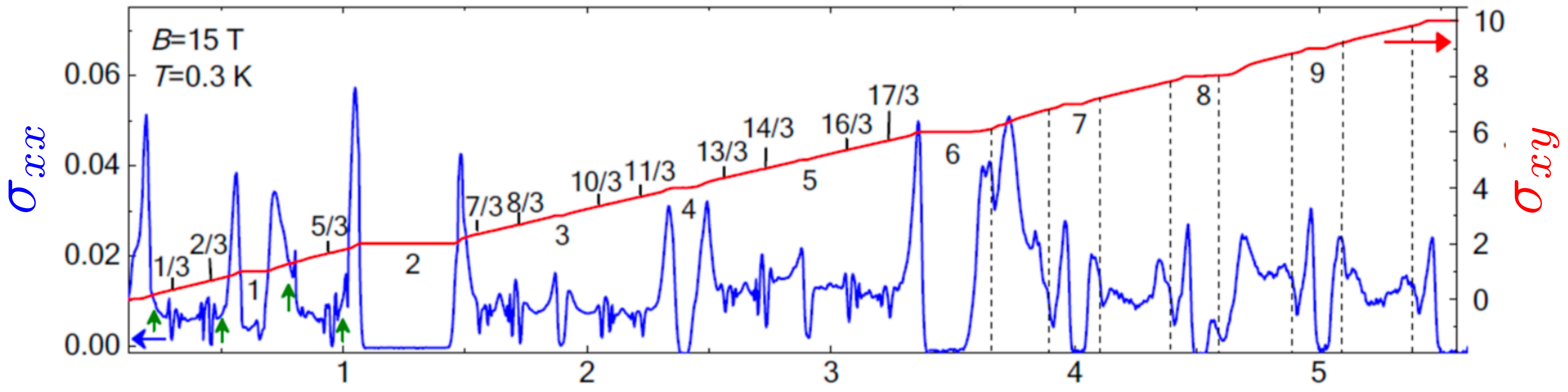


**In materials**: typical unit cell scale $a \sim 0.3\,\mathrm{nm}$, so $n \sim \frac{1 \text{ electron}}{(10 \text{ nm})^2} \sim \frac{10^{-3} \text{ electron}}{(\text{unit cell})}$

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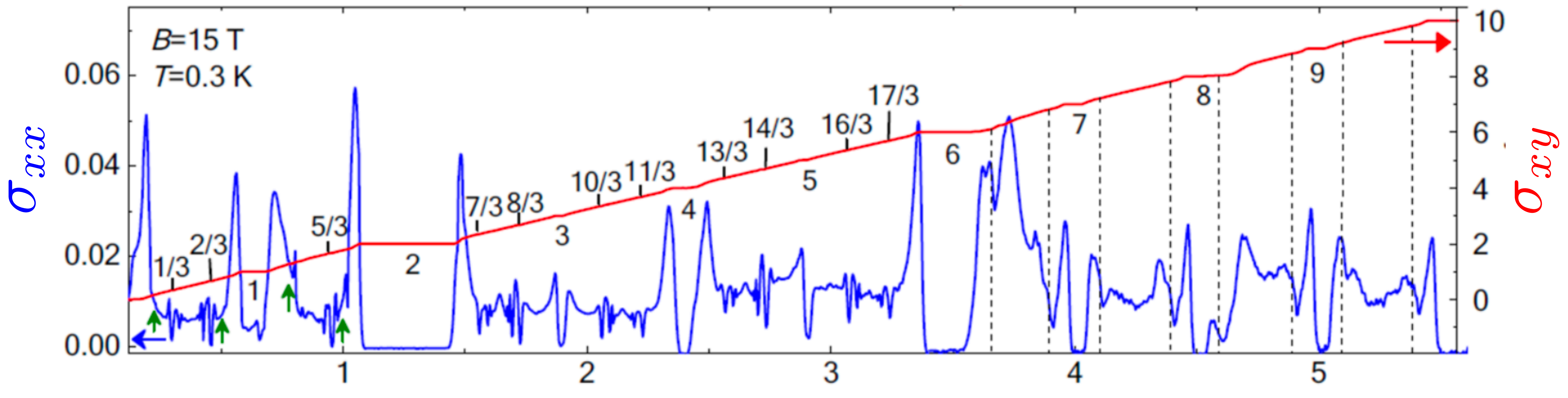


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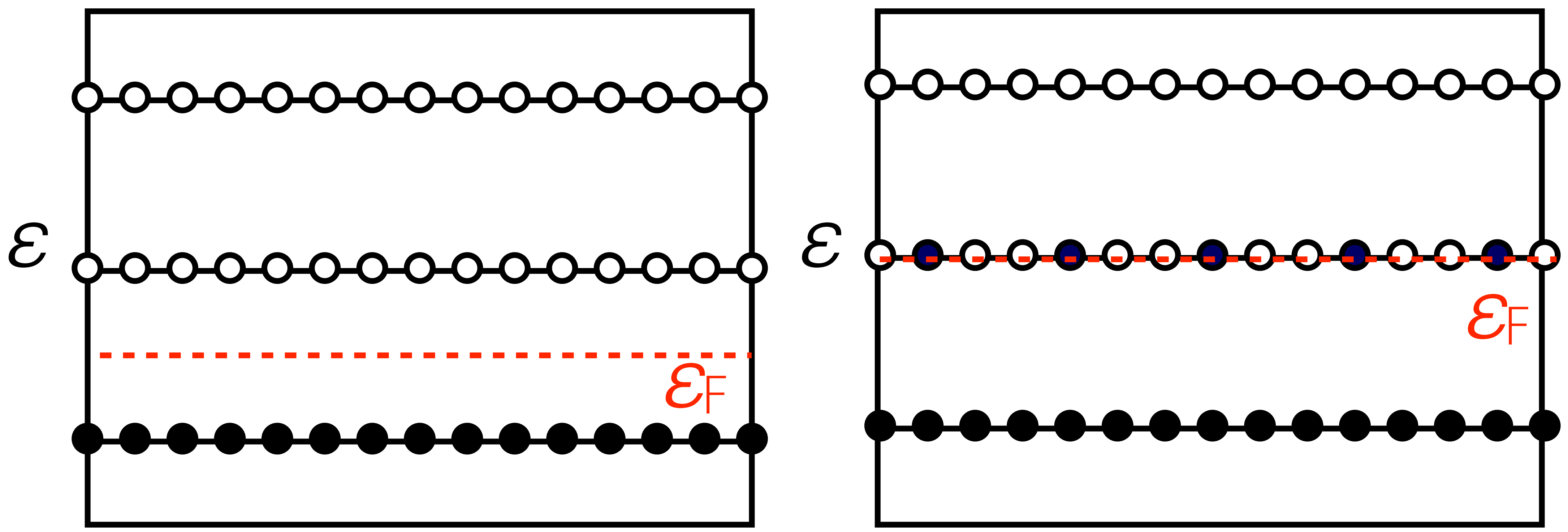
So even **if** we realise a Chern band in a regular crystal, very difficult to access sensible fractional fillings!

# State of Play: Landau Levels vs. Chern Bands

**Landau Levels**

$$\nu = \frac{\text{number of electrons}}{\text{number of flux quanta}}$$

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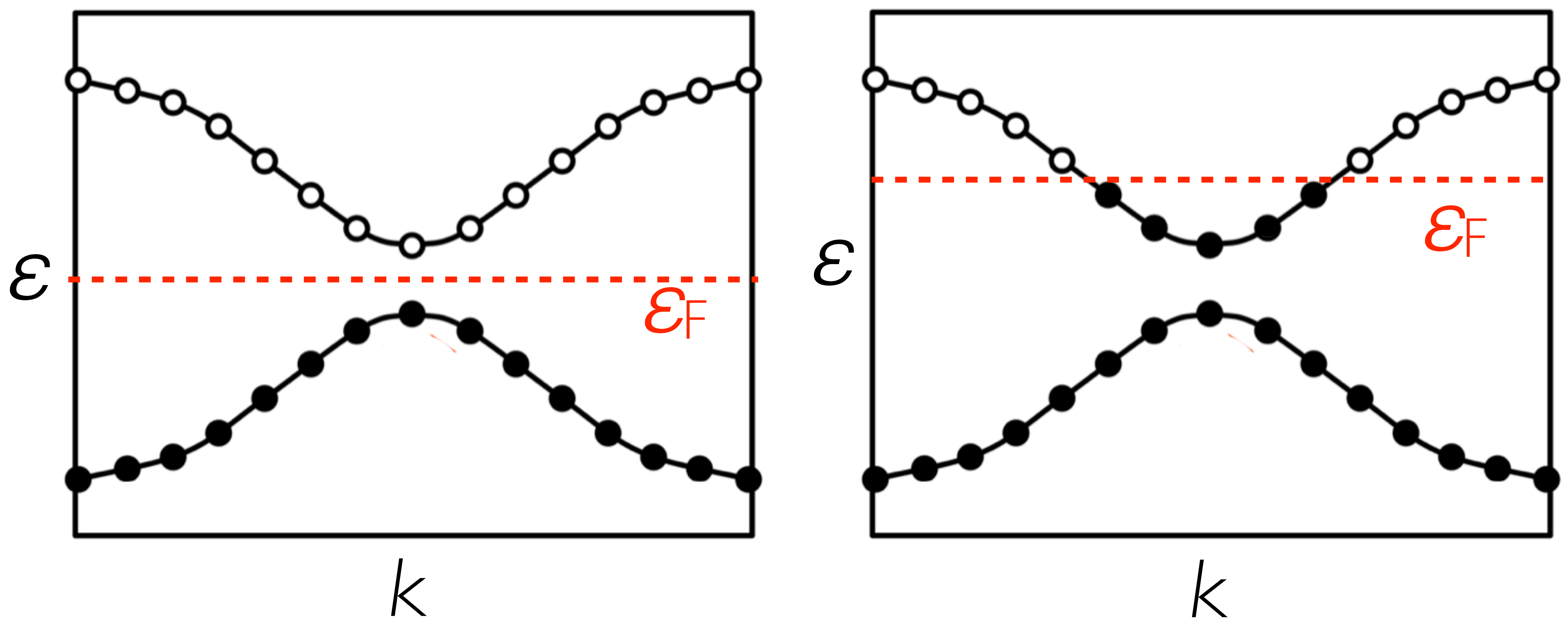

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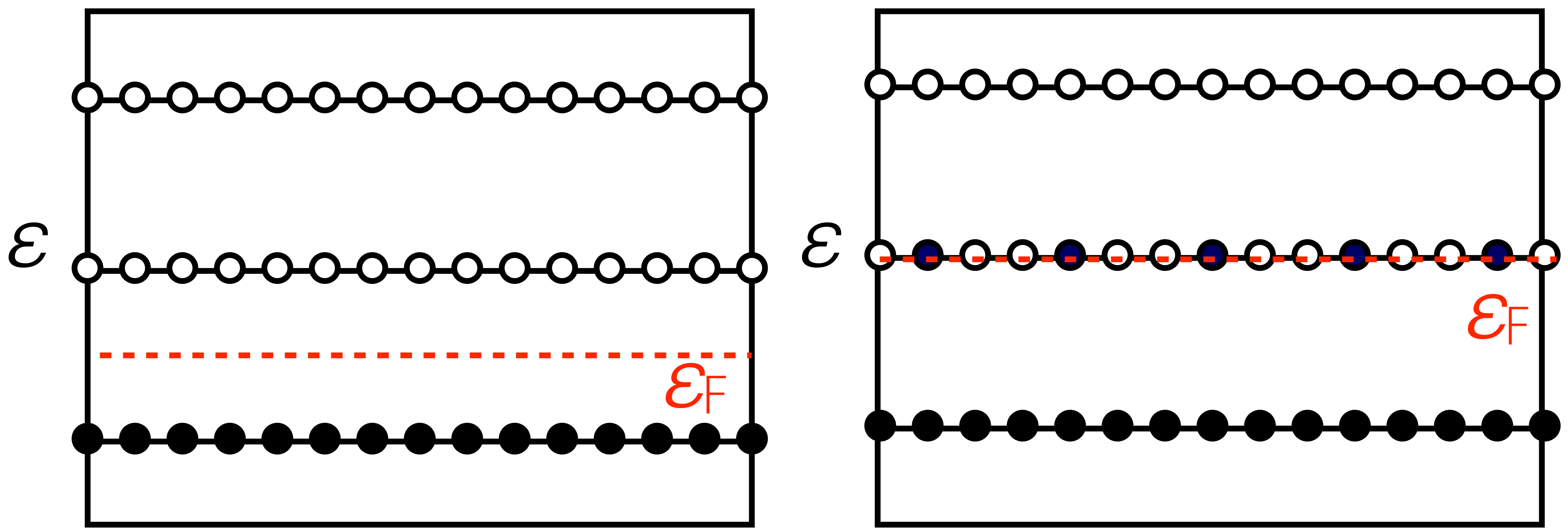


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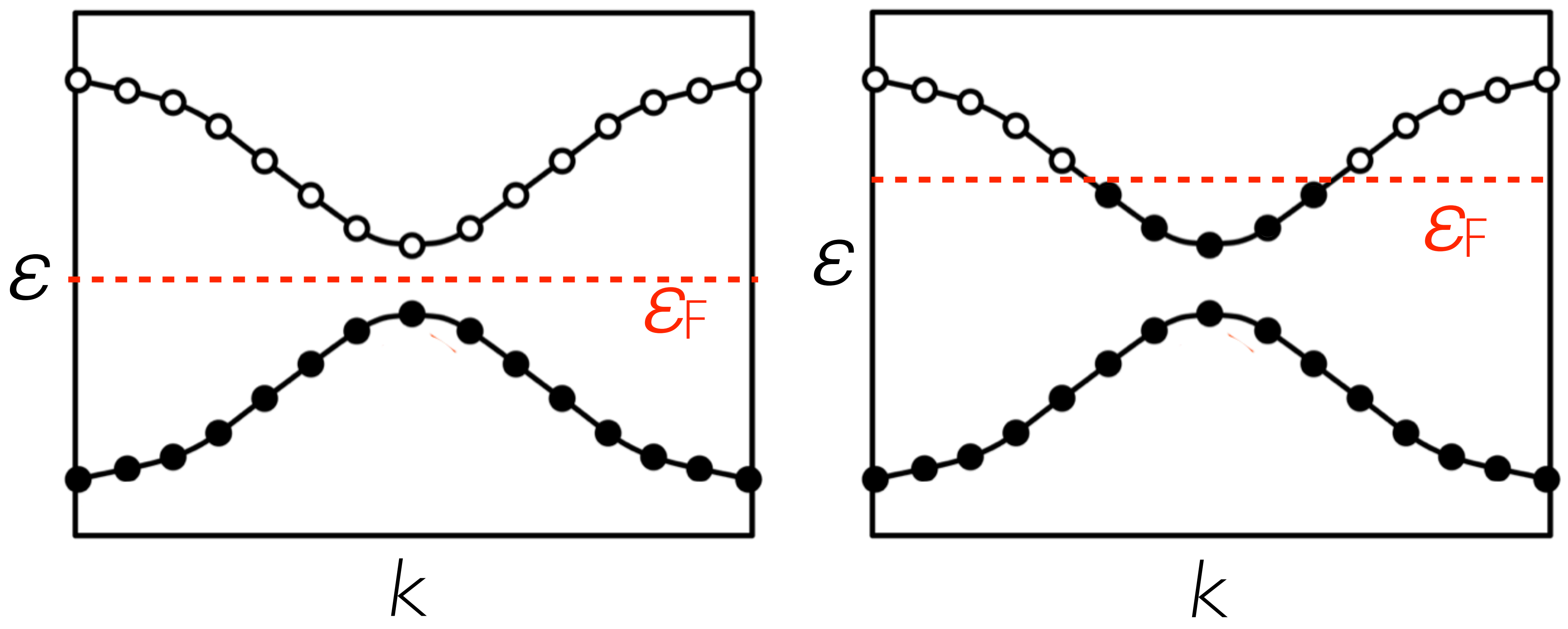

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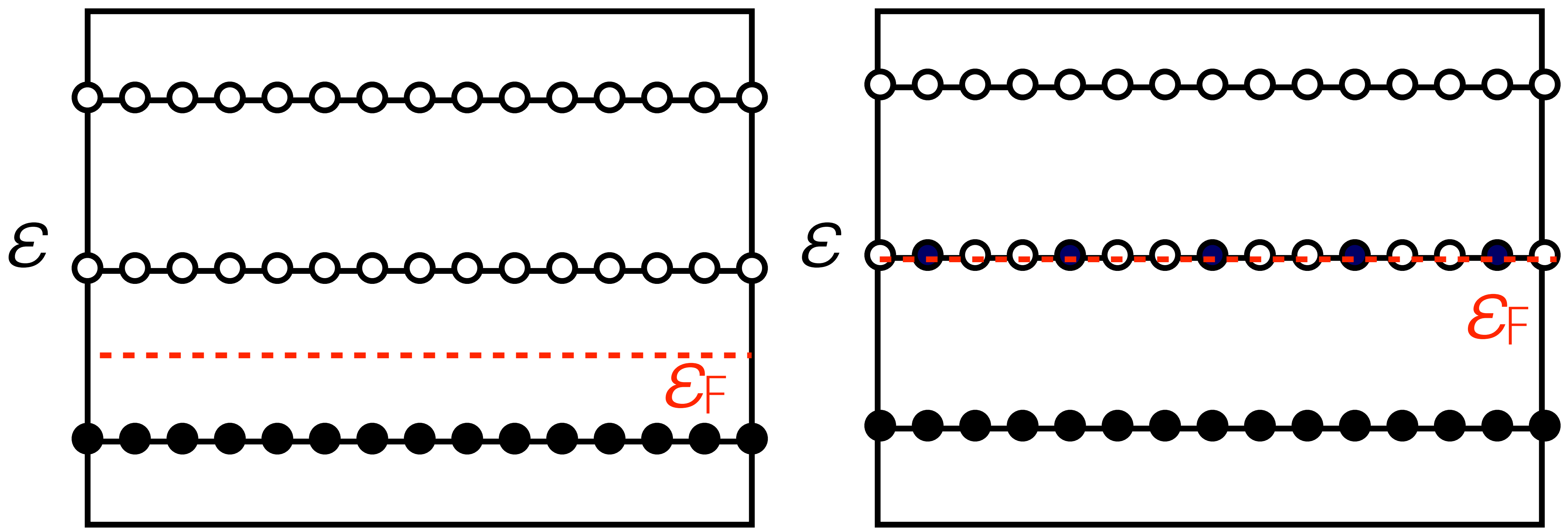


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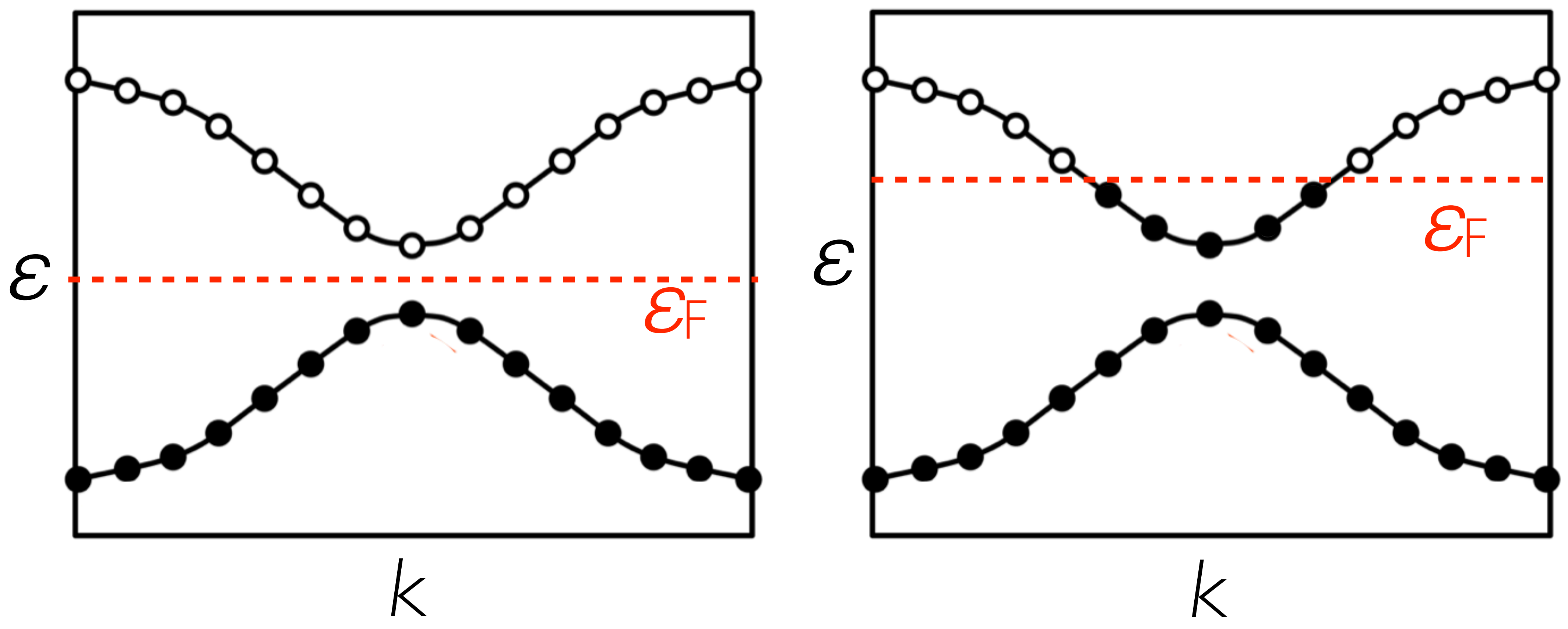


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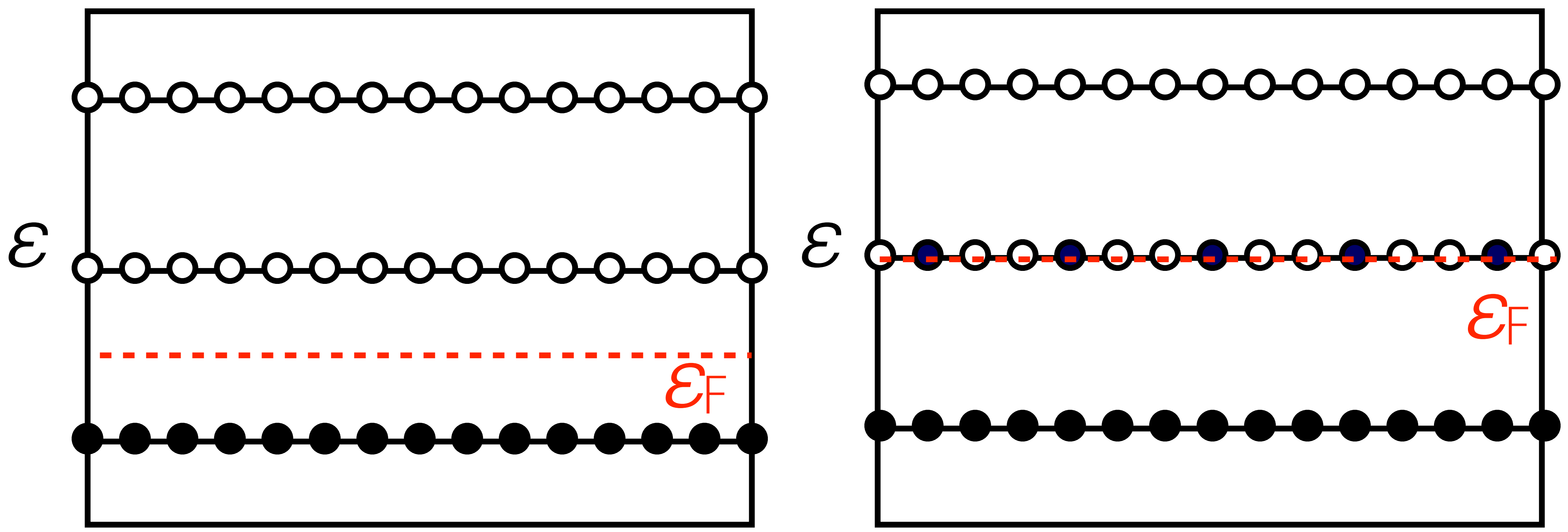


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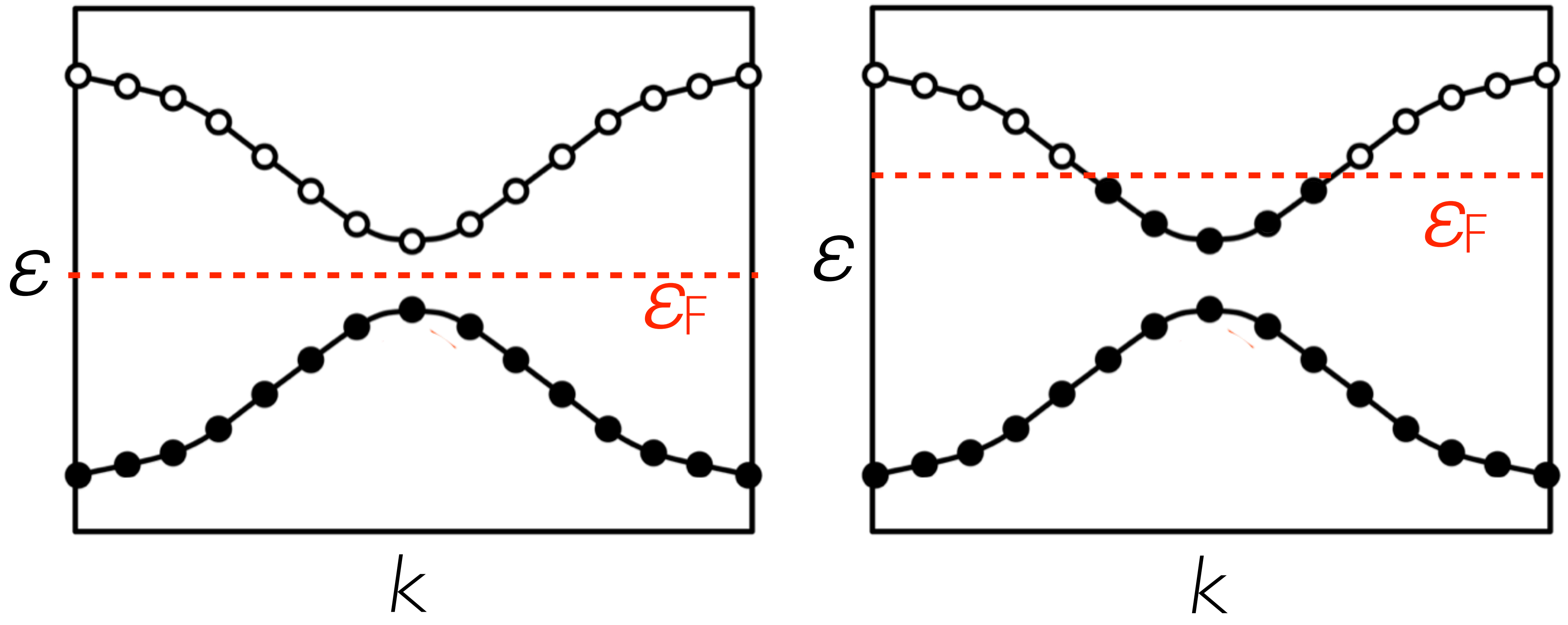


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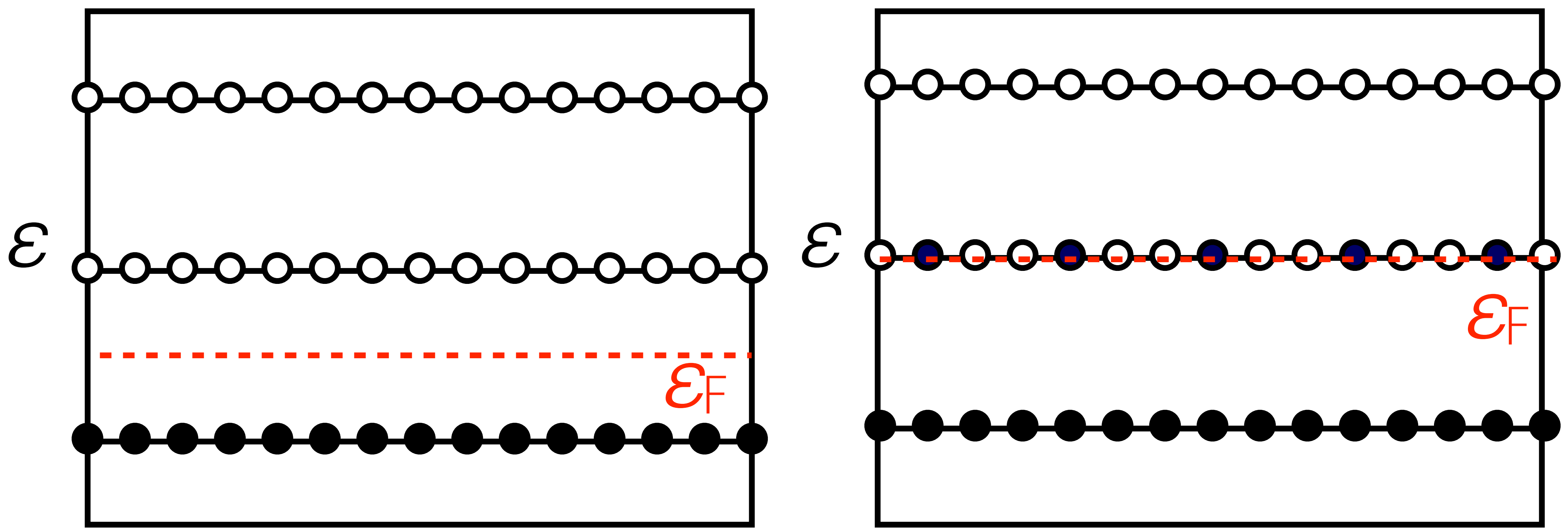


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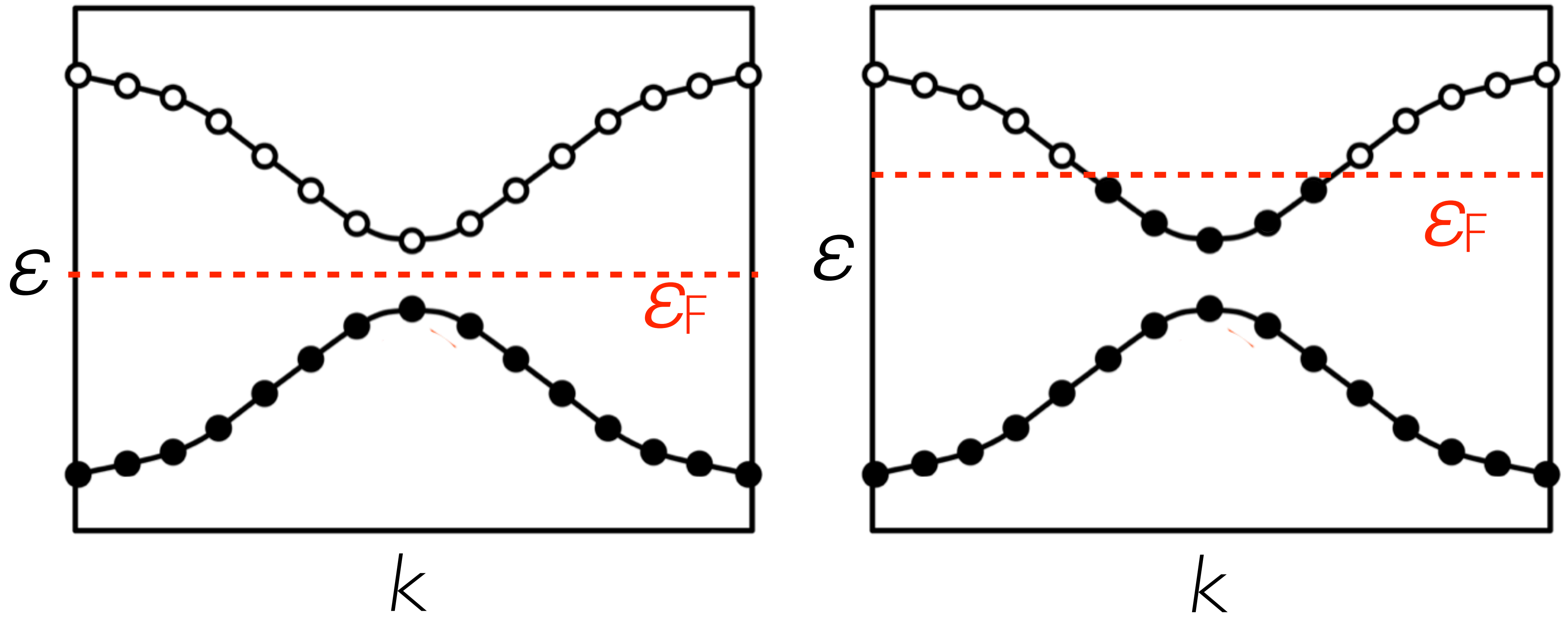


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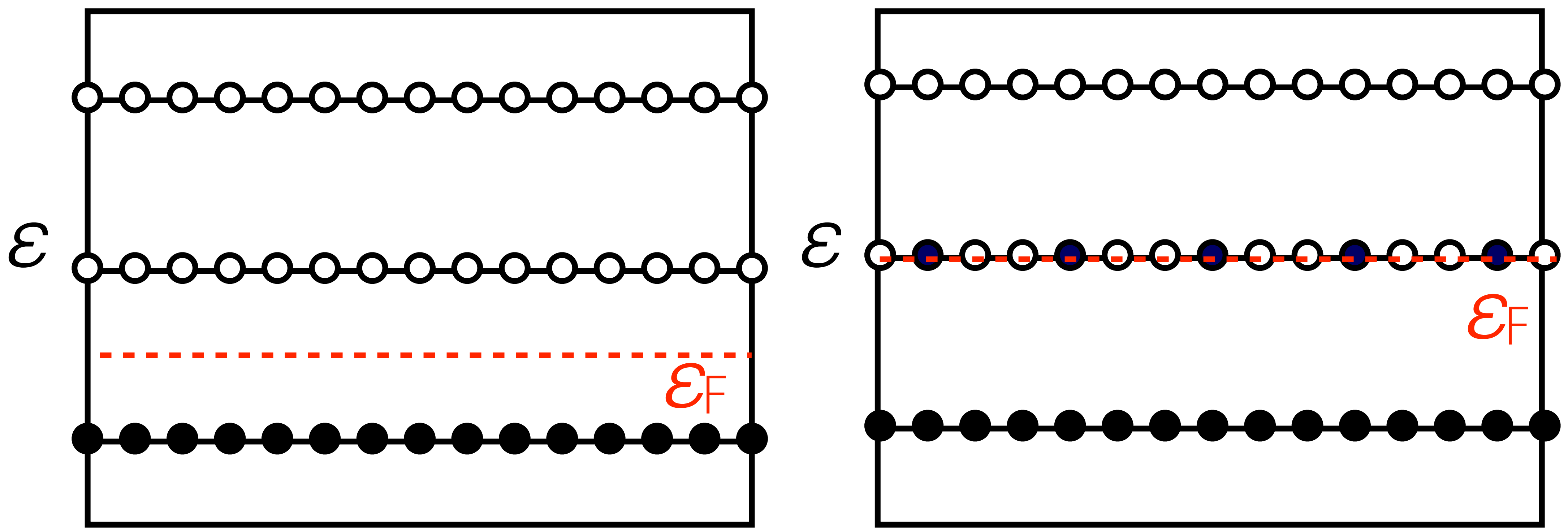


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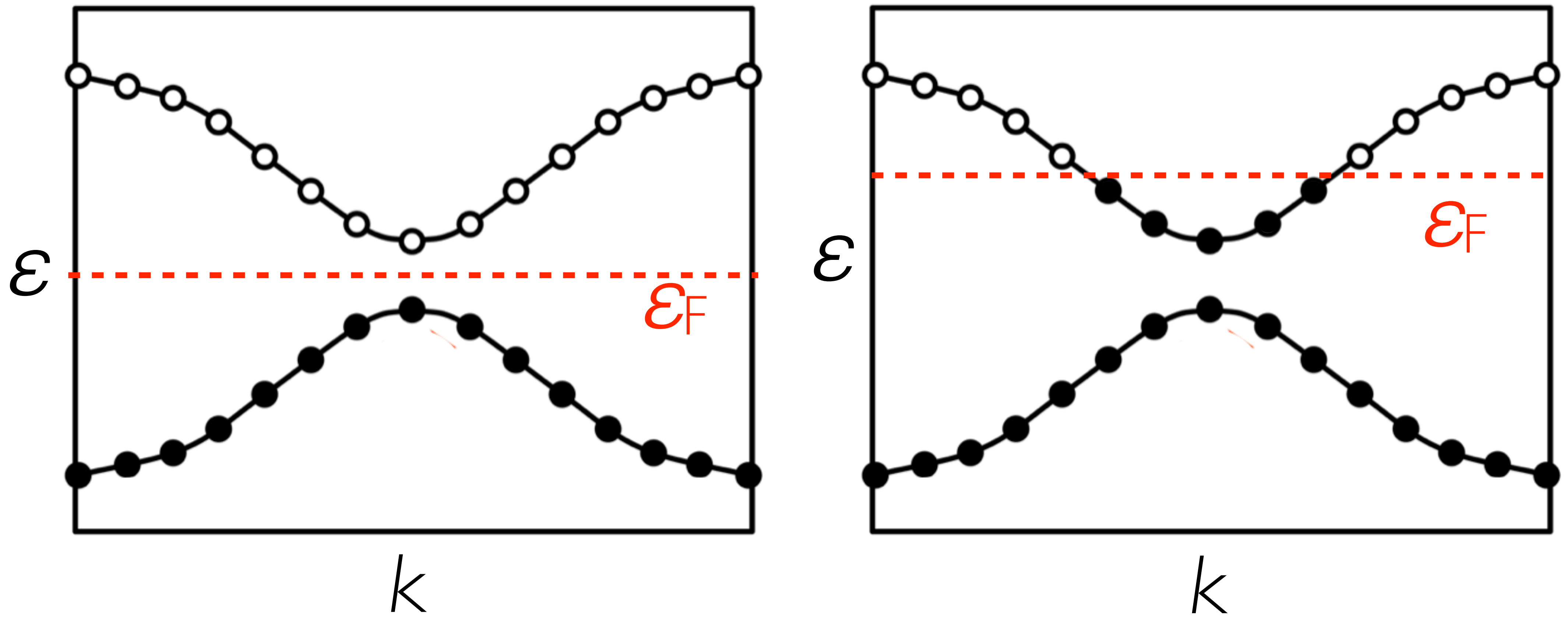


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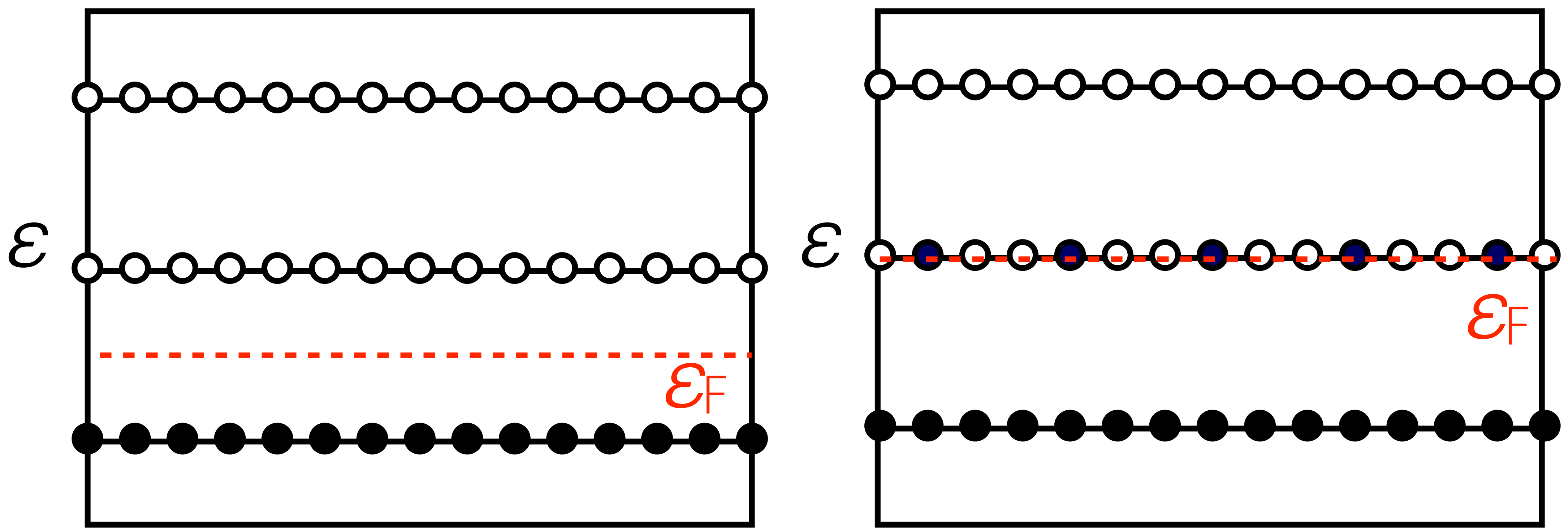


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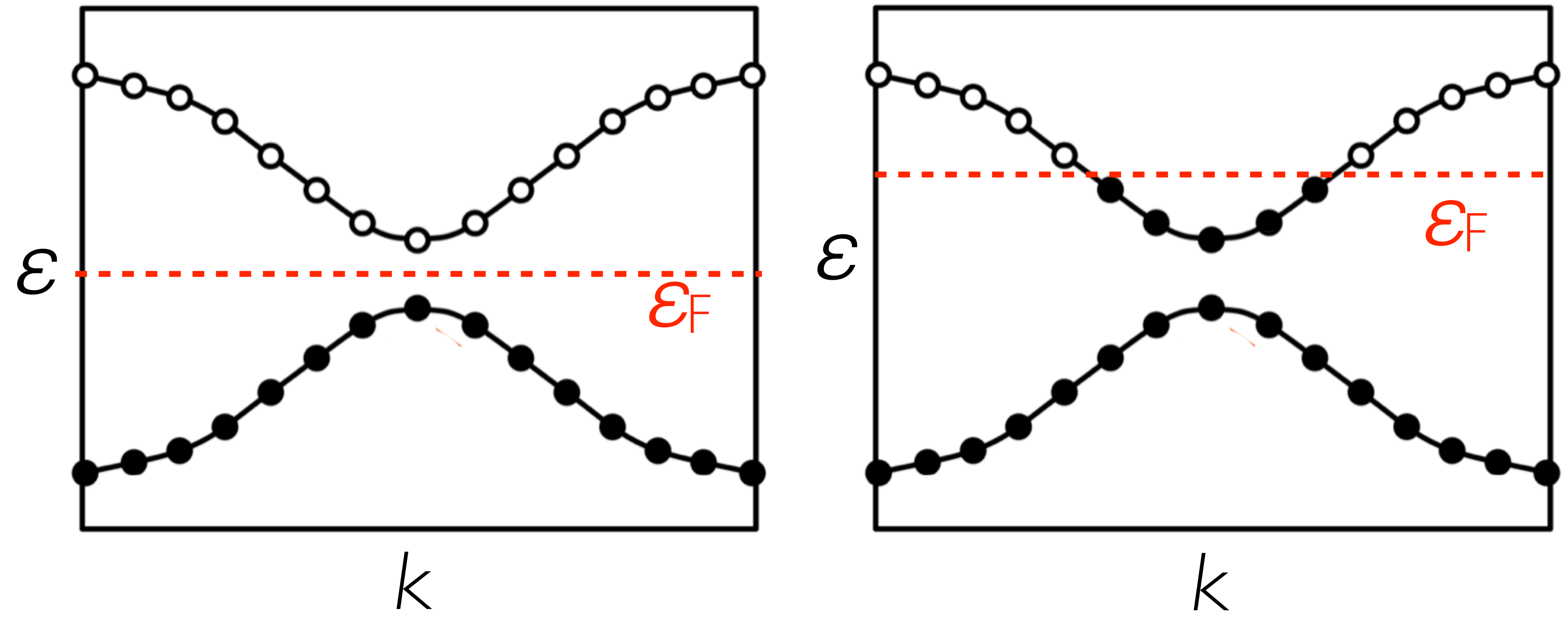


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- Dispersive ⇒ intermediate coupling
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## So, the challenge is clear: need crystalline systems with

- **topology:** energy bands with Chern numbers
- **correlations:** interactions enhanced relative to kinetic energy
- **tunability:** ability to easily change filling by $\pm O(1)$ electron per unit cell

# What happened next… the Hollywood version

Stills from Walt Disney's Silly Symphony: The Tortoise and the Hare, Youtube

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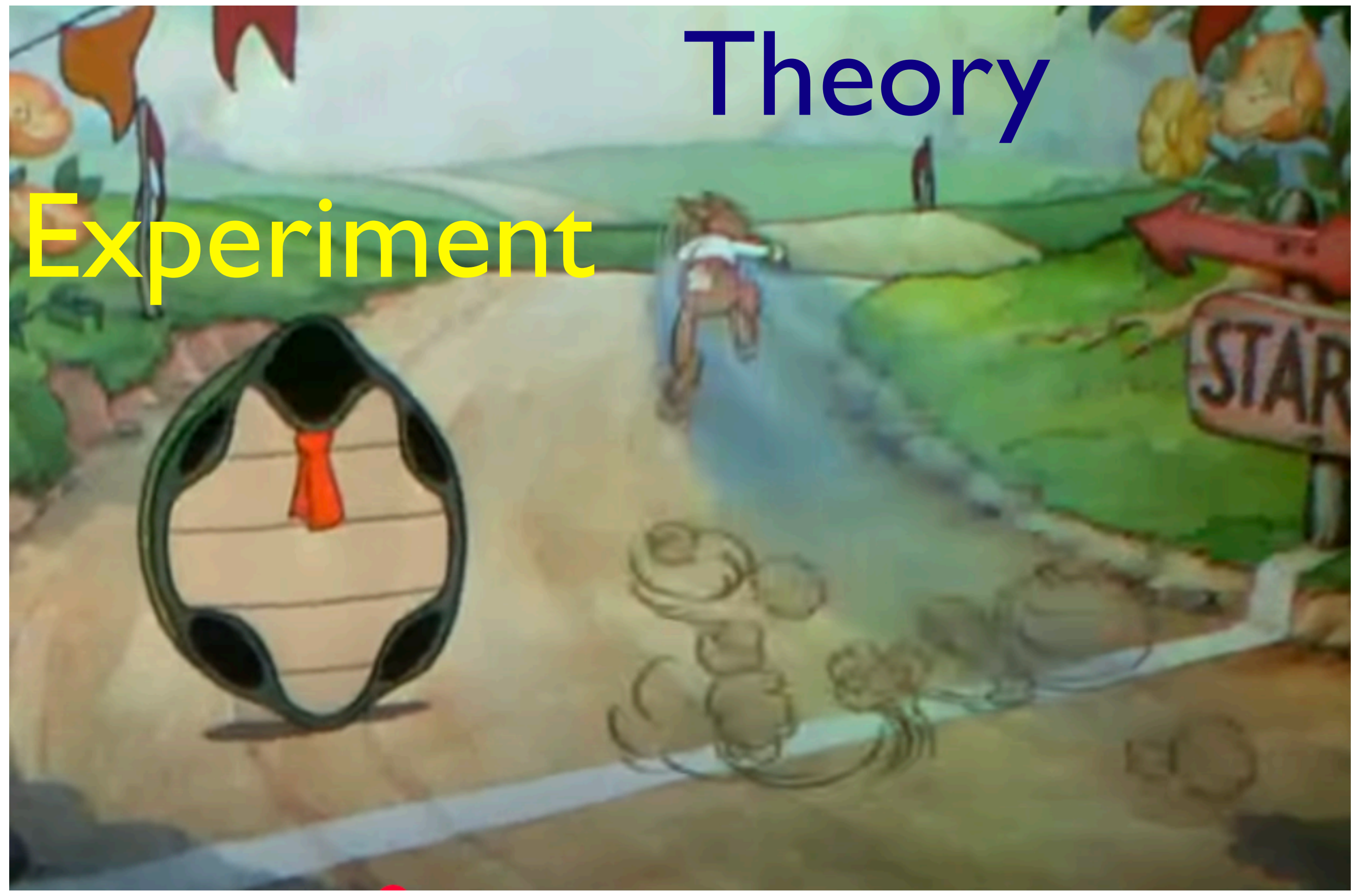


c. 2013

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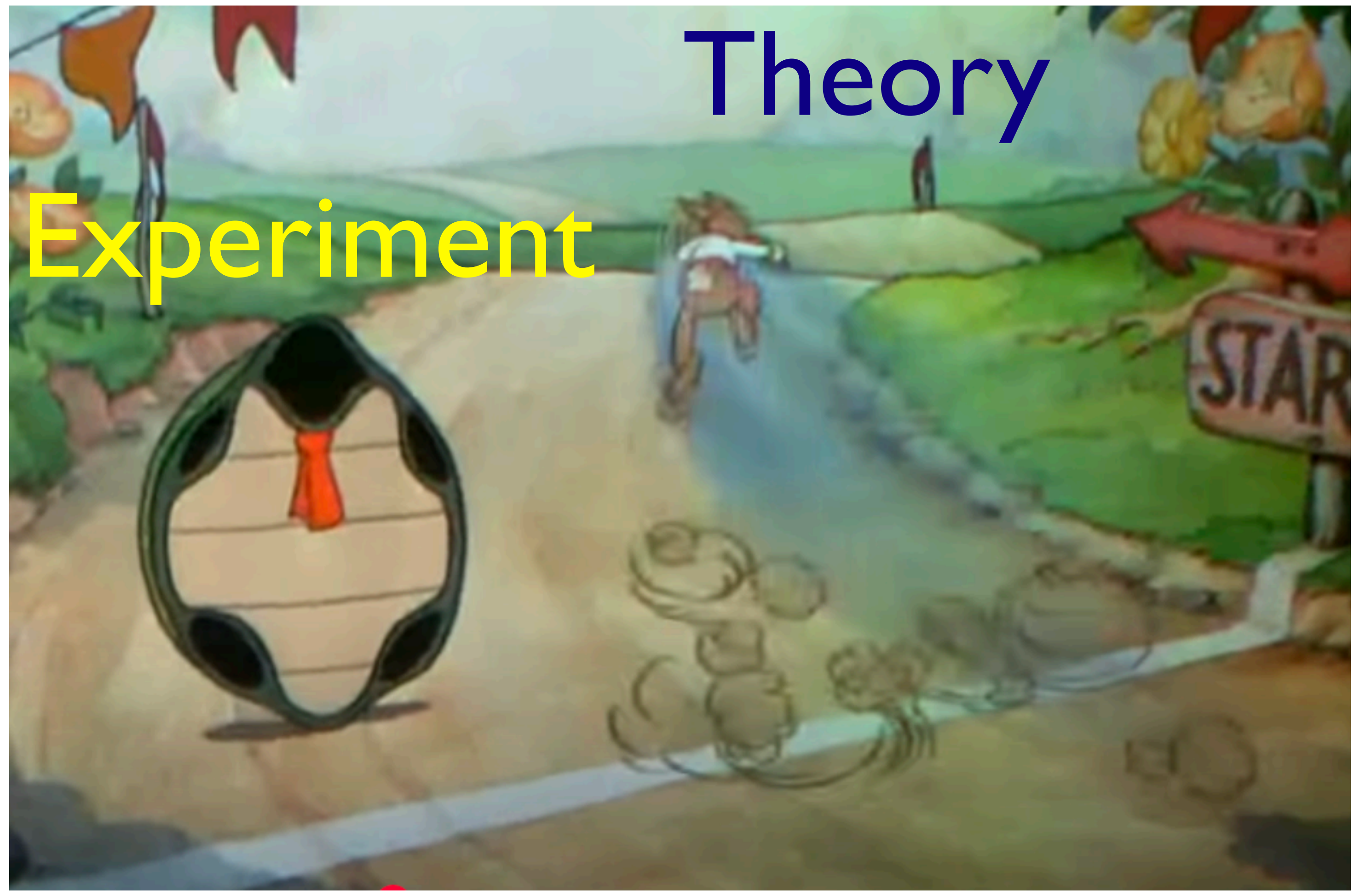


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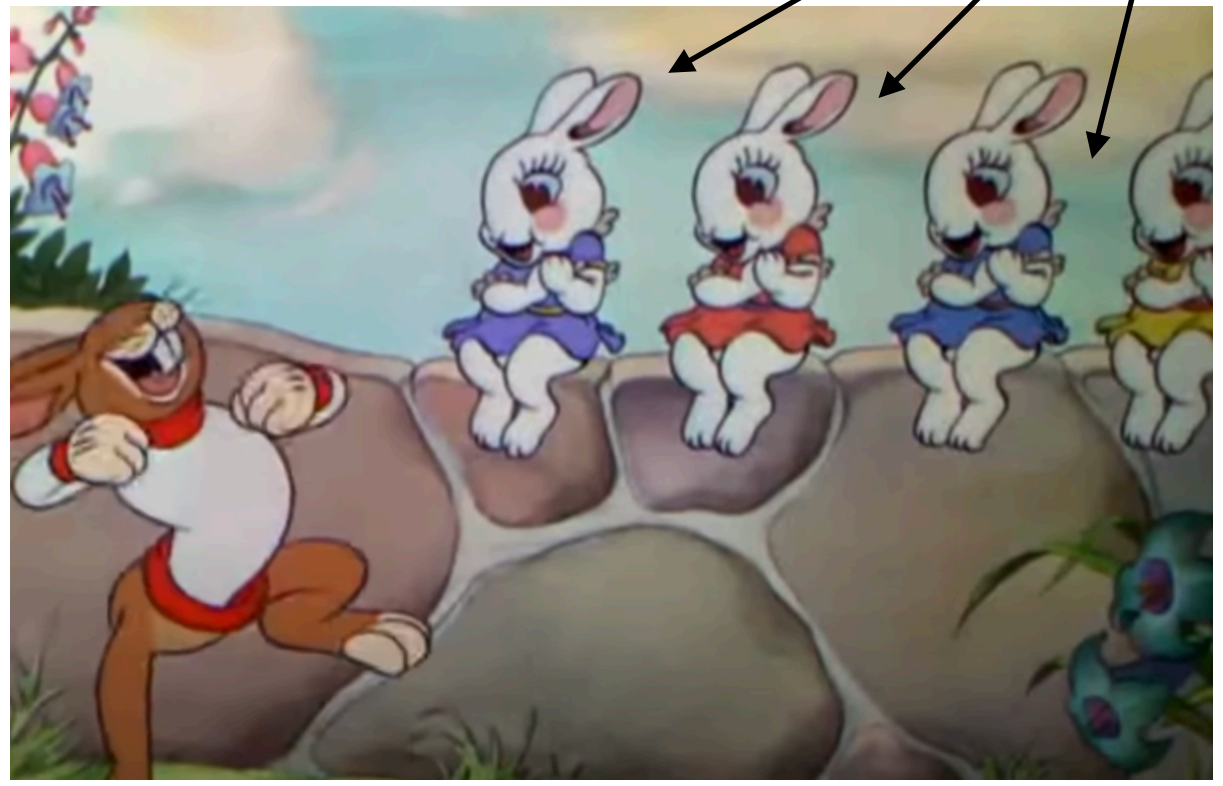

2013-2018

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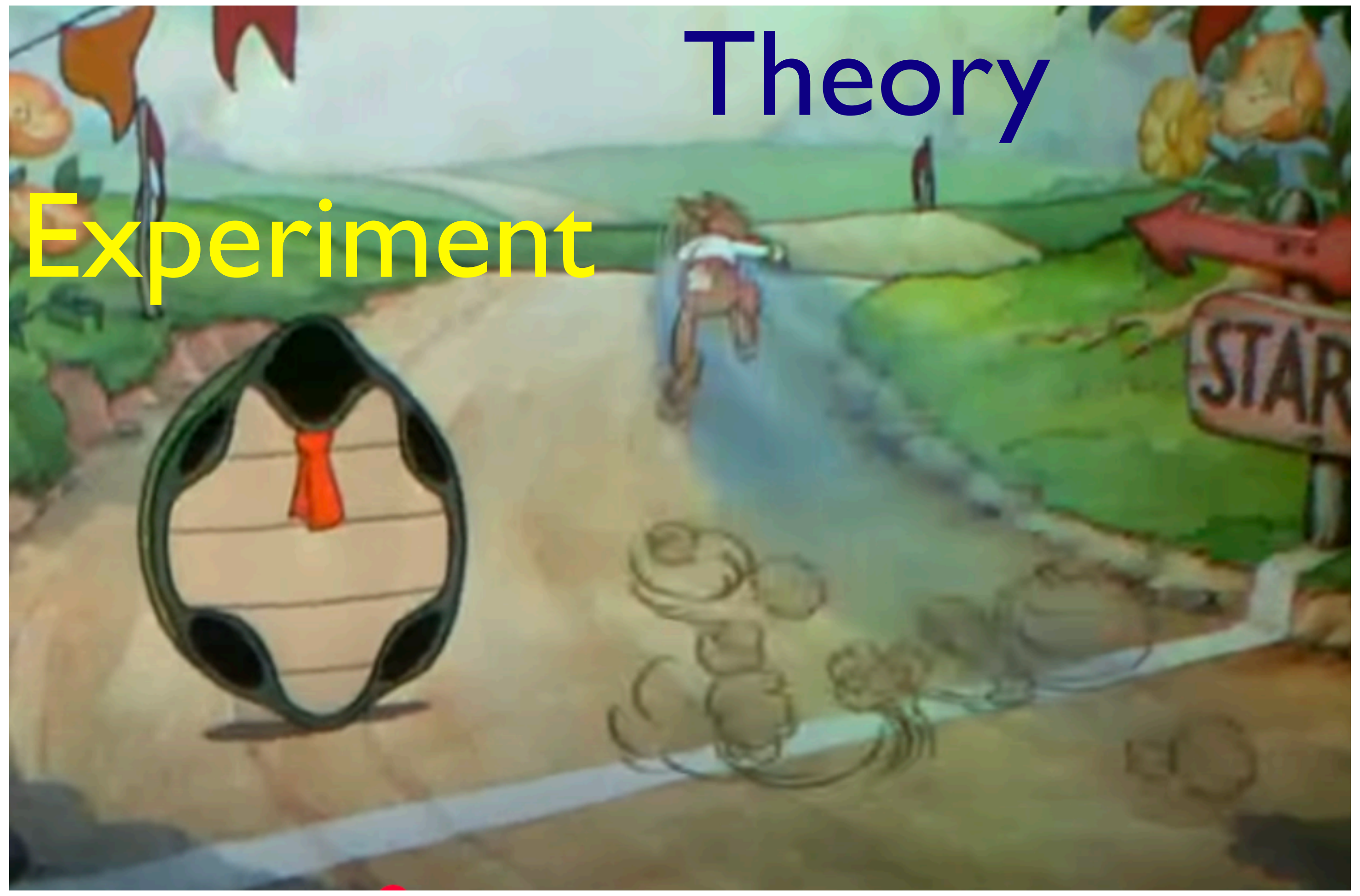


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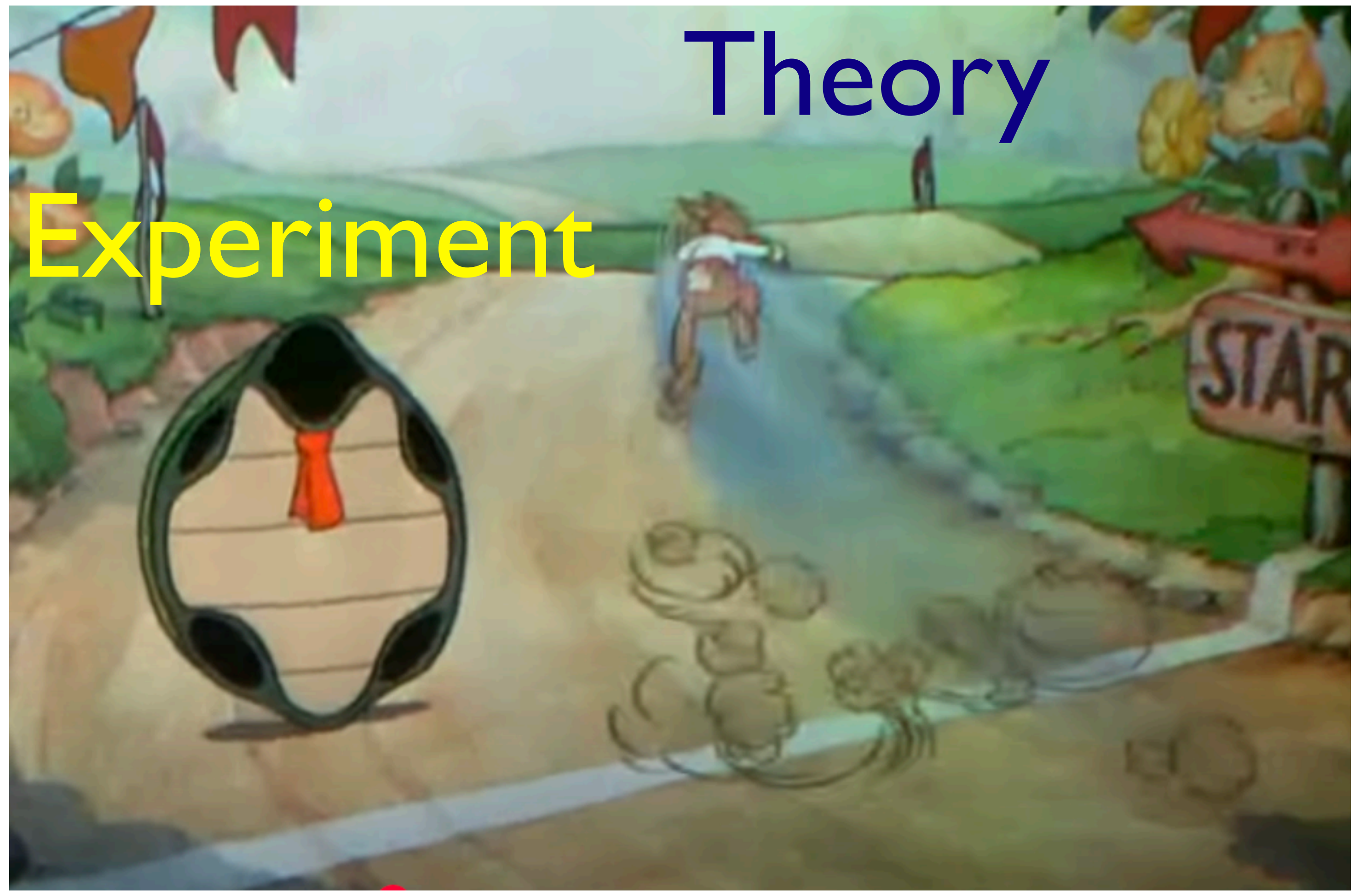


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2013-2018

New physics!


c. 2018

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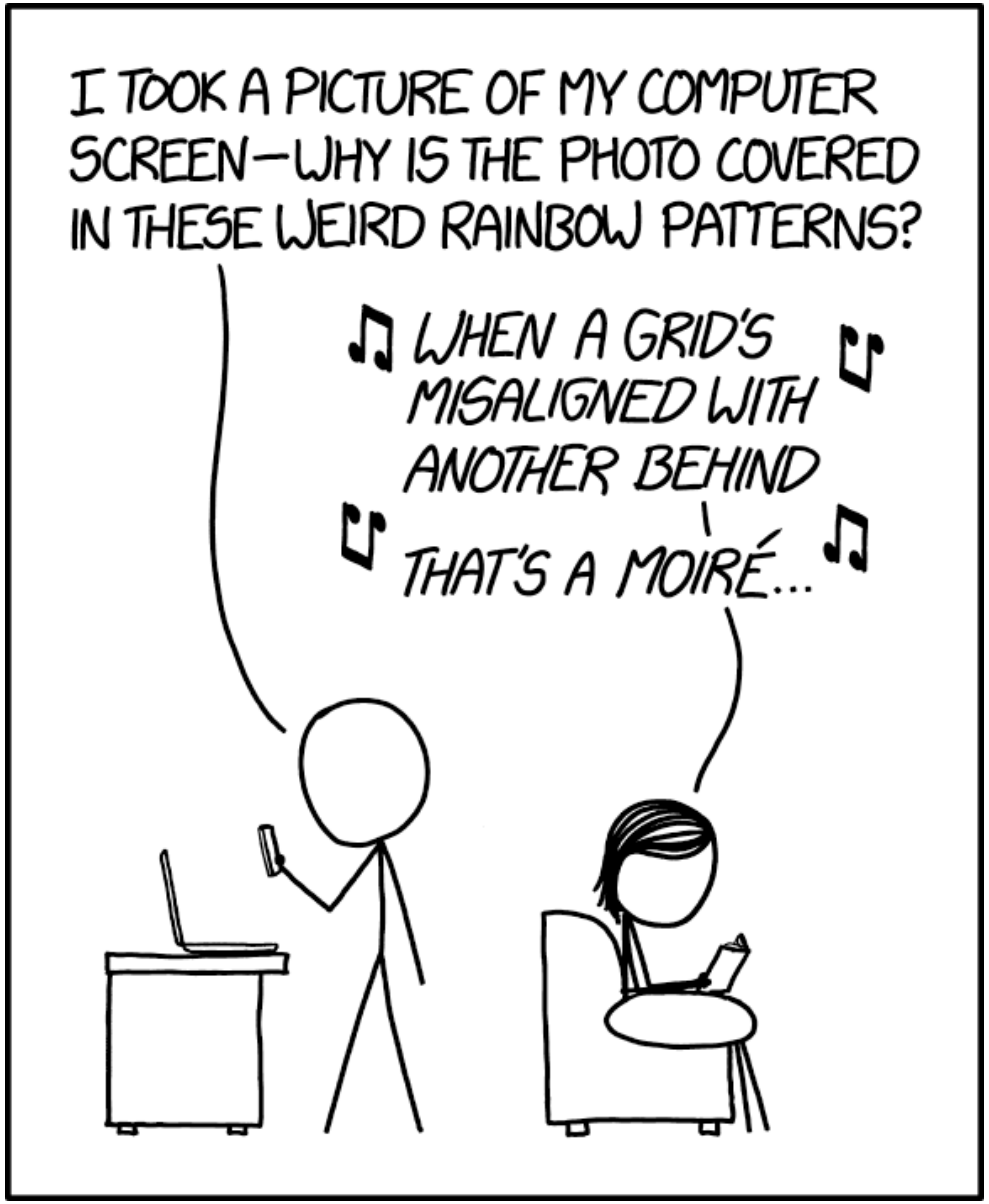
# New Physics: the "Moiré Effect"




xkcd

(with apologies to Dean Martin)


Spotify

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**moiré** — from textile industry

~ moirer (French) ~ mohair (English) ~ mukhayyar (Arabic = "chosen")


A. Zwicky's blog

**"beating" of periodic structures w/ slightly offset period or orientation**

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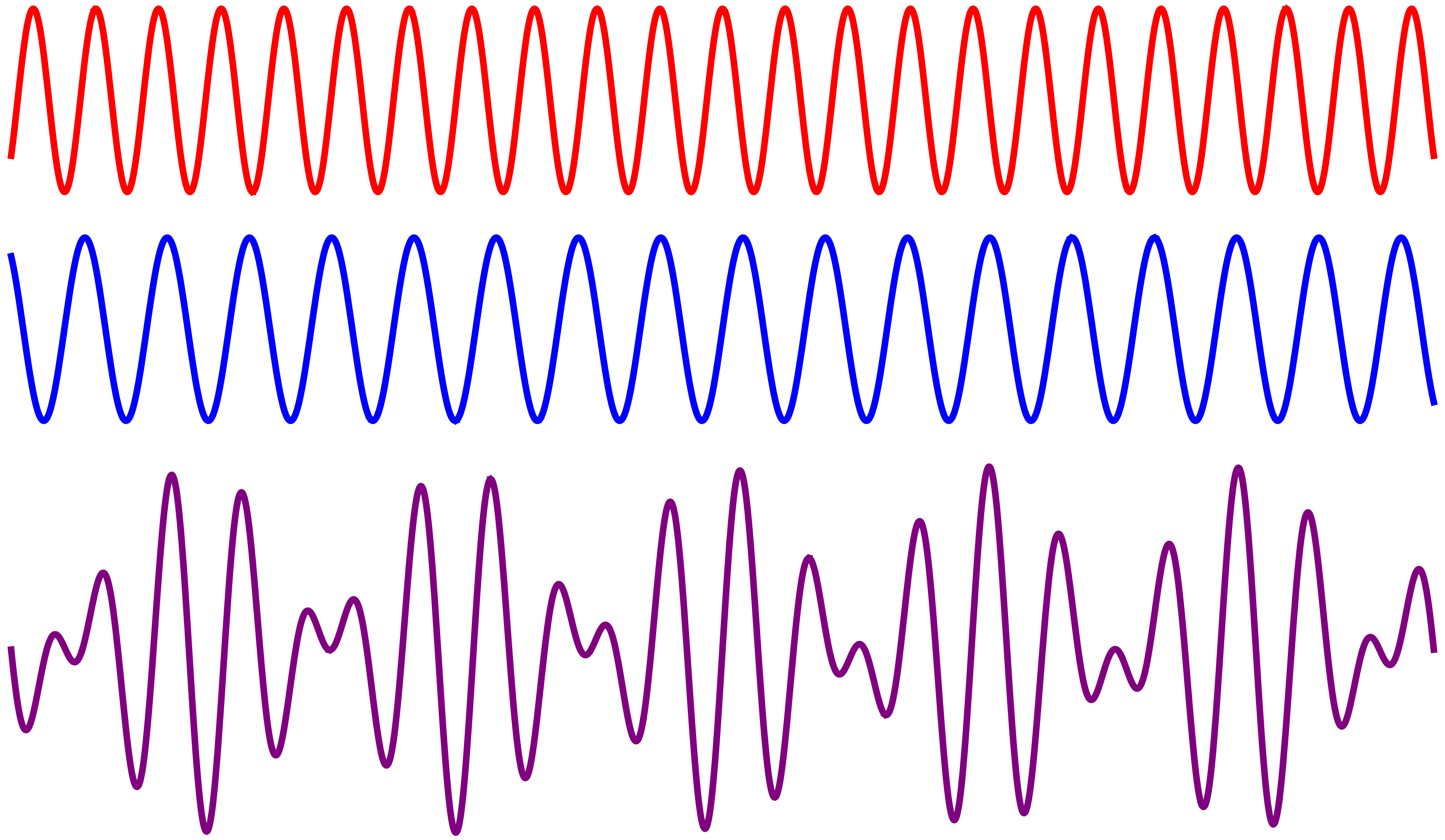
## 1D example:

$$V_\pm(x) = \sin[(k \pm \delta)x]$$



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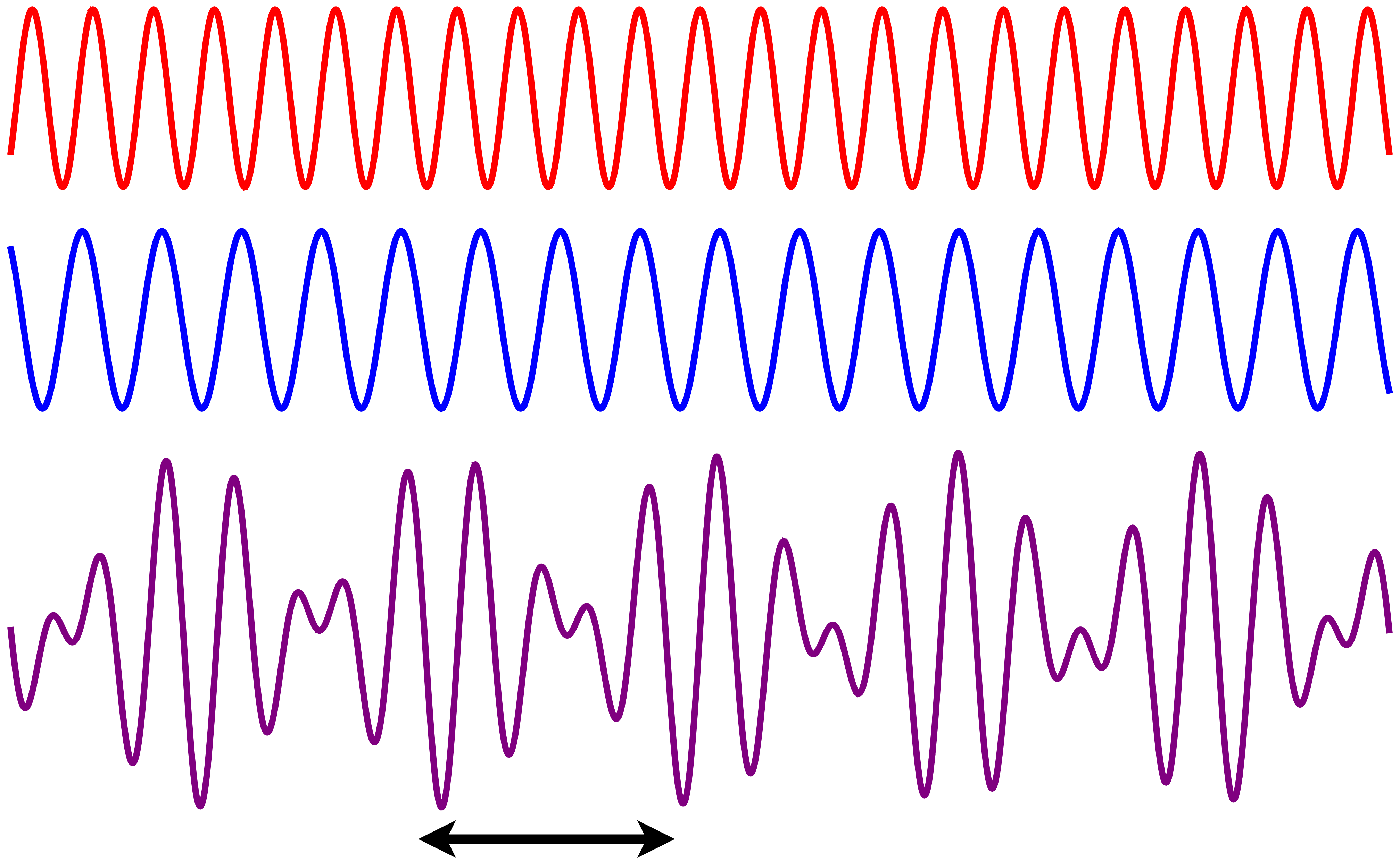
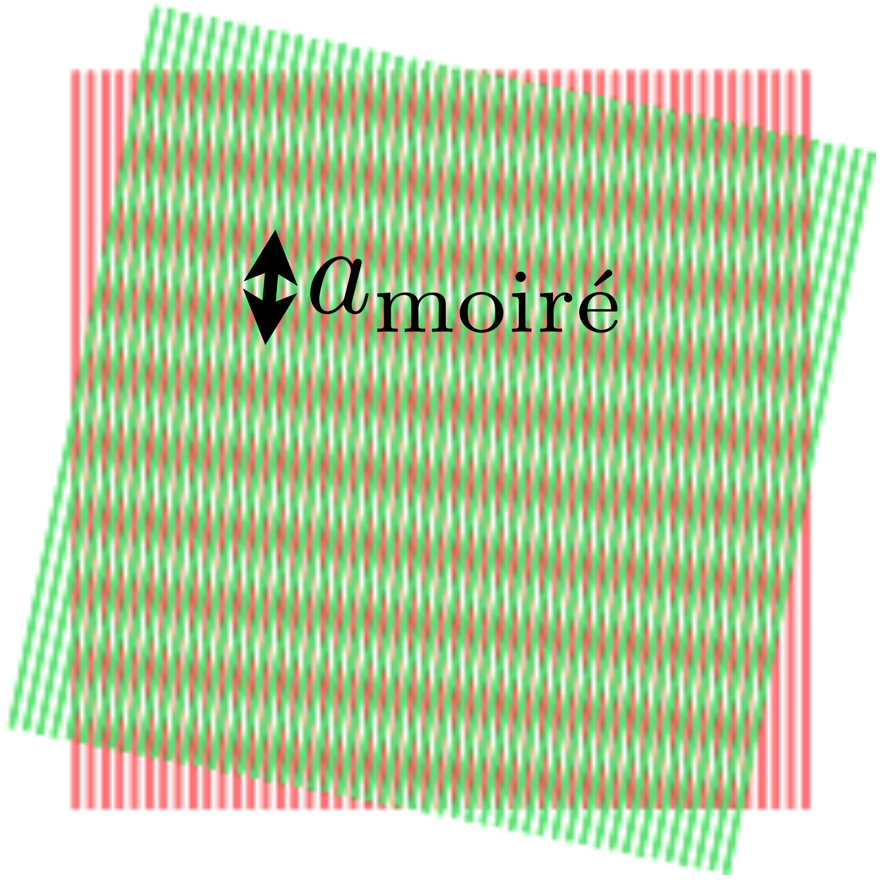


"superlattice" w/ approximate period $a_{\text{moiré}} = \dfrac{2\pi}{\delta} \gg \dfrac{2\pi}{k}$

**2D materials** (e.g. graphene) held together by van der Waals forces: **moiré from twisting**

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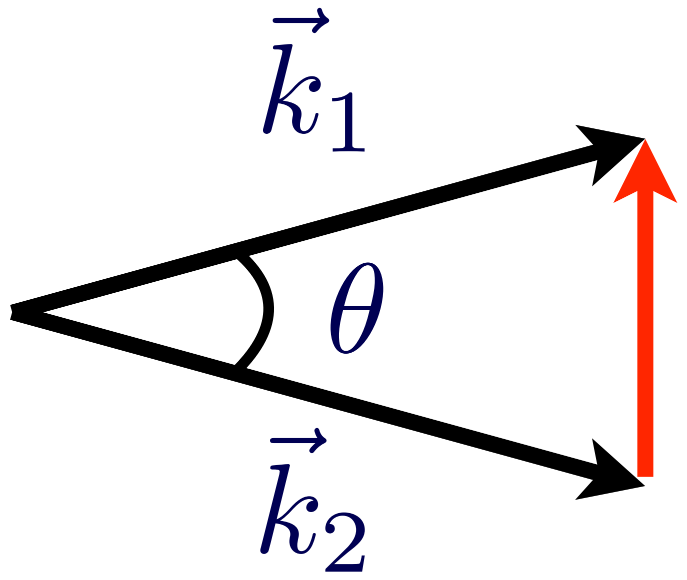
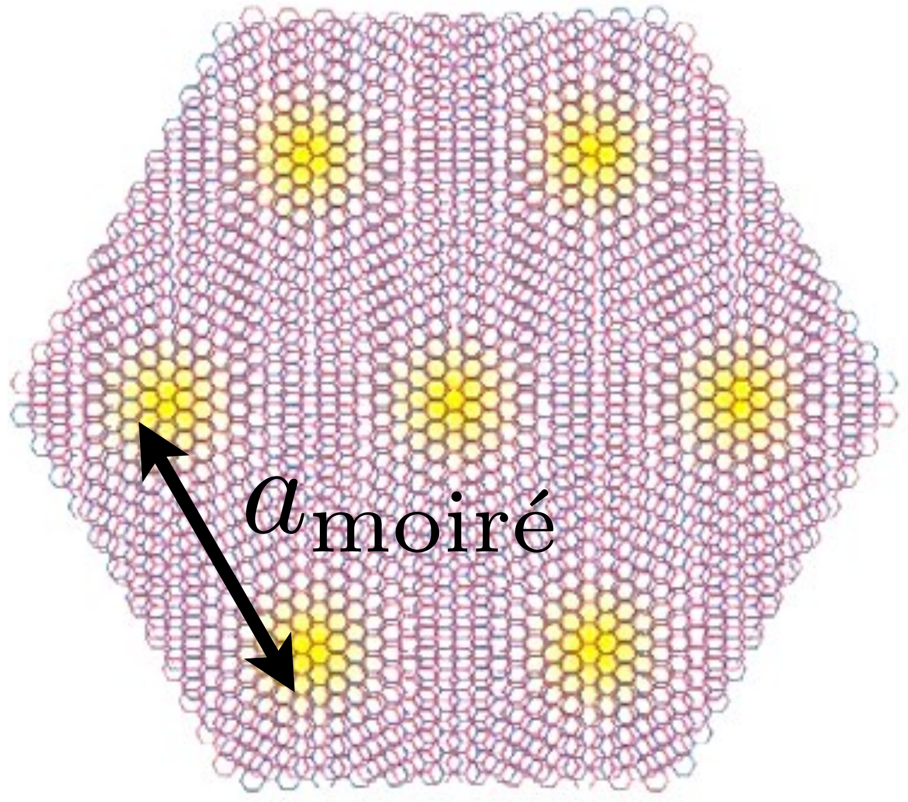



$$\delta = |\vec{k}_1 - \vec{k}_2| \sim 2|\vec{k}_i| \sin\frac{\theta}{2} \approx |\vec{k}_i|\theta$$



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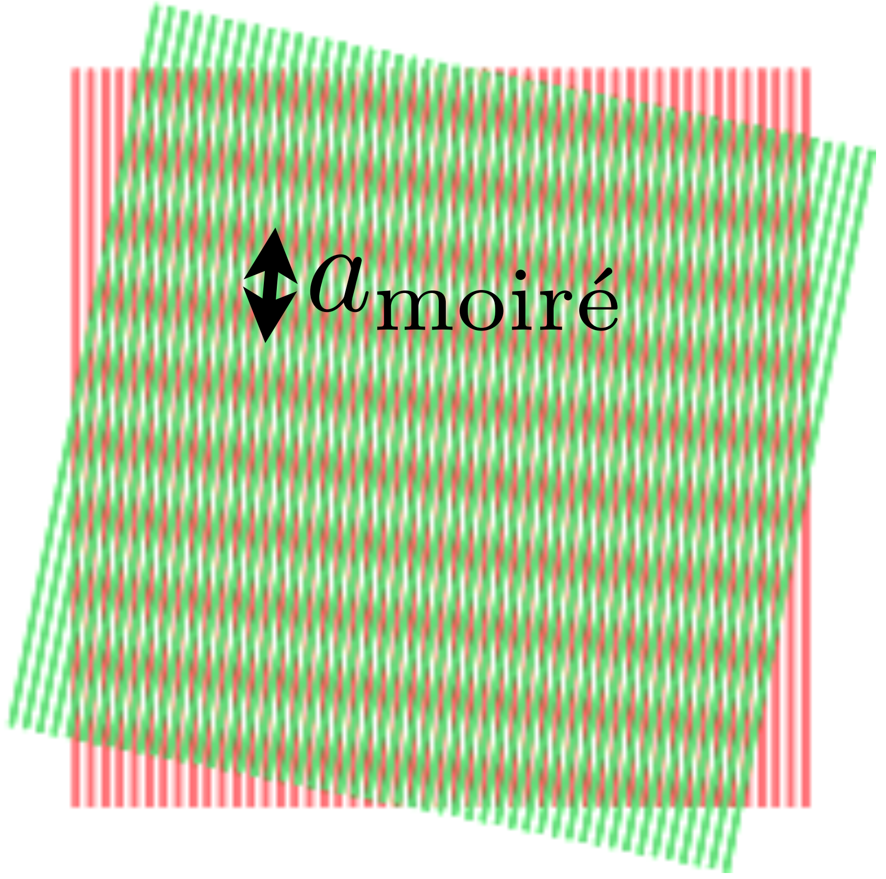


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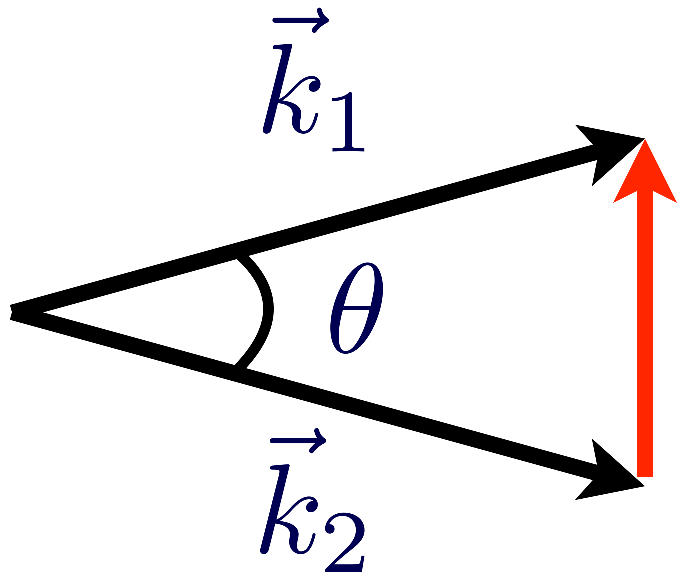


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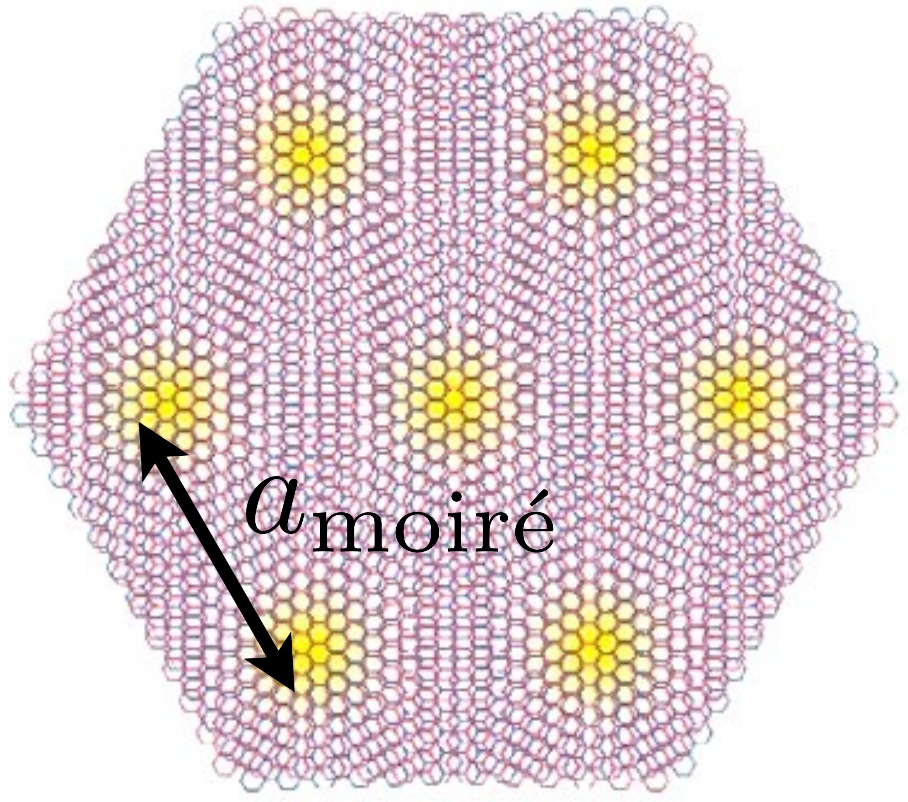


using $\quad |\vec{k}_i| \sim \dfrac{2\pi}{a_{\text{lattice}}}$

$$a_{\text{lattice}} \sim 0.3 \text{ nm}$$

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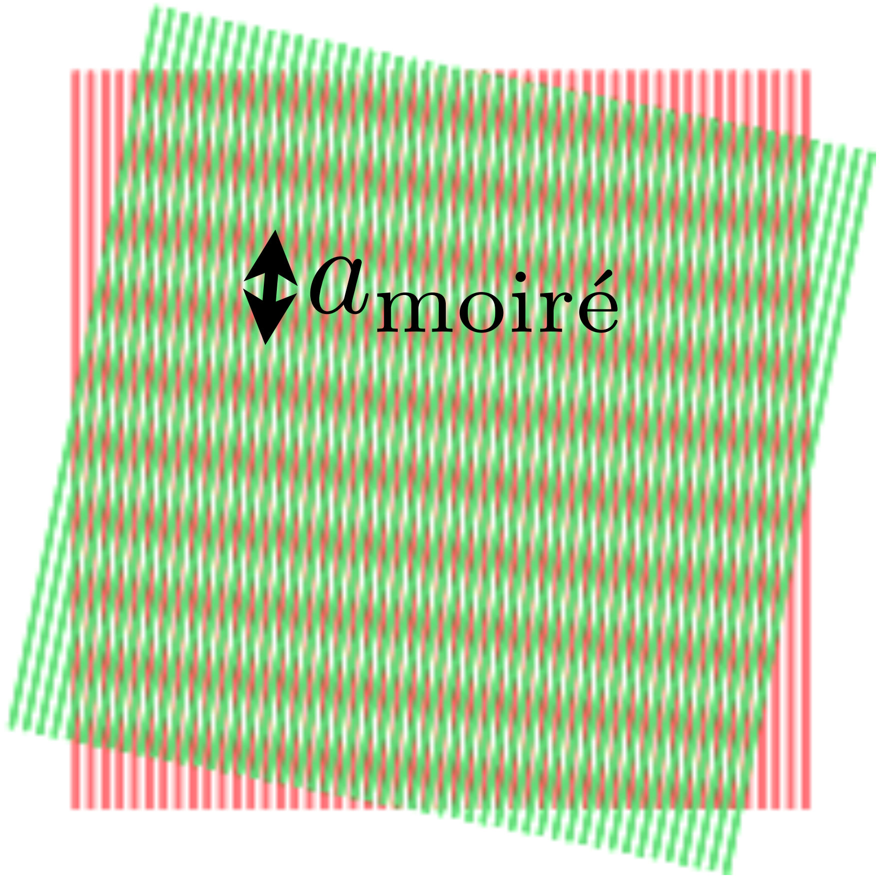


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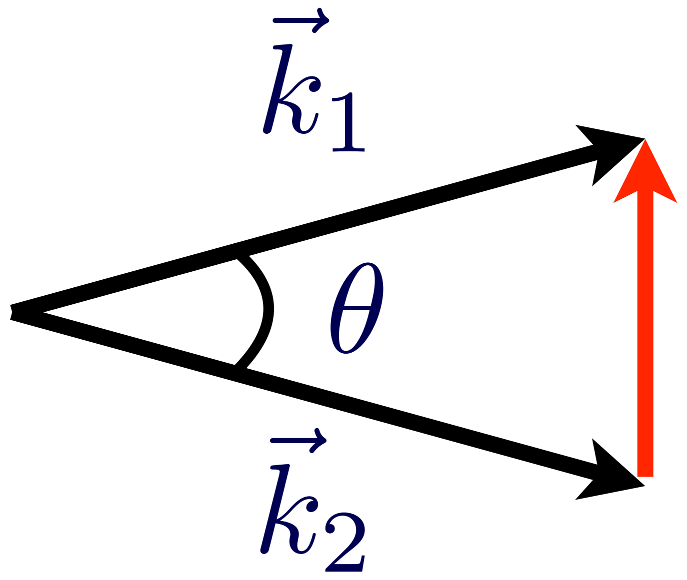


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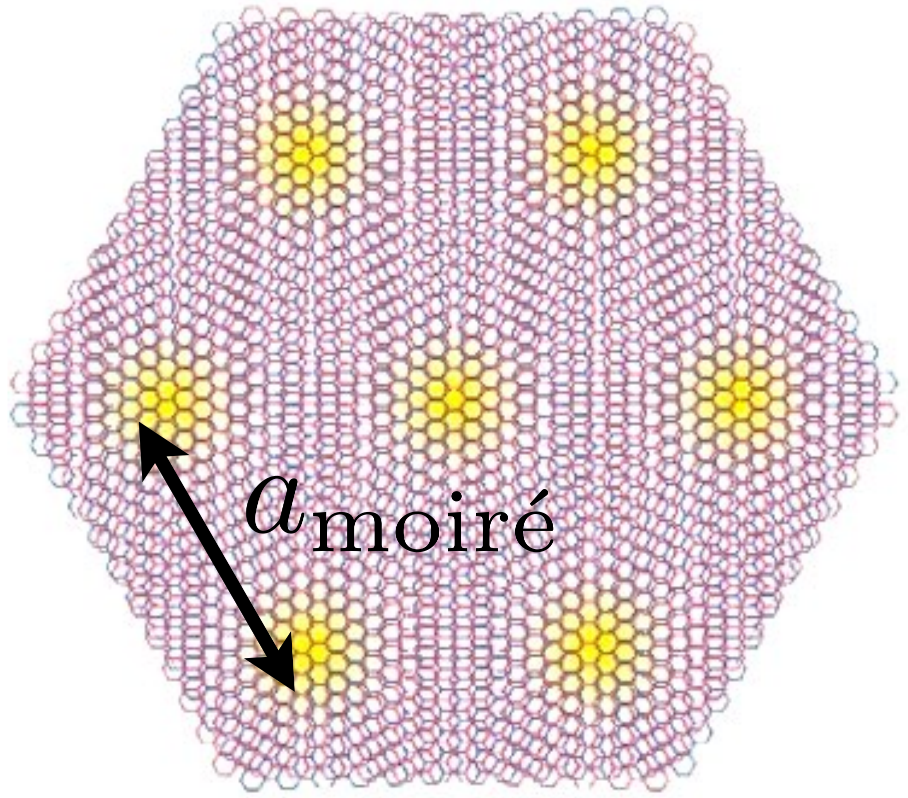


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Just as magnetic field modifies the kinetic energy of free electrons, moiré modifies ε(k)

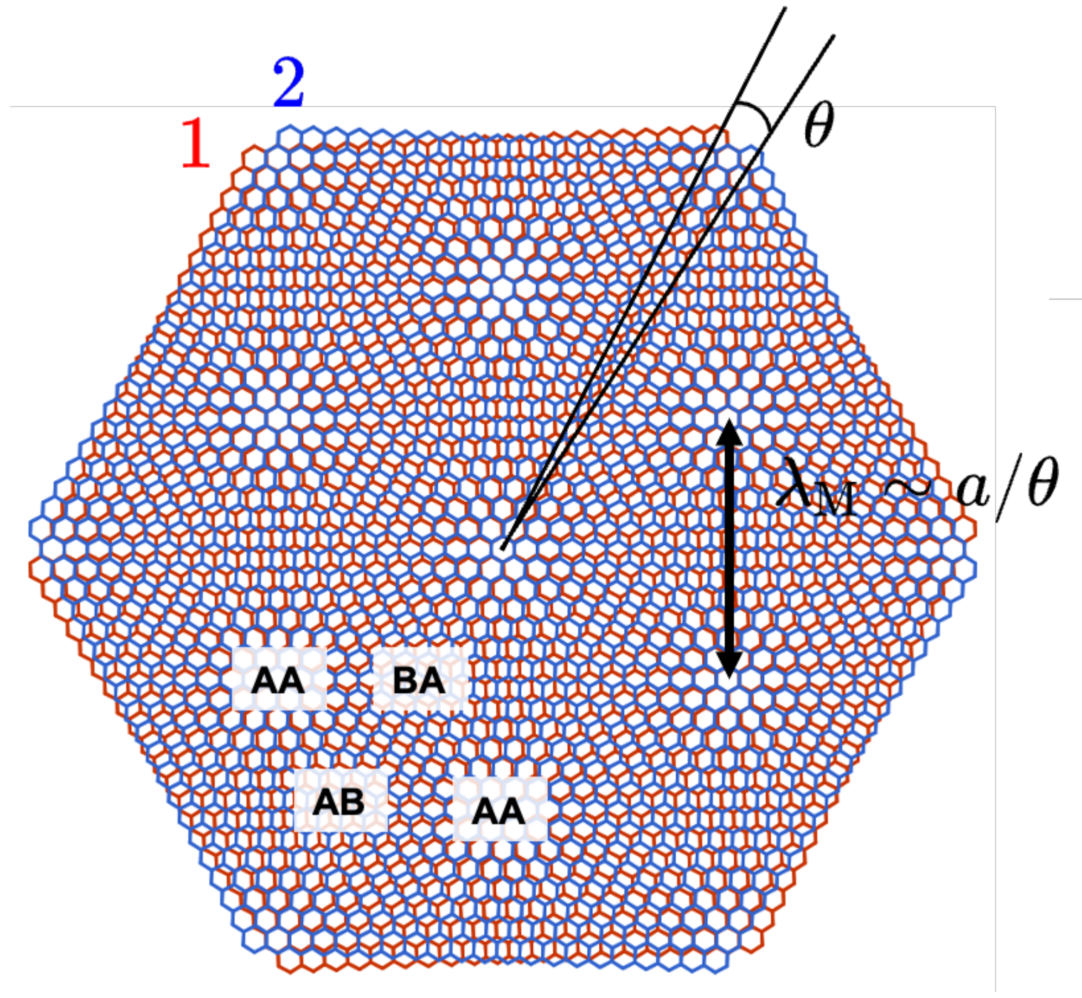
⇒ “flat” bands w/ small kinetic energy ⇒ strong correlations

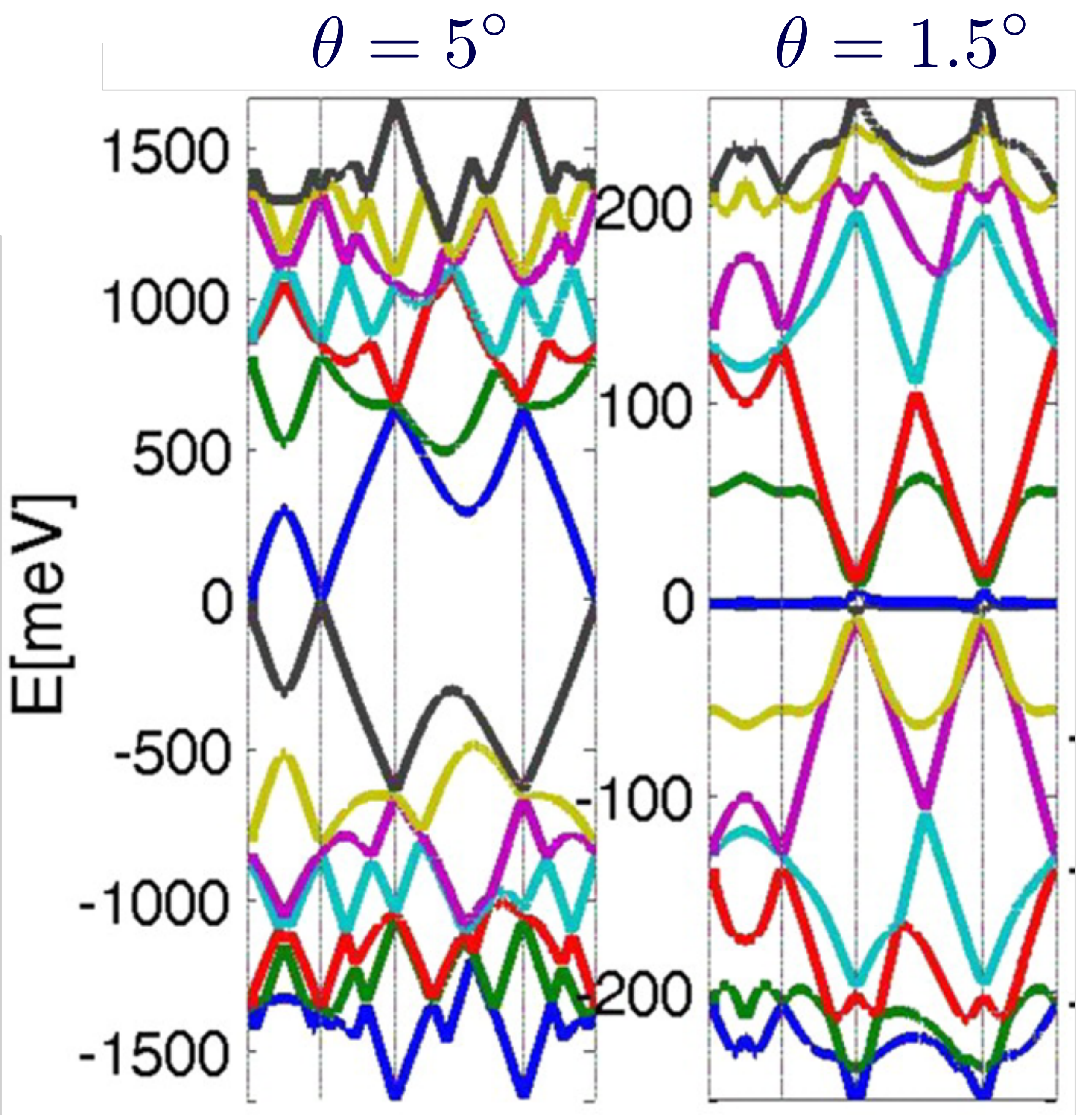
**Many possible combinations & parameters — routes to new physics!**

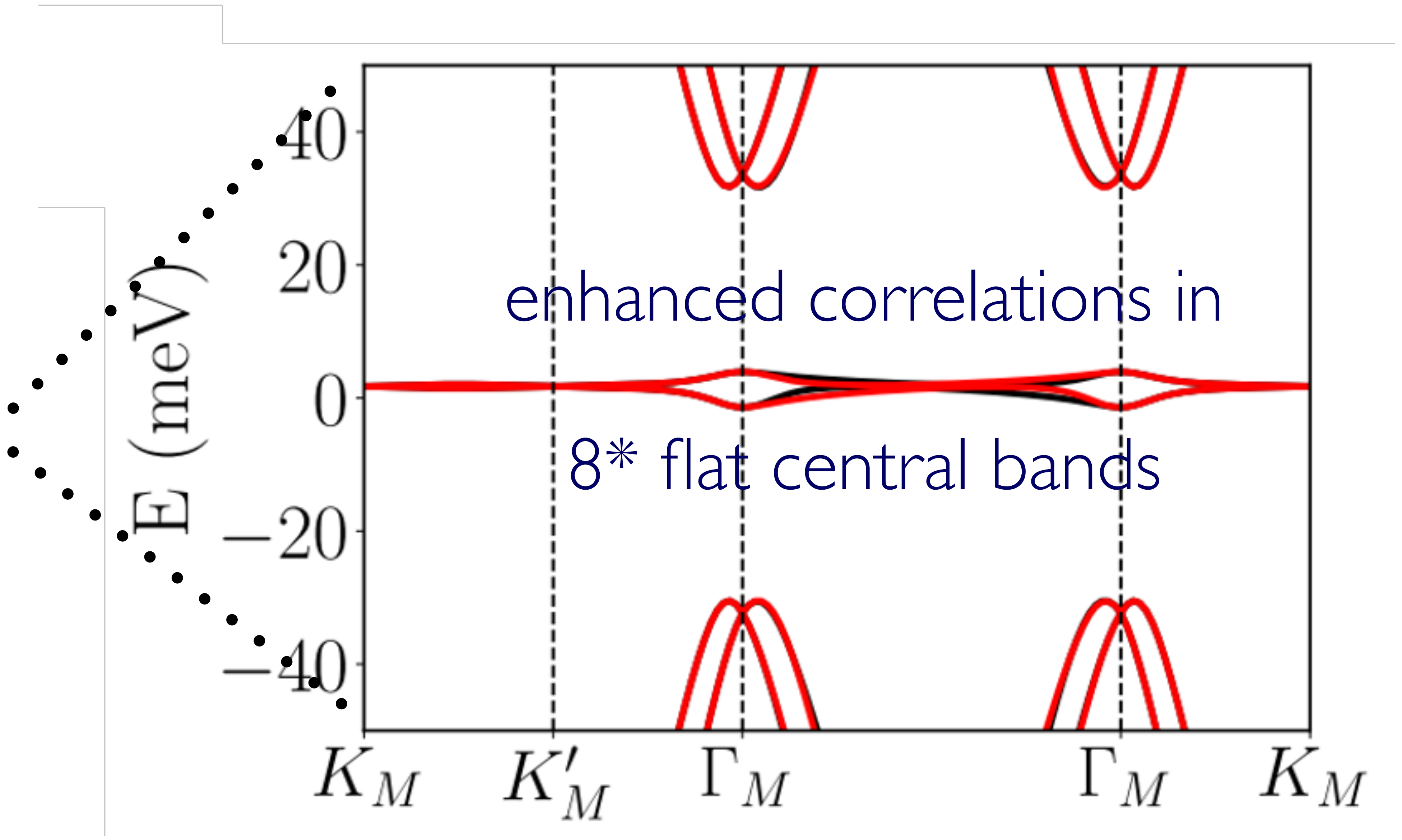
# “Hydrogen Atom” of Moiré Materials: Twisted Bilayer Graphene (TBG)

Linear “Dirac” dispersion gives special structure to twisted moiré multilayers of graphene

moiré-reconstructed TBG bands **almost perfectly flat** near “magic” twist angle $\theta \sim 1.05°$




[Bistritzer & Macdonald, PNAS’11]



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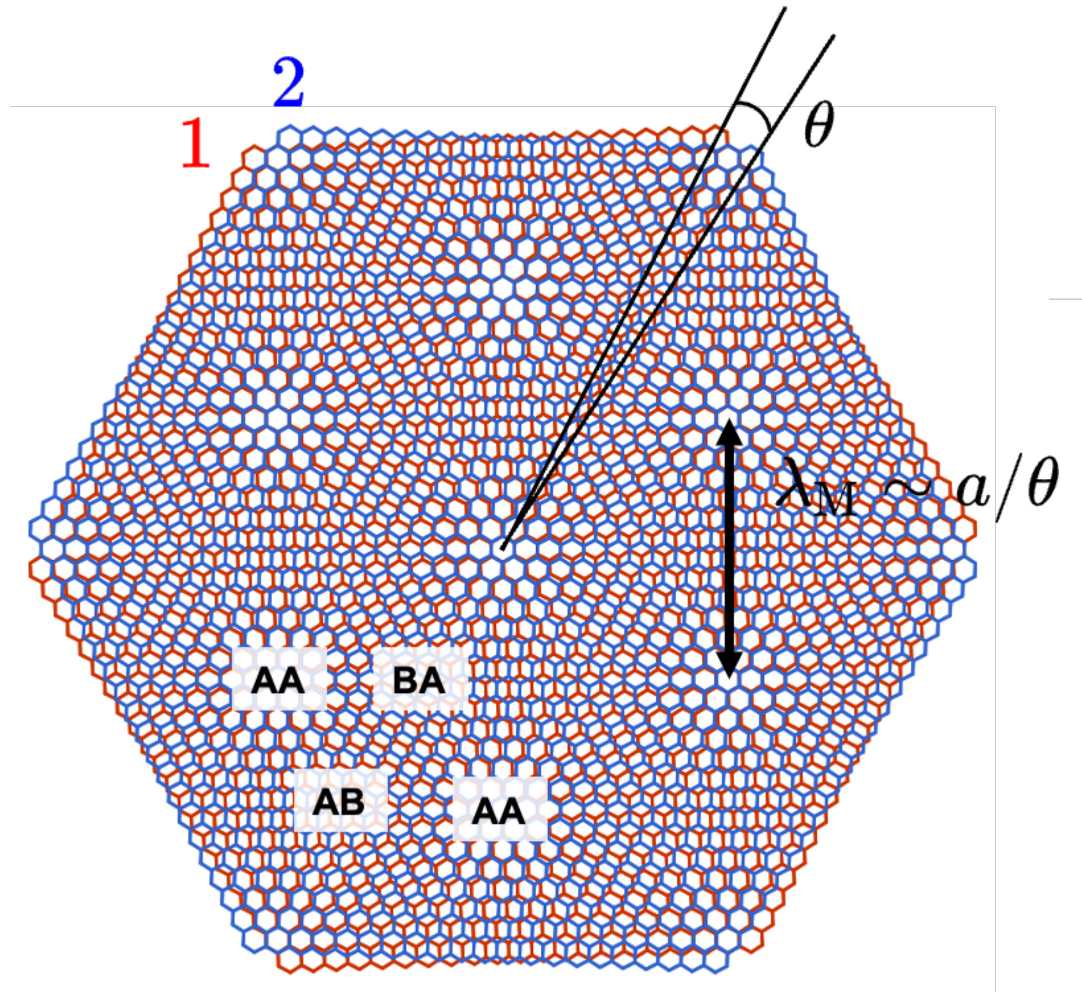


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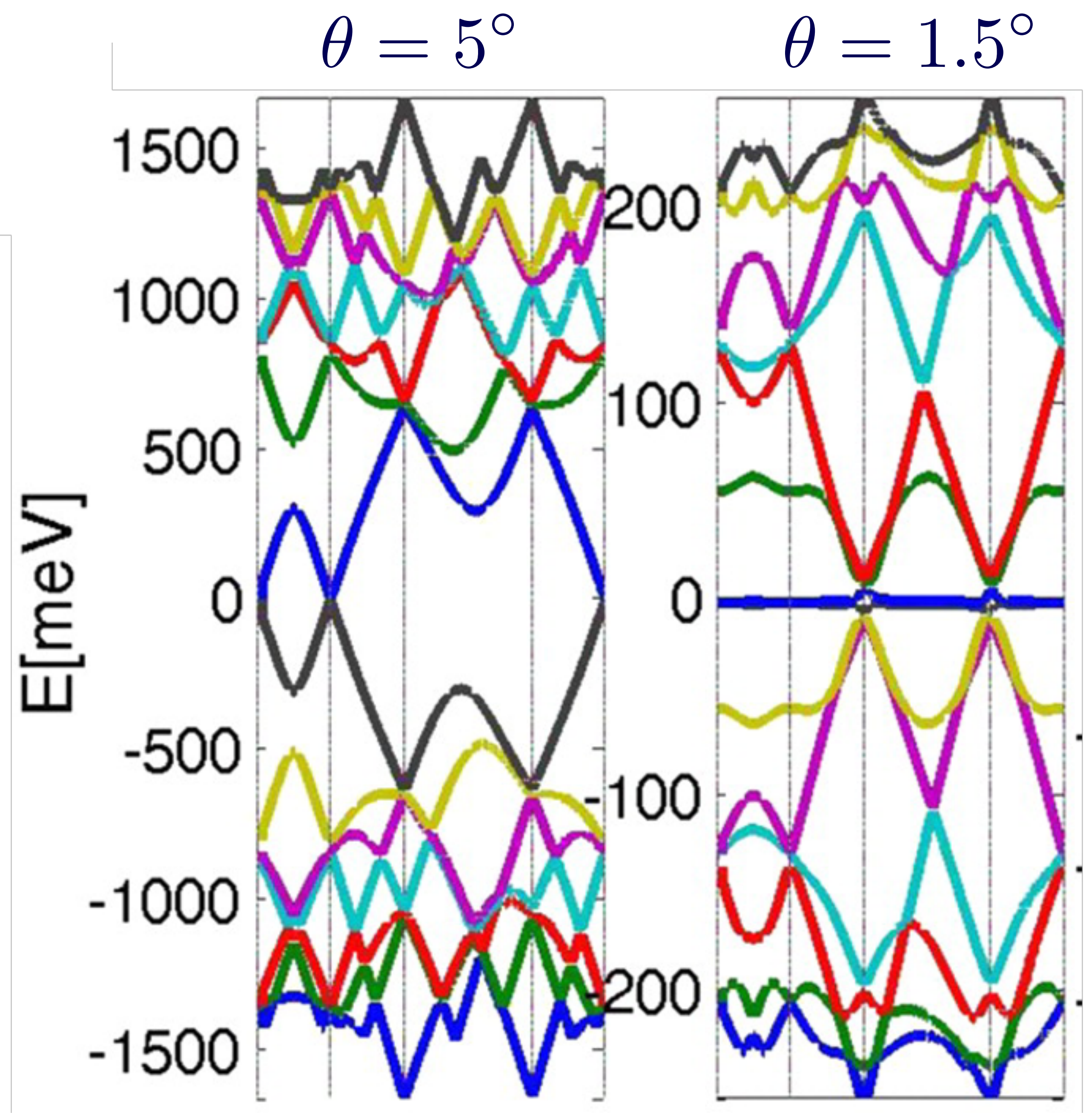


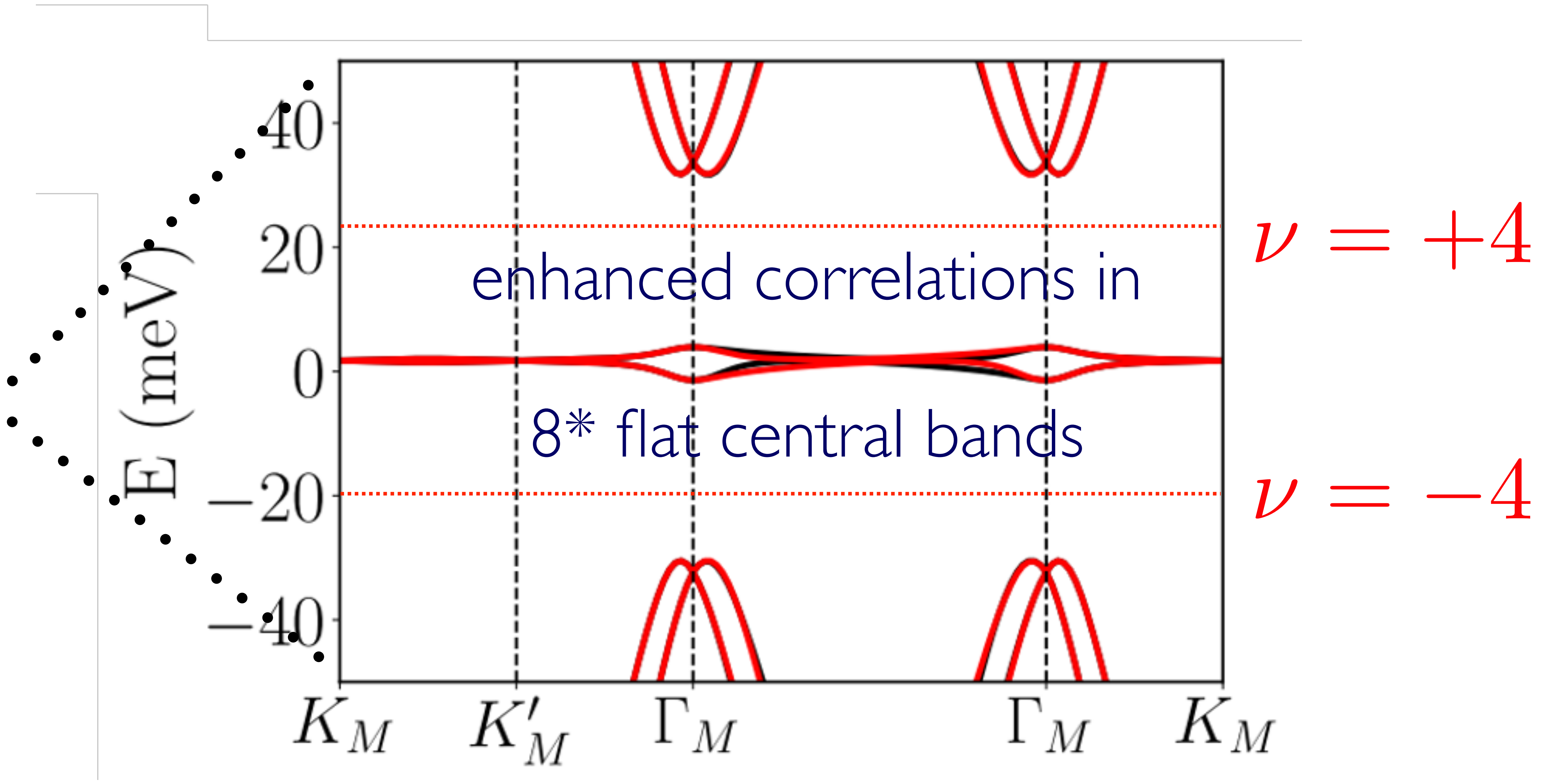




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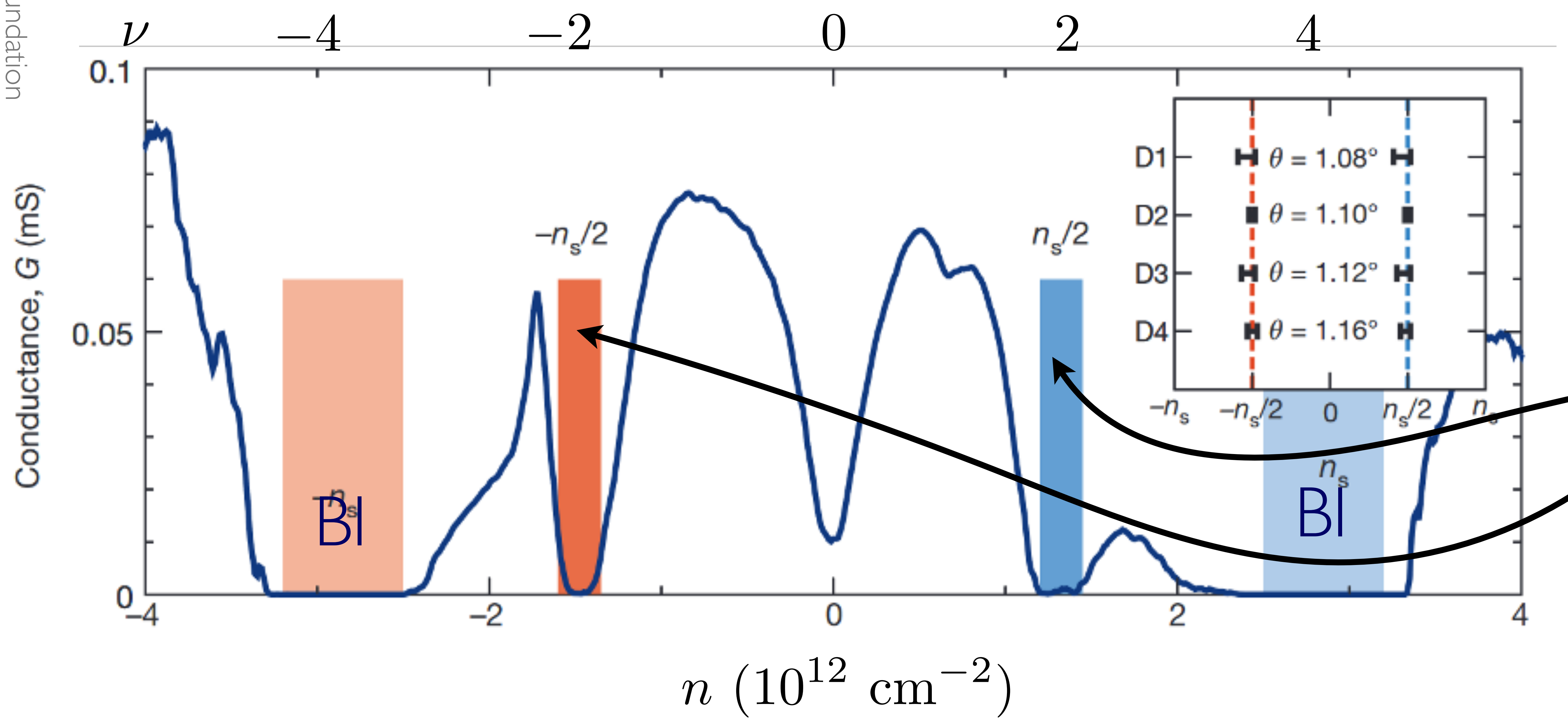
Pablo Jarillo-Hererro (MIT): experiments on magic-angle graphene bilayers

[Cao *et al* Nature '18a,b]

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(band theory predicts metal)

Many proposed explanations:
**correct one**: SP, SH Simon + Oxford students/postdocs (2021)
[experimental confirmation in 2023!]

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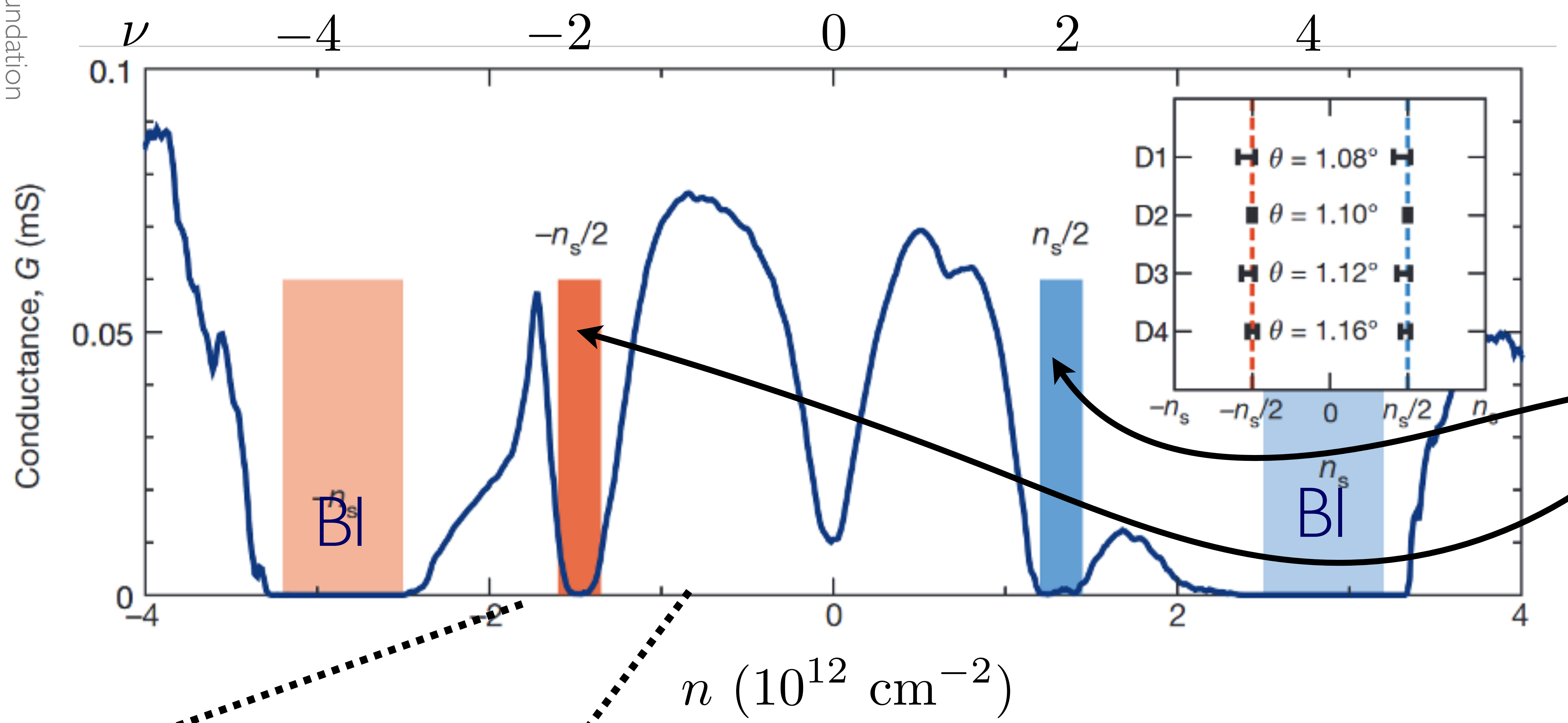


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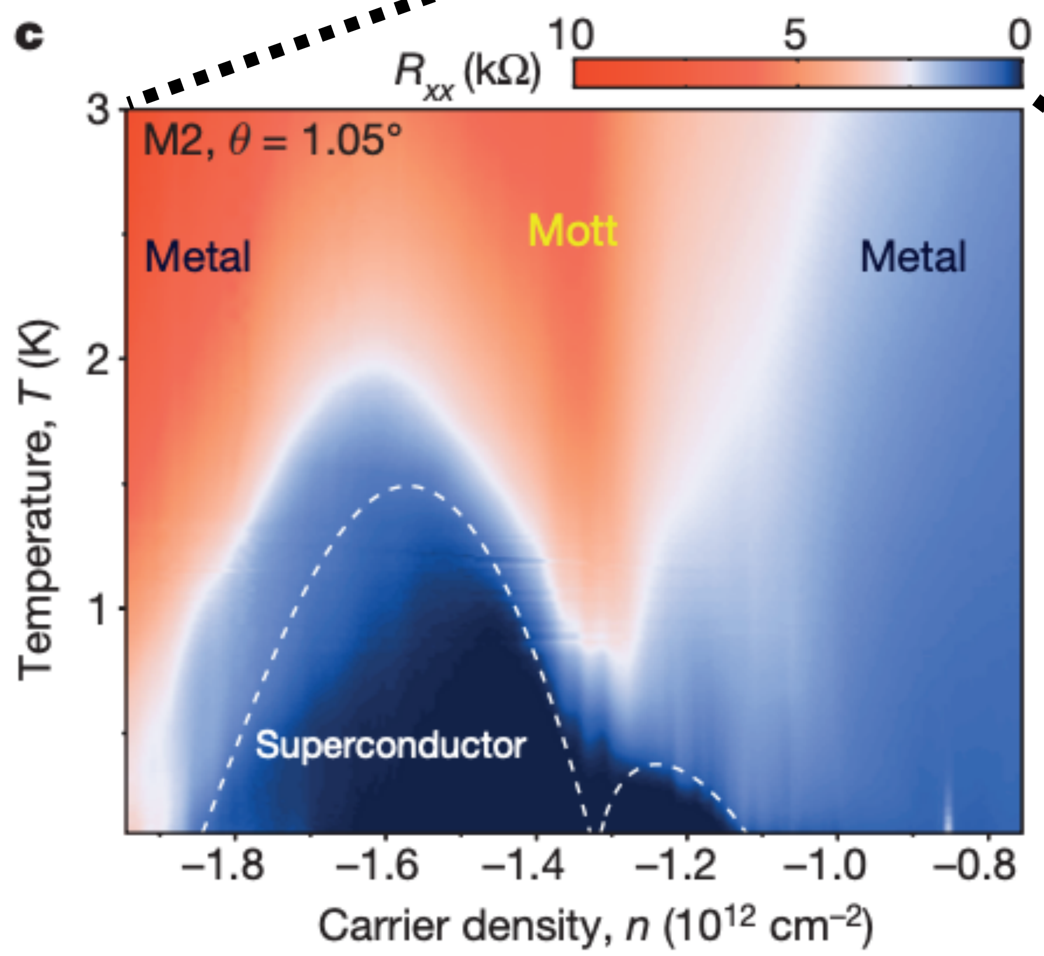


gate-tunable
superconductivity

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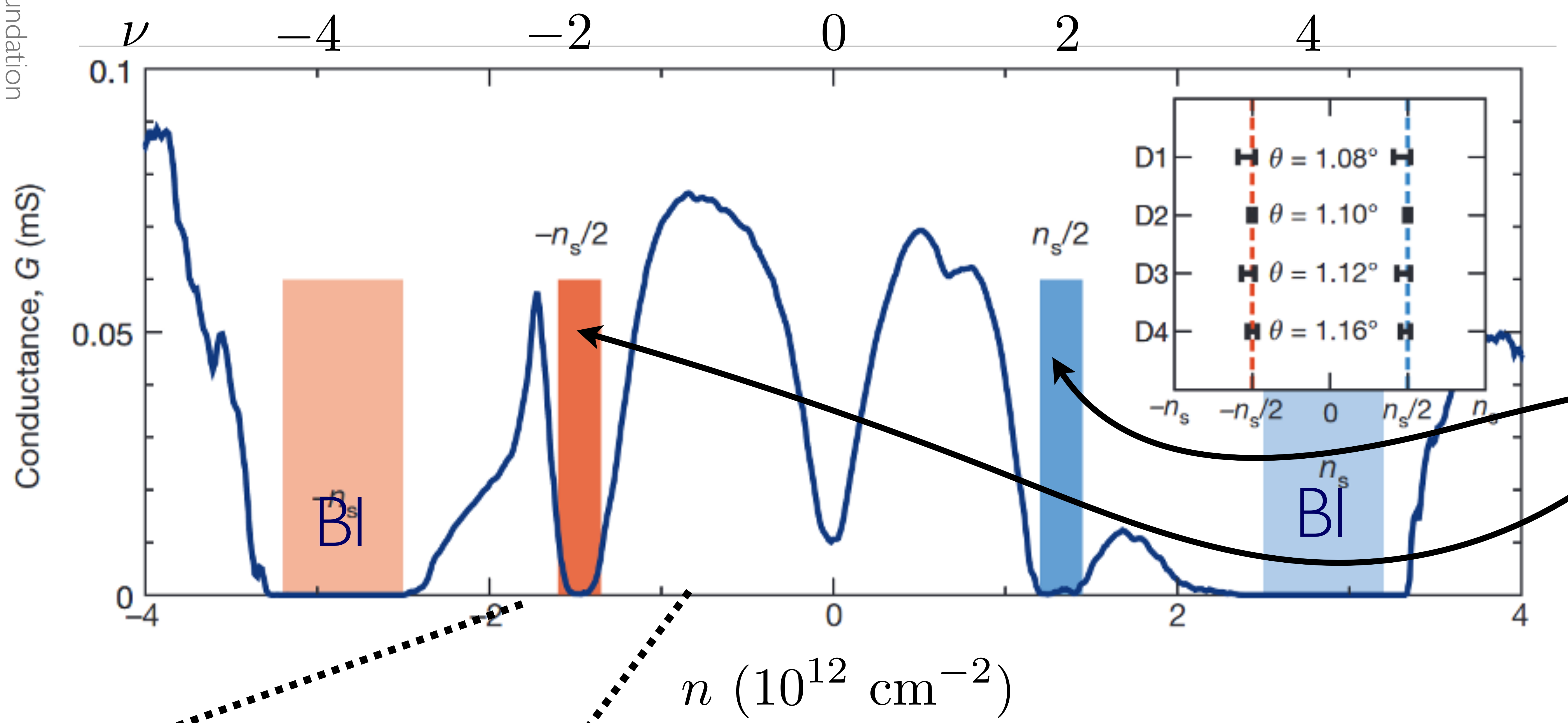
Wolf Foundation

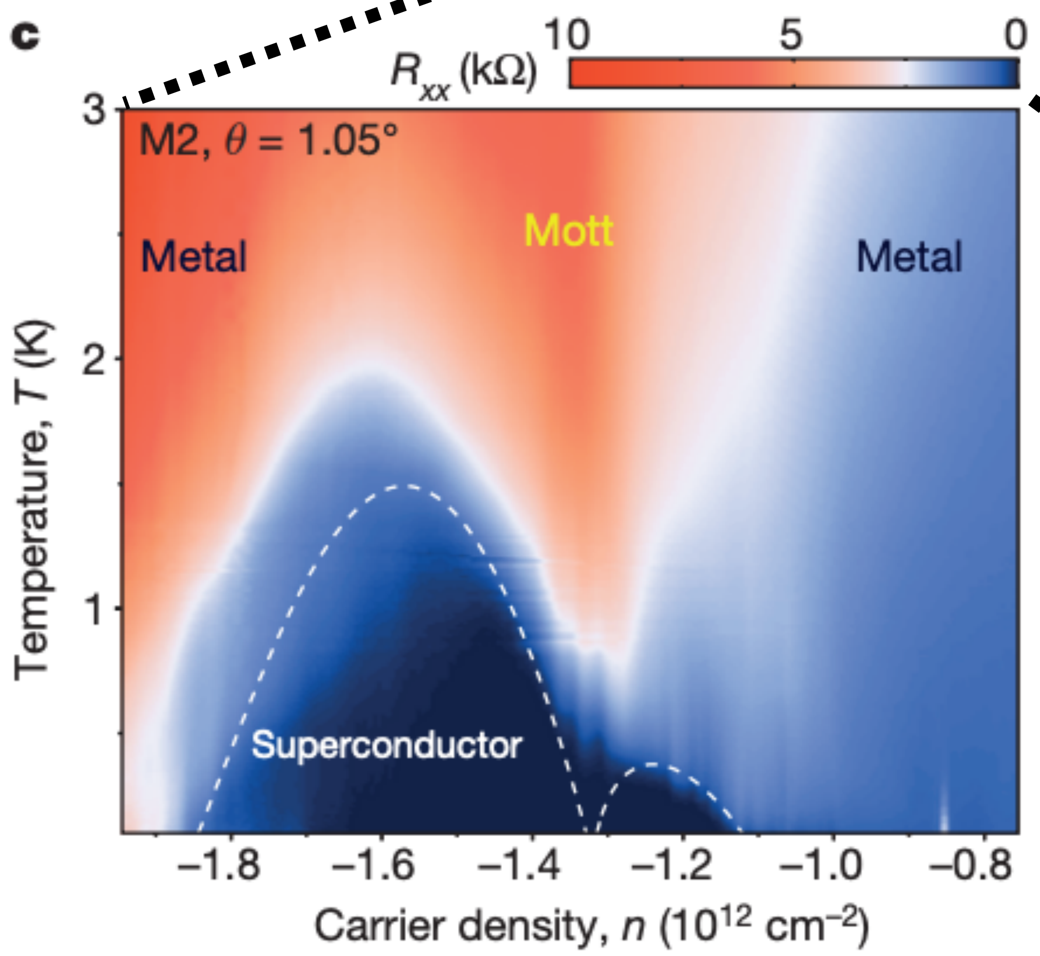
Pablo Jarillo-Hererro (MIT): experiments on magic-angle graphene bilayers

[Cao *et al* Nature ’18a,b]




“correlated insulators”
(band theory predicts metal)

Many proposed explanations:
**correct one**: SP, SH Simon + Oxford students/postdocs (2021)
[experimental confirmation in 2023!]




gate-tunable
superconductivity

**Wide range of other phenomena**:
including some hints of topology…
but difficult to get topological states reliably

So we found a nice system with correlations and tunability…

but only tantalizing hints of topology

Can we do better?

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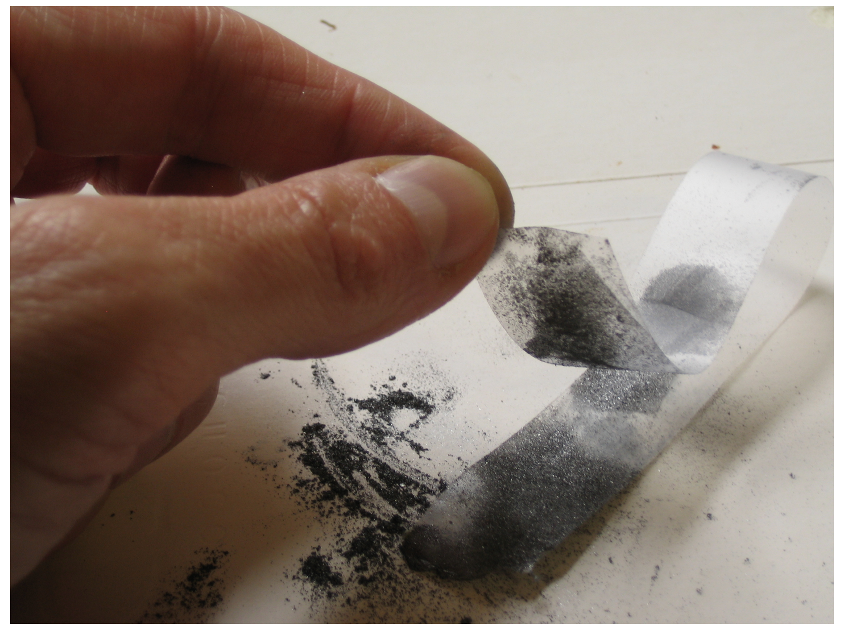

lostinscience.wordpress.com/

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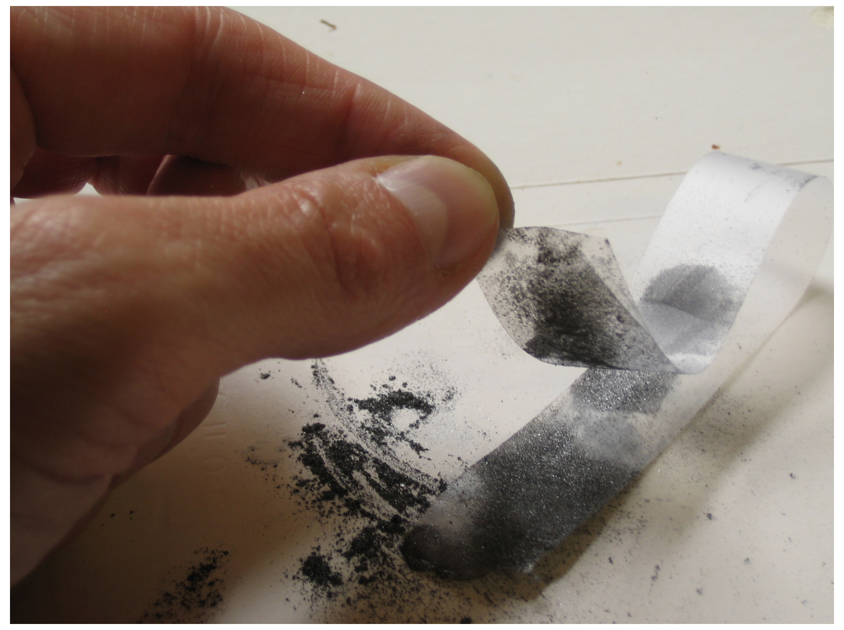

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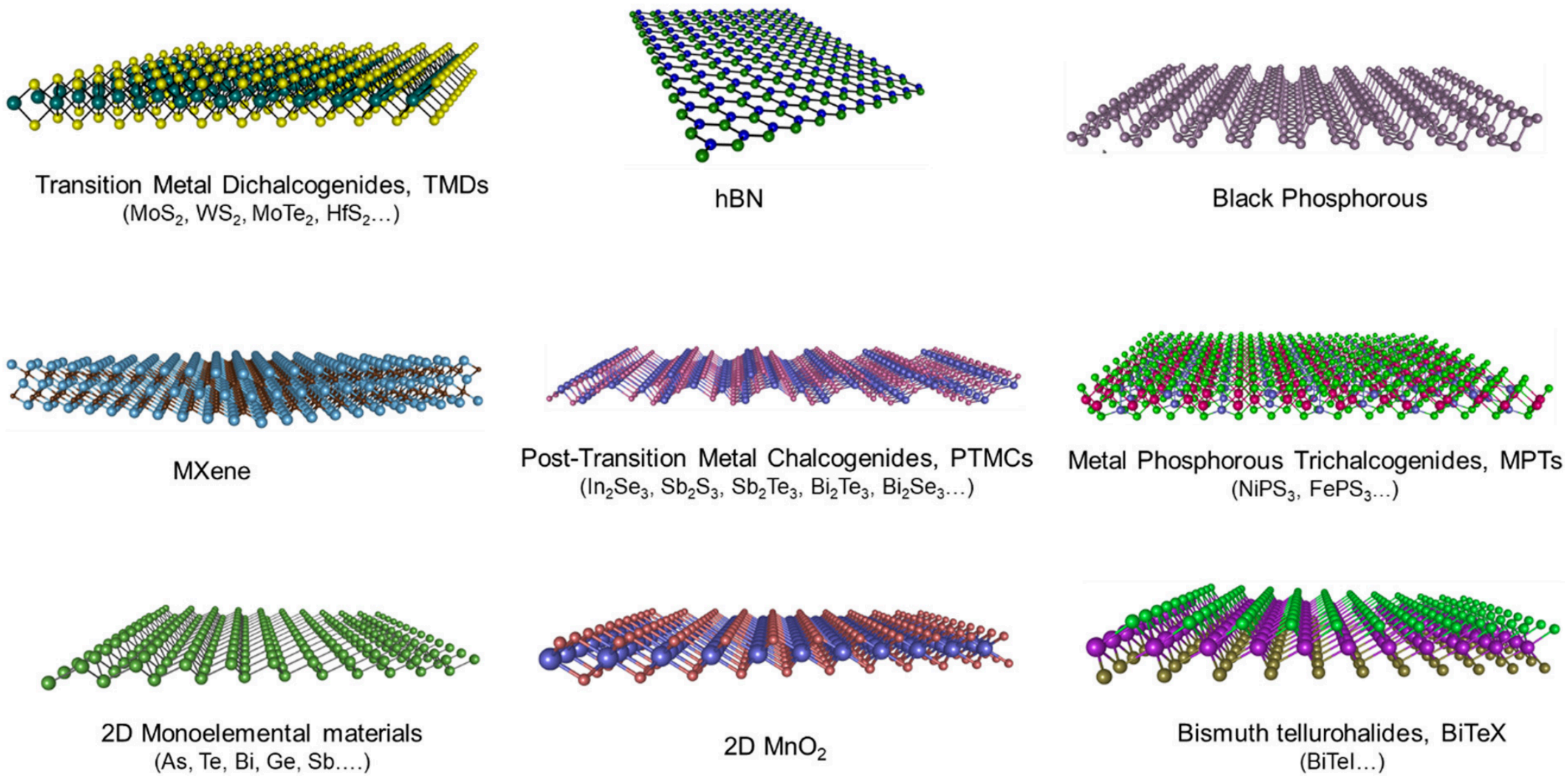


[Zhao *et al* Chem. Soc. Rev. '24]

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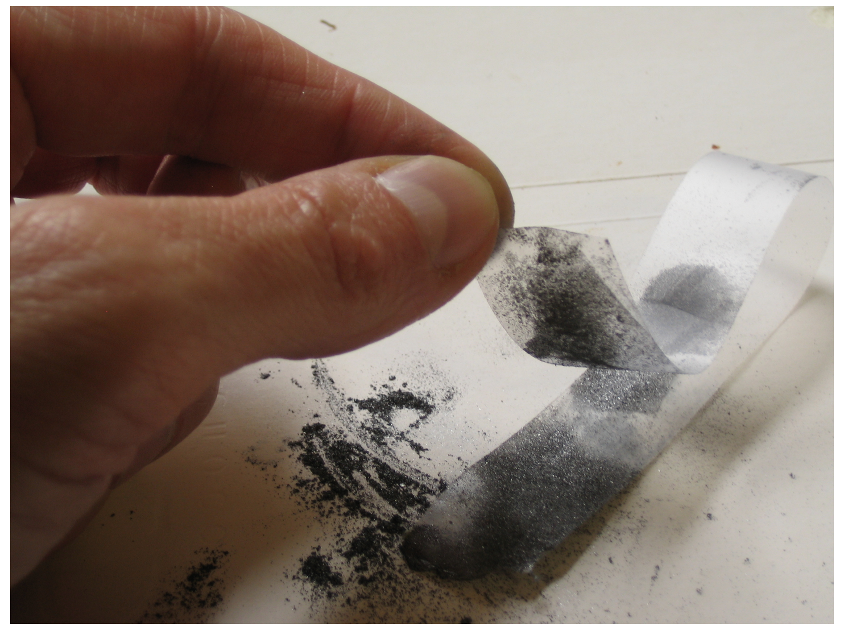

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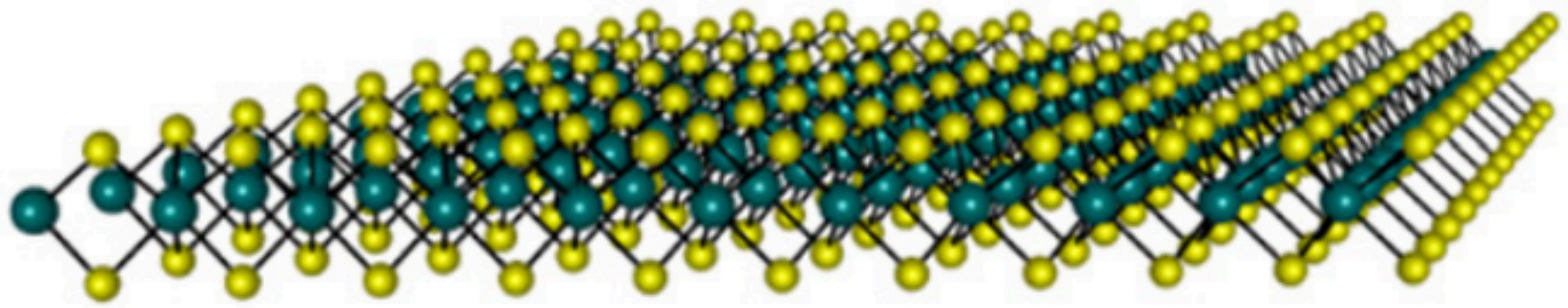
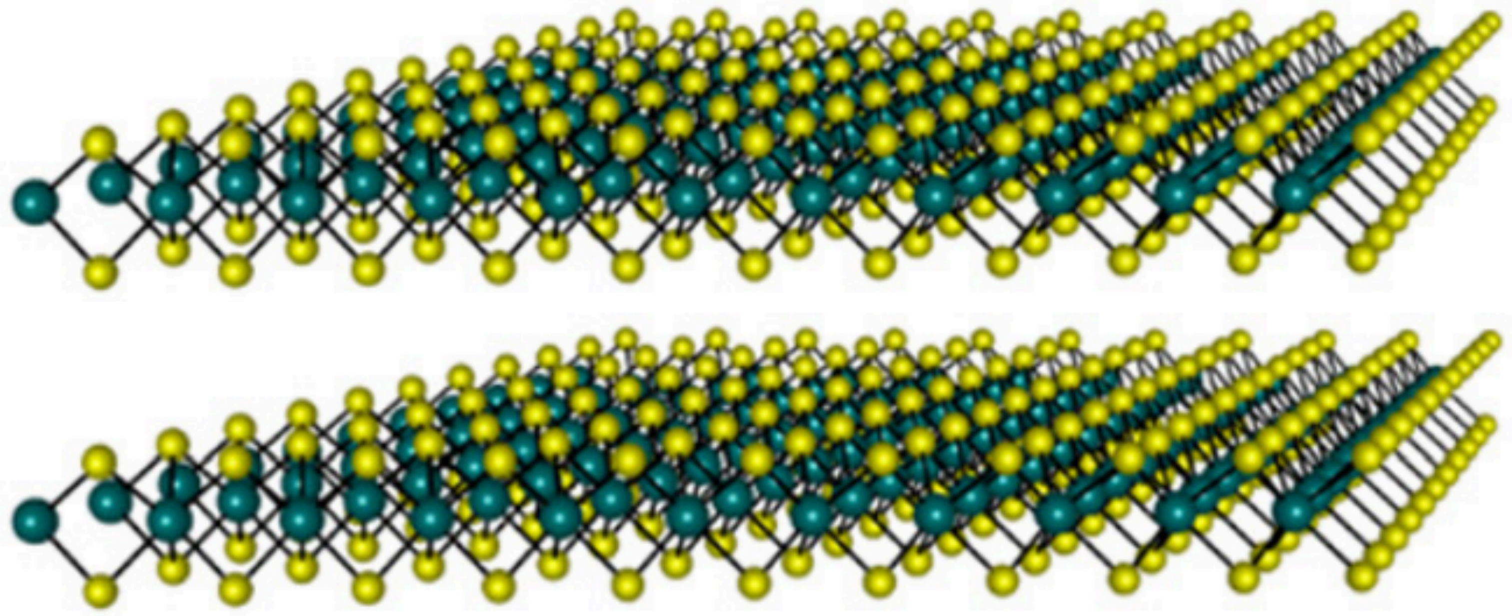


Transition Metal Dichalcogenides, TMDs
($MoS_2$, $WS_2$, $MoTe_2$, $HfS_2$…)

Focus on one family in particular:
"twisted TMDs"

# Topology in Twisted TMD Bilayers



 = Transition Metal (Mo, W, Hf,…)

 = Chalcogen (S, Se, Te,…)

Parabolic dispersion per layer + complex interlayer tunneling amplitude/layer potential $\boldsymbol{\Delta}(\boldsymbol{r})$

$$H = \frac{\hbar^2 \boldsymbol{k}^2}{2m^*} + \boldsymbol{\Delta}(\boldsymbol{r}) \cdot \boldsymbol{\sigma}$$ where $\boldsymbol{\sigma}$ = layer "pseudospin"

[**Wu et al '19;** Pan et al '20; Devakul et al '21; Morales-Duran et al '24]

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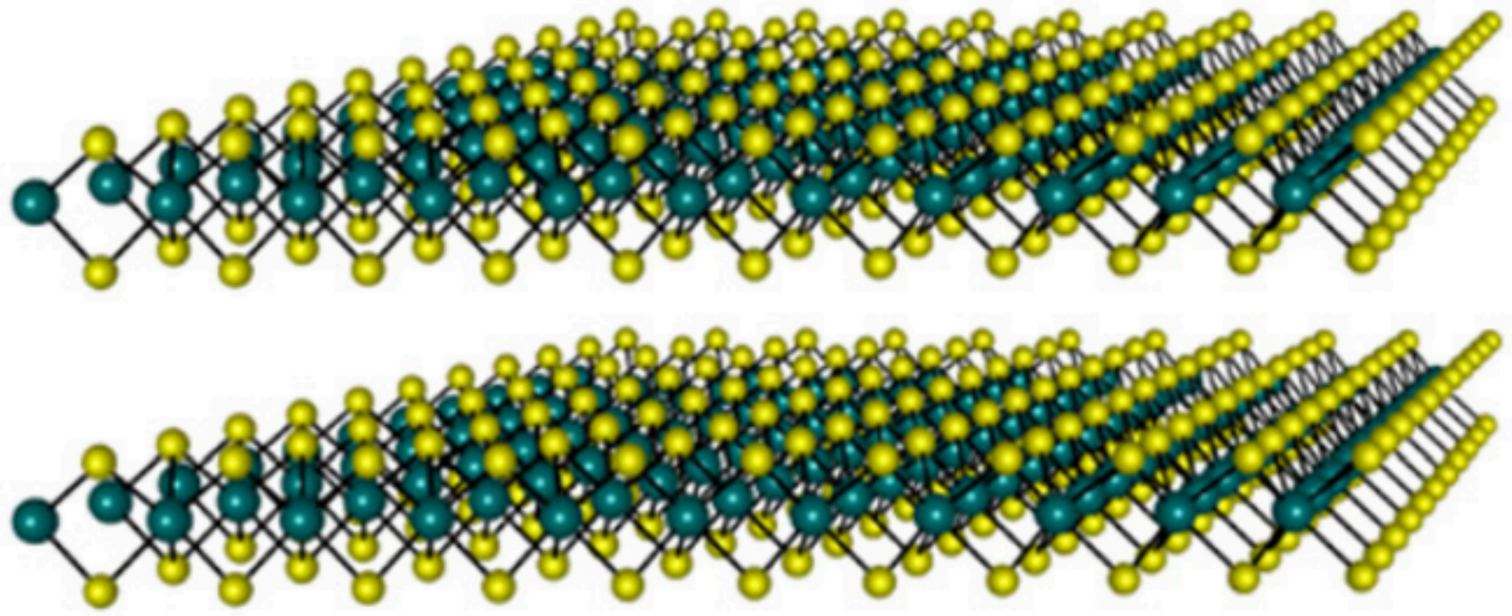

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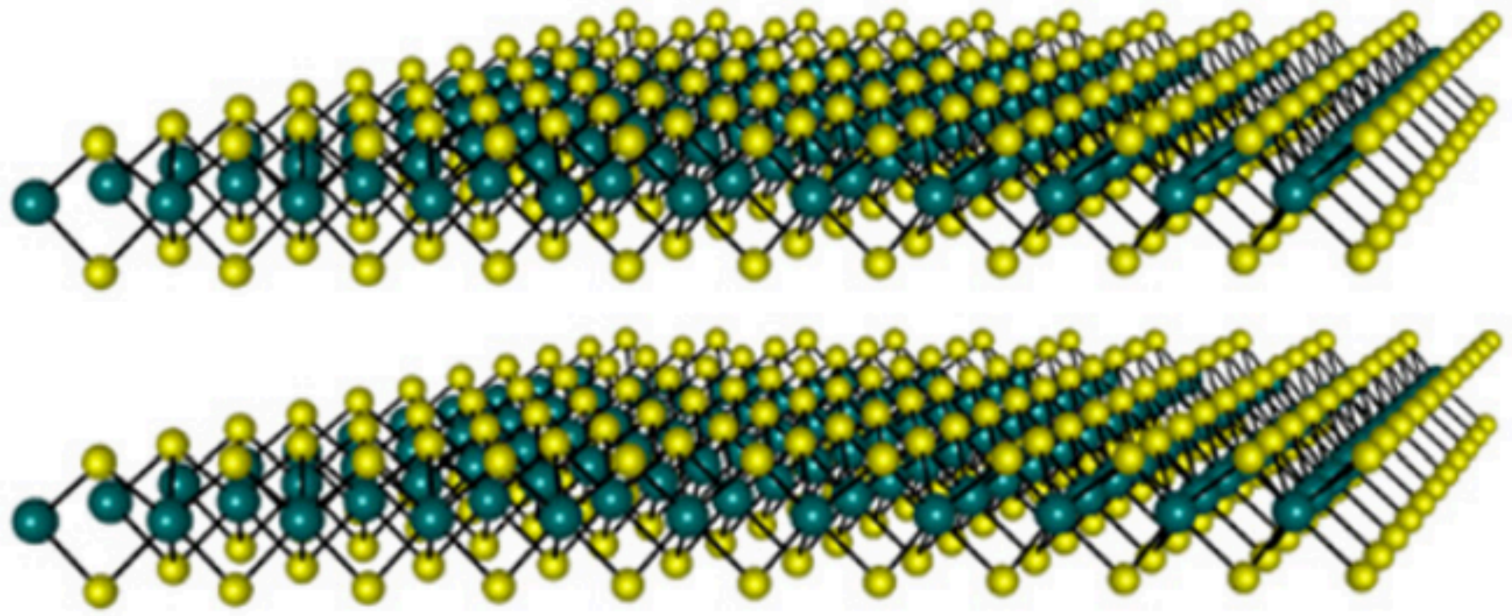

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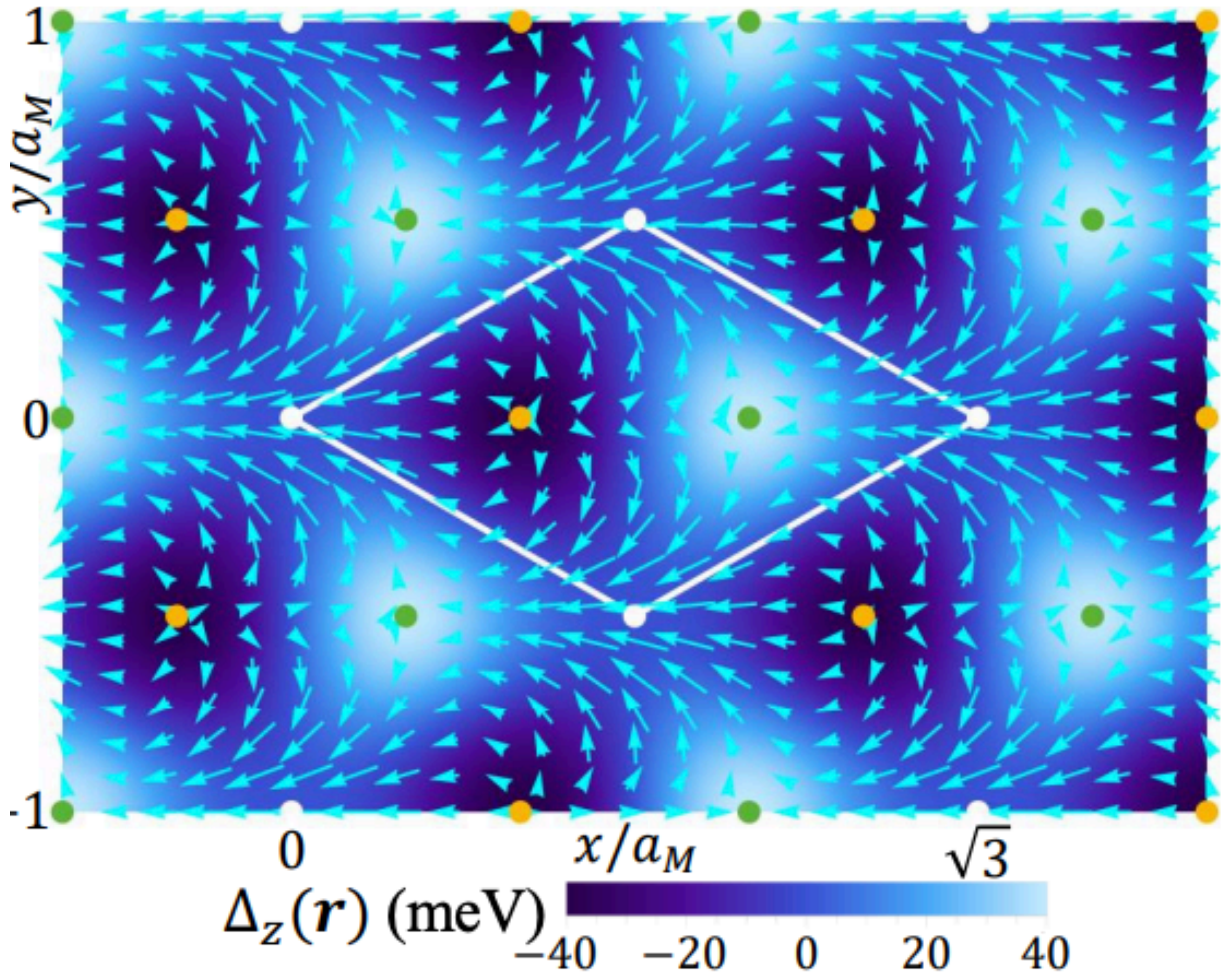
Near special "magic" angles $\boldsymbol{\Delta}(\boldsymbol{r})$ forms a "skyrmion lattice"

What are the consequences for electrons?

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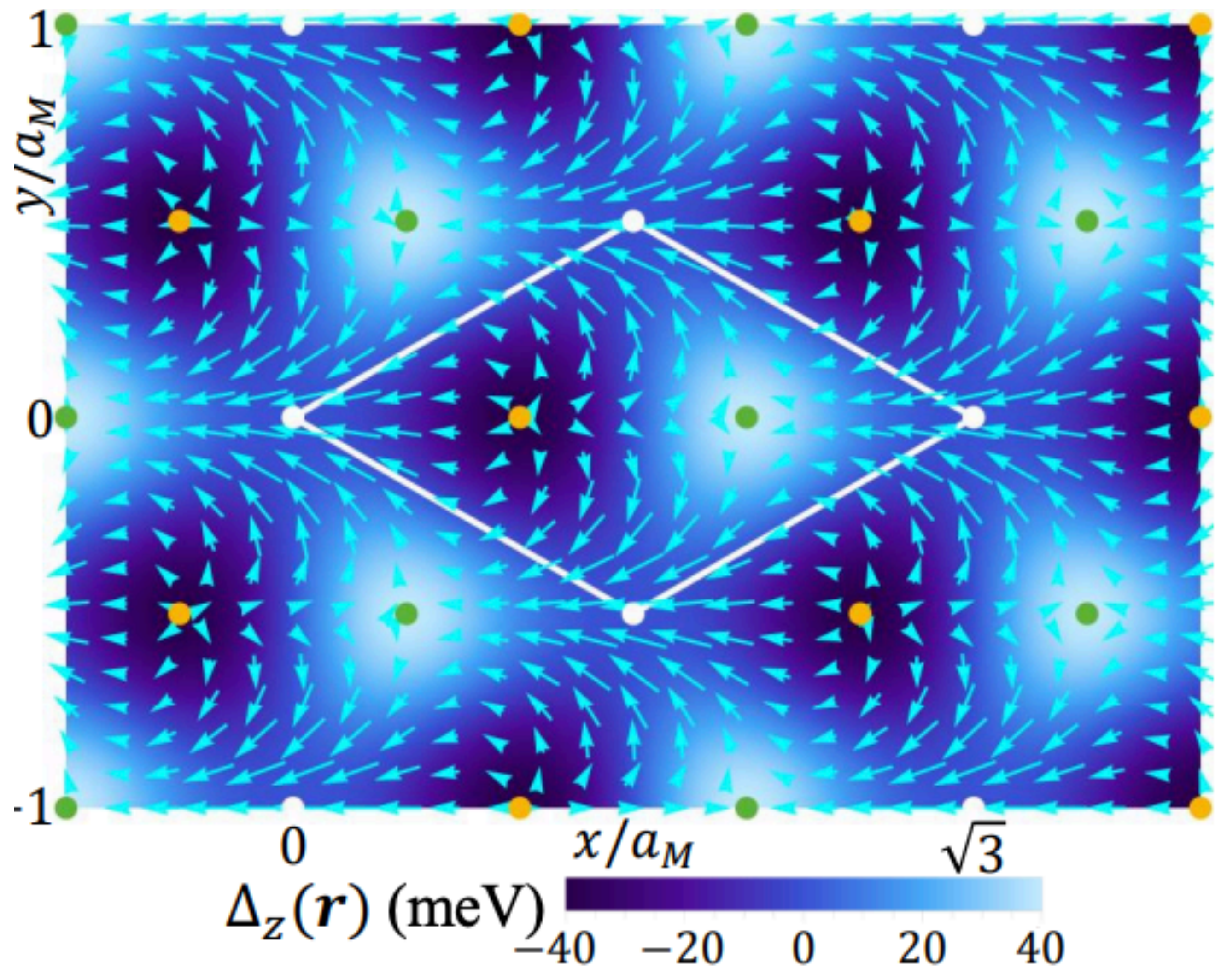


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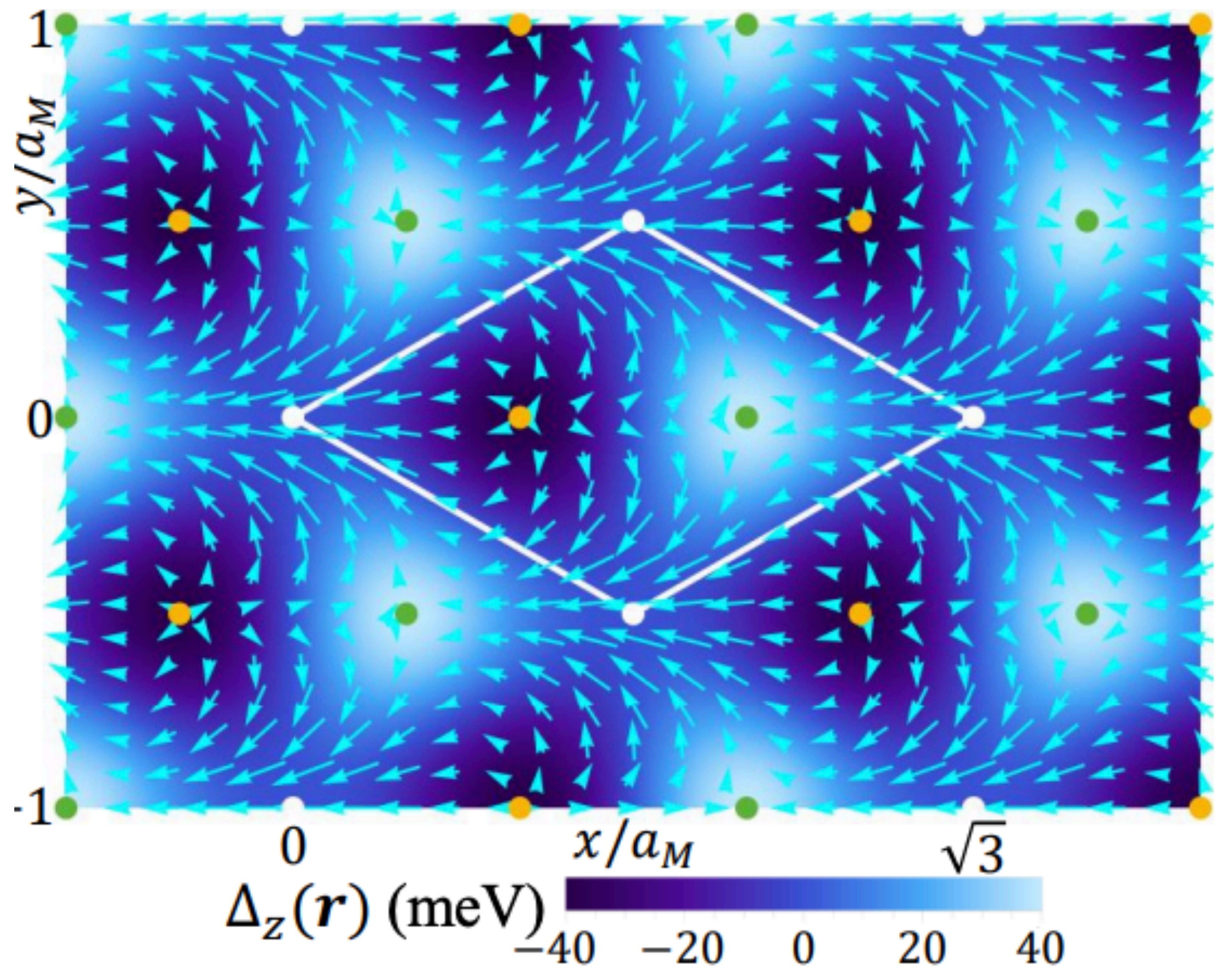


Change the local axes so $\boldsymbol{\sigma} \parallel \boldsymbol{\Delta}(\boldsymbol{r})$

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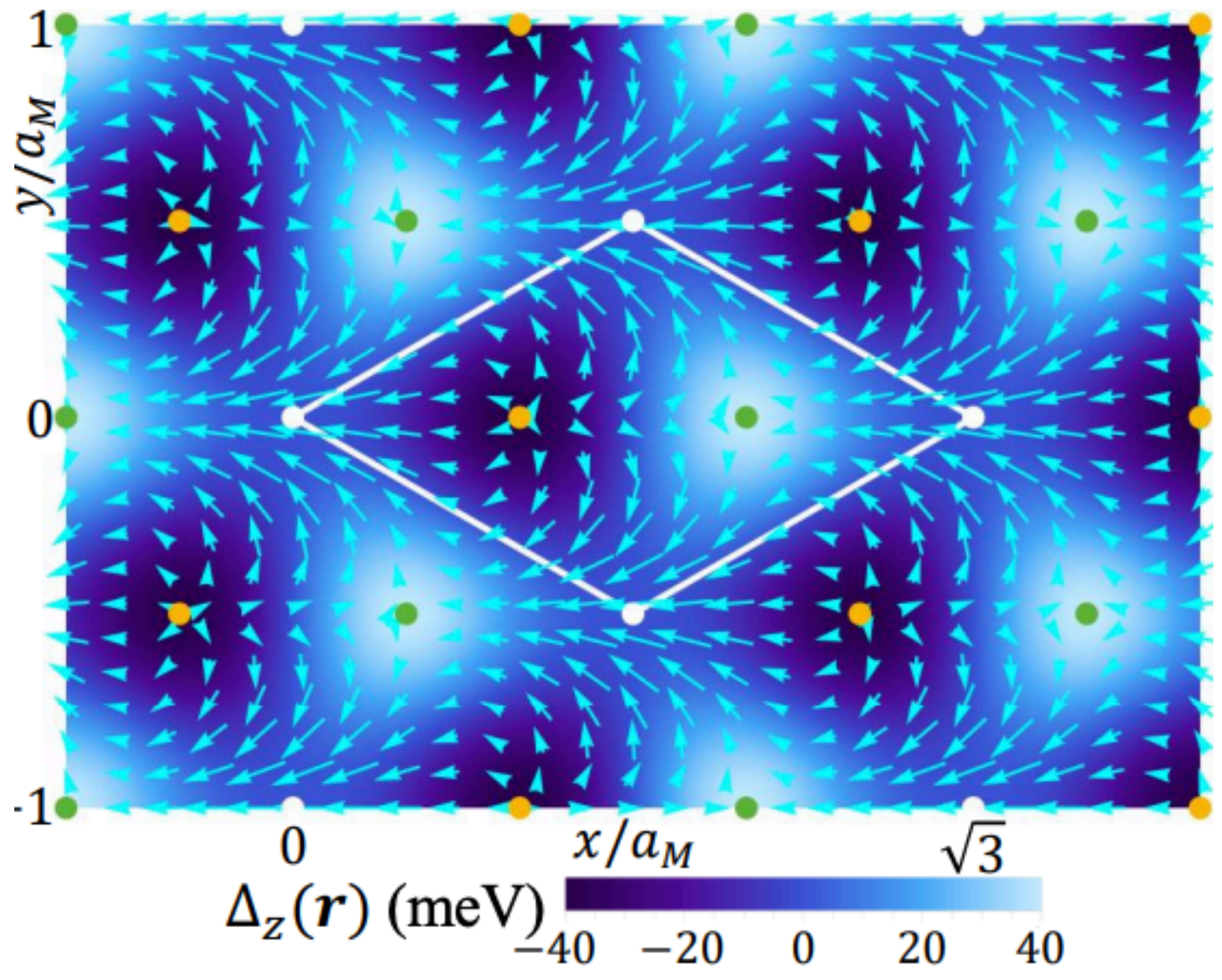


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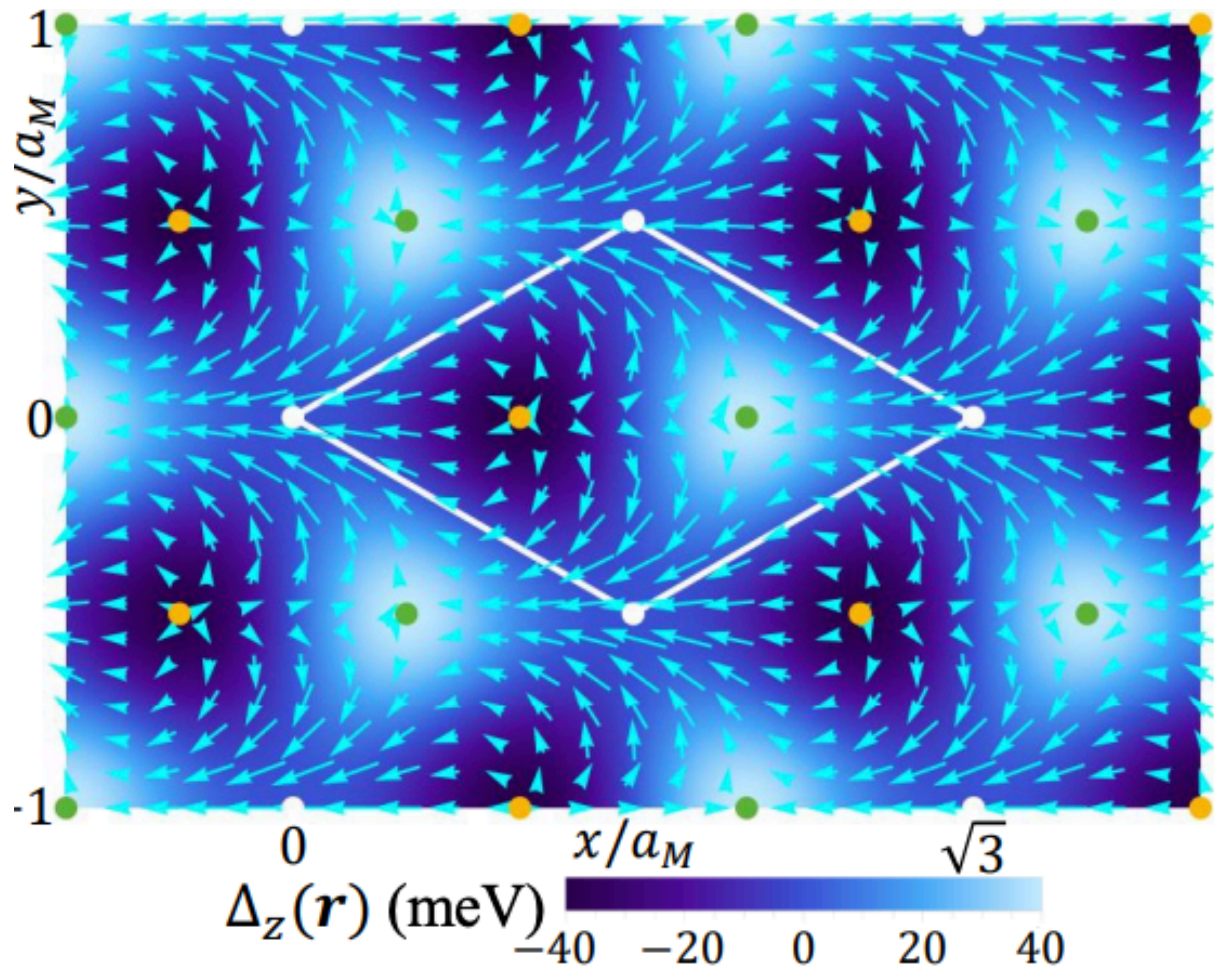


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If $\boldsymbol{\Delta}(\boldsymbol{r})$ has 1 skyrmion per unit cell, $\boldsymbol{B}(\boldsymbol{r}) = \nabla \times \boldsymbol{A}(\boldsymbol{r})$ has 1 flux quantum per unit cell

**This gives a $C = 1$ Chern band which is pretty flat!**
($\approx$ Landau level in weak periodic potential!)

[**Wu et al '19;** Pan et al '20; Devakul et al '21; Morales-Duran et al '24]

# Checking off our Wishlist

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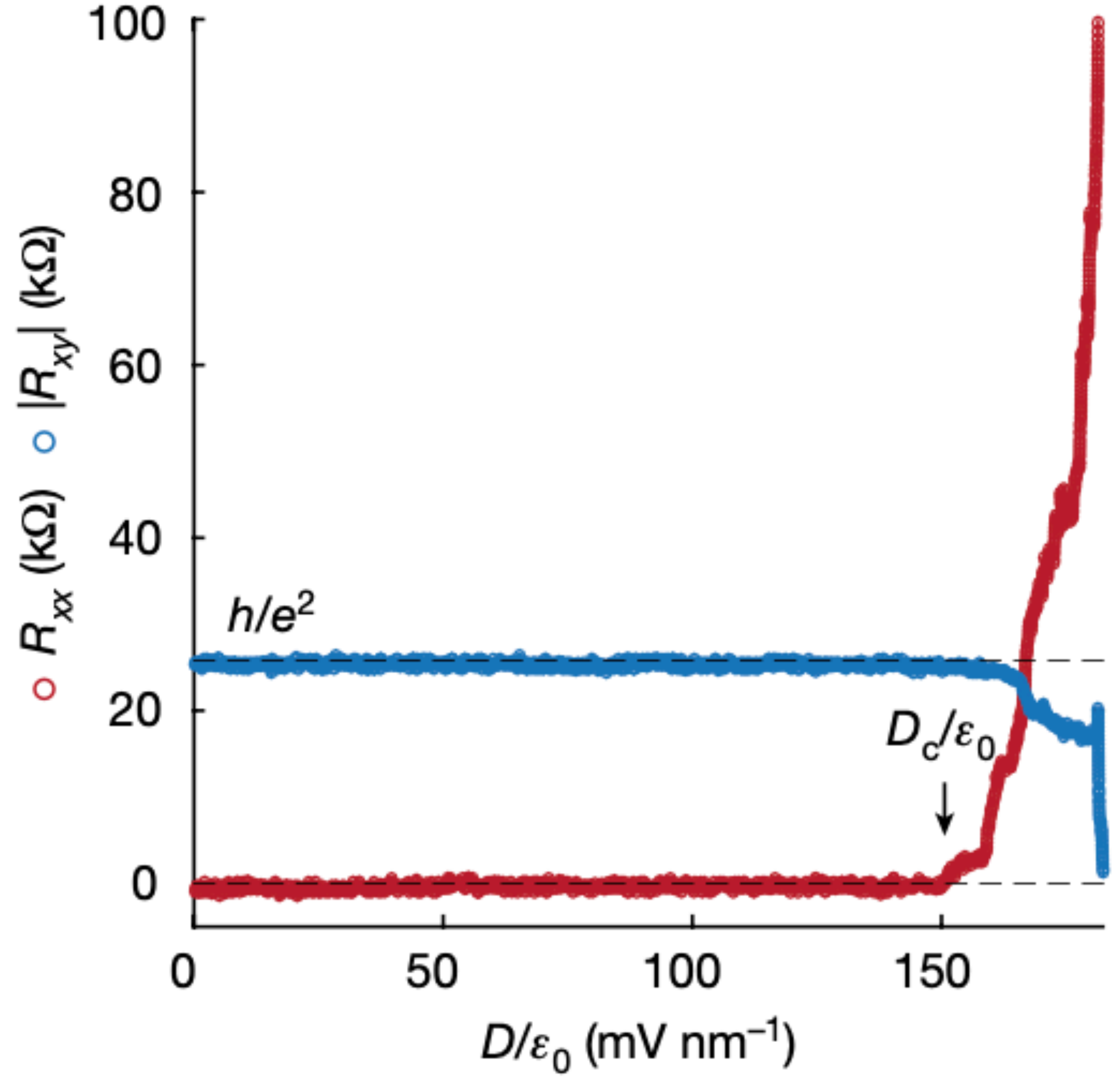
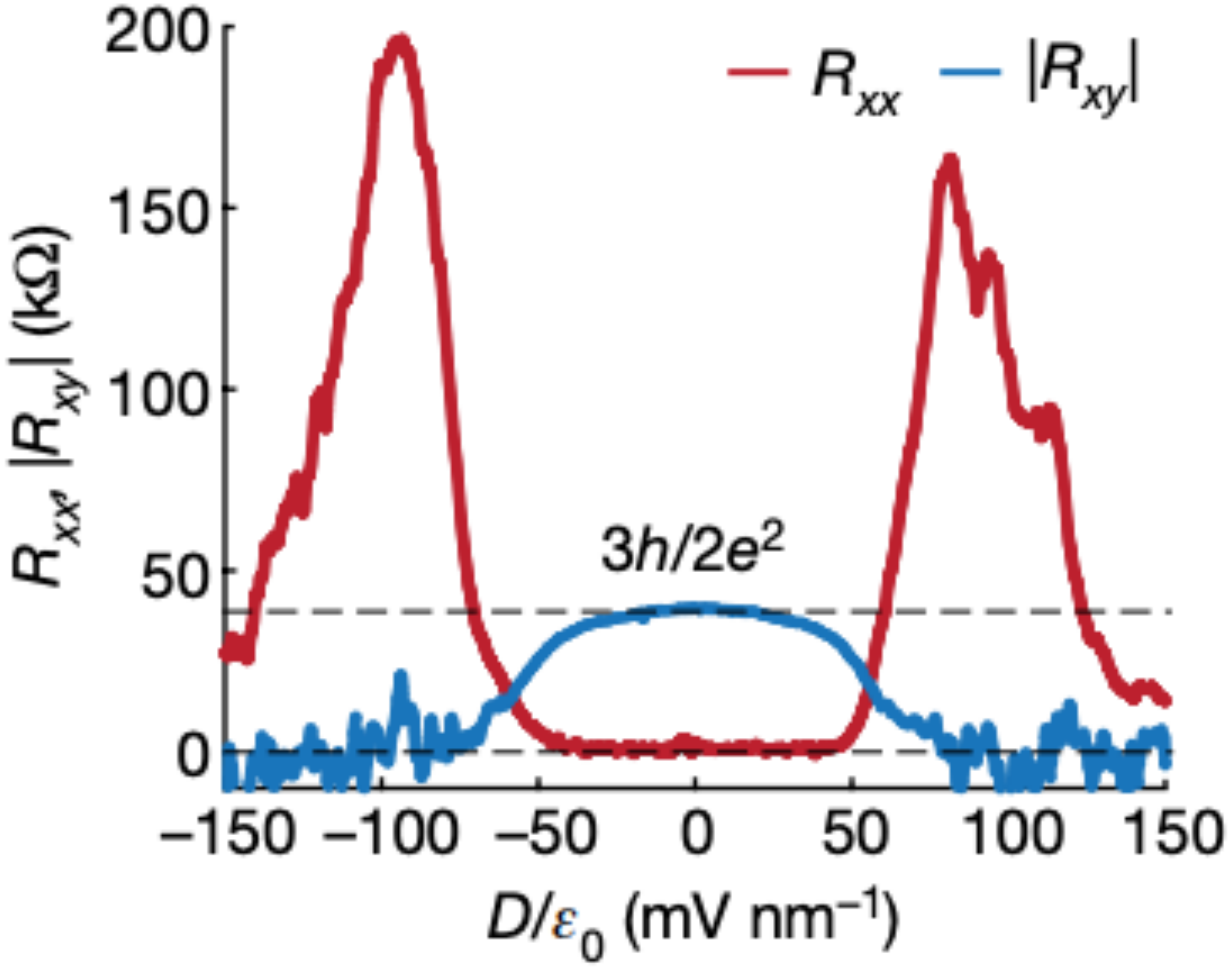
- Lattice spacing $a \sim 0.6\,\mathrm{nm} \Rightarrow$ moiré length $a_M \sim 10\,\mathrm{nm}$ ✔ tunability

  Bonus: interlayer potential ($D \sim \Delta_z$) tunes skyrmion number
  (can use it to switch from $C = 1$ to $C = 0$ as a test)

# Experiments!

Transport experiments* on $tMoTe_2$ at the appropriate densities shows quantization of the Hall response at integer and fractional values, switchable by tuning the interlayer potential!



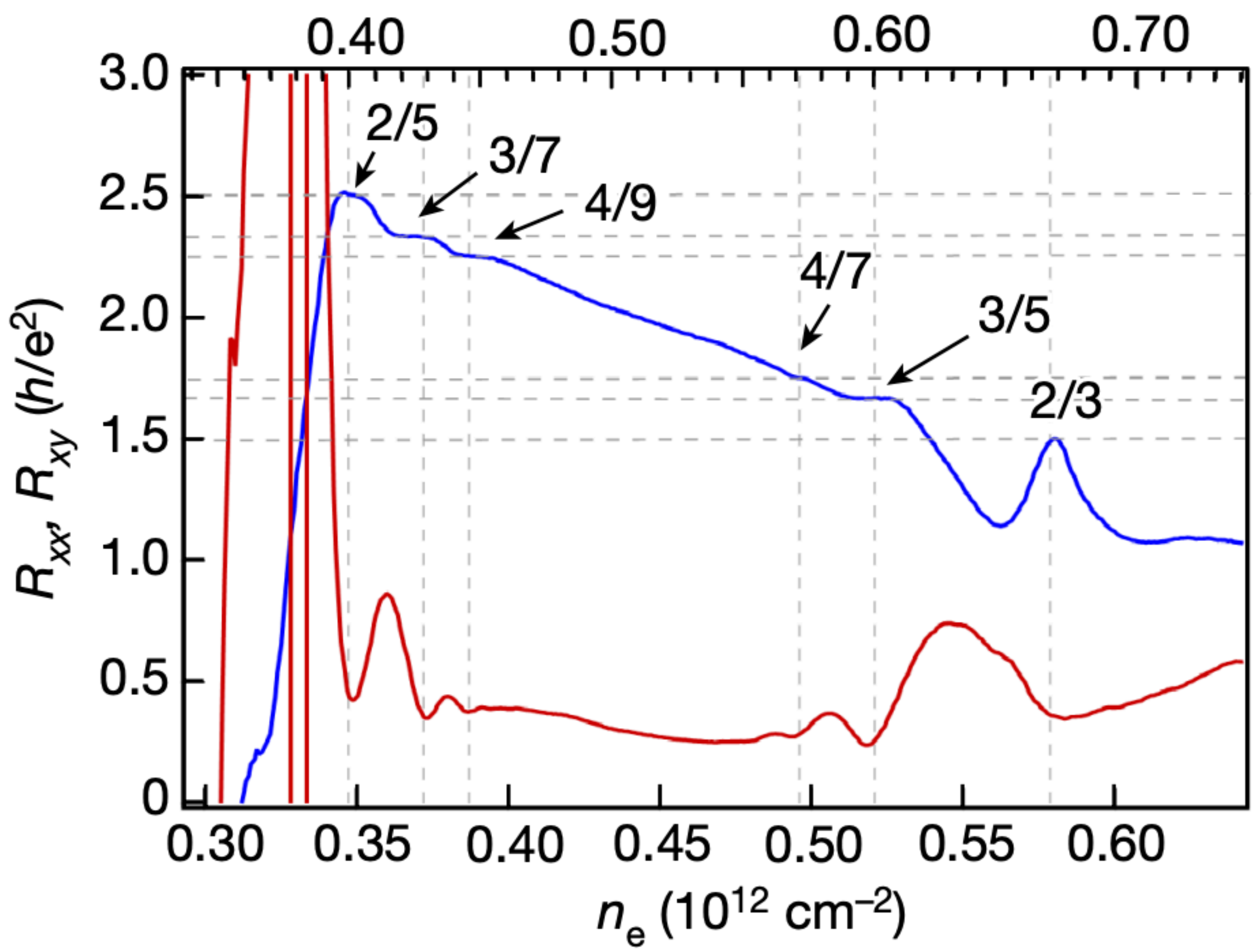

Active search for "fractional topological insulators", non-Abelian anyons, …

*first experiments were indirect and involved optics — more in a moment

[Park et al Nature '23 ]

# New Surprises!

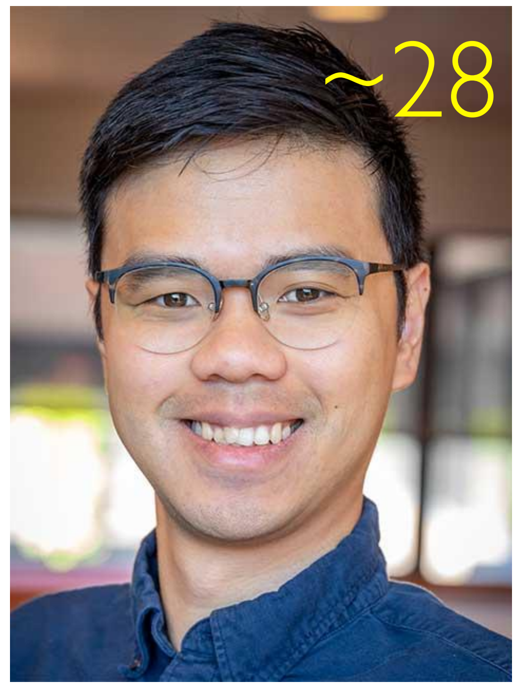
rhombohedral 5-layer graphene on hBN — second FCI system, a few months after $tMoTe_2$




[Lu et al Nature '24 ]

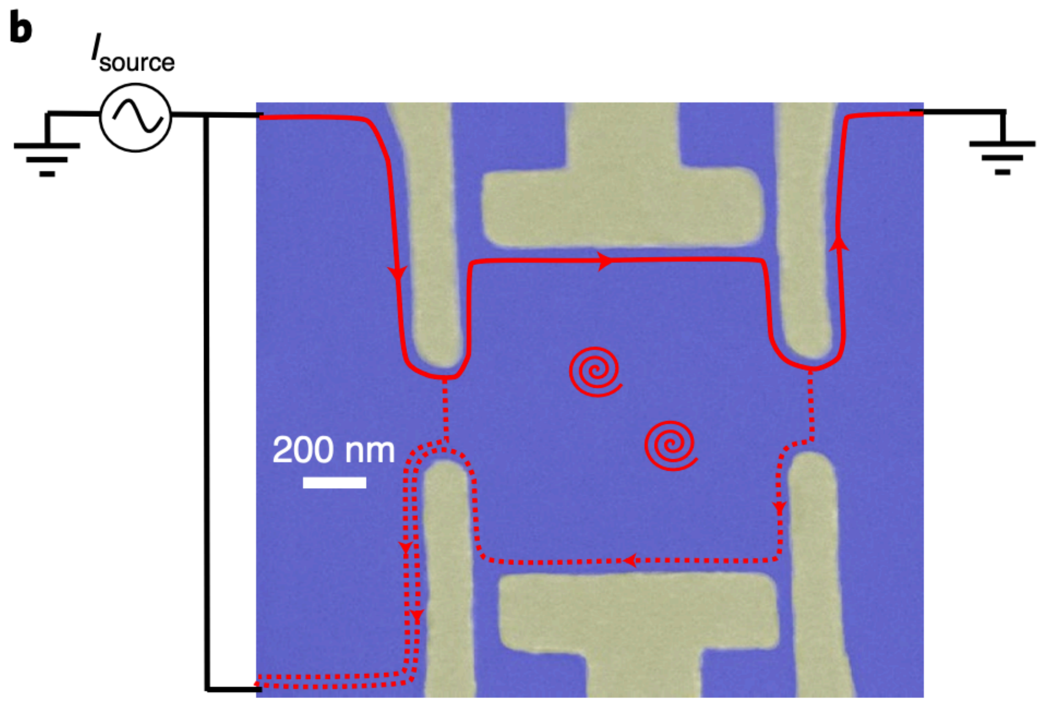
- Rich physics but poorly understood
- Moiré effect is much weaker and seems to have a different role — stabilizing a state where interactions make electrons *spontaneously* crystallize into a Chern insulator, rather than simply filling existing Chern bands!

**Key Player:** Yves Kwan
(Oxford DPhil '22→Princeton Postdoc)



# The Future: "Seeing" Fractional Statistics?

Signature of FQHE: anyonic statistics
(intermediate between bosons/fermions) [Arovas et al PRL '84]




In Landau levels— "edge state interferometry" (cf Steve Simon's talk)

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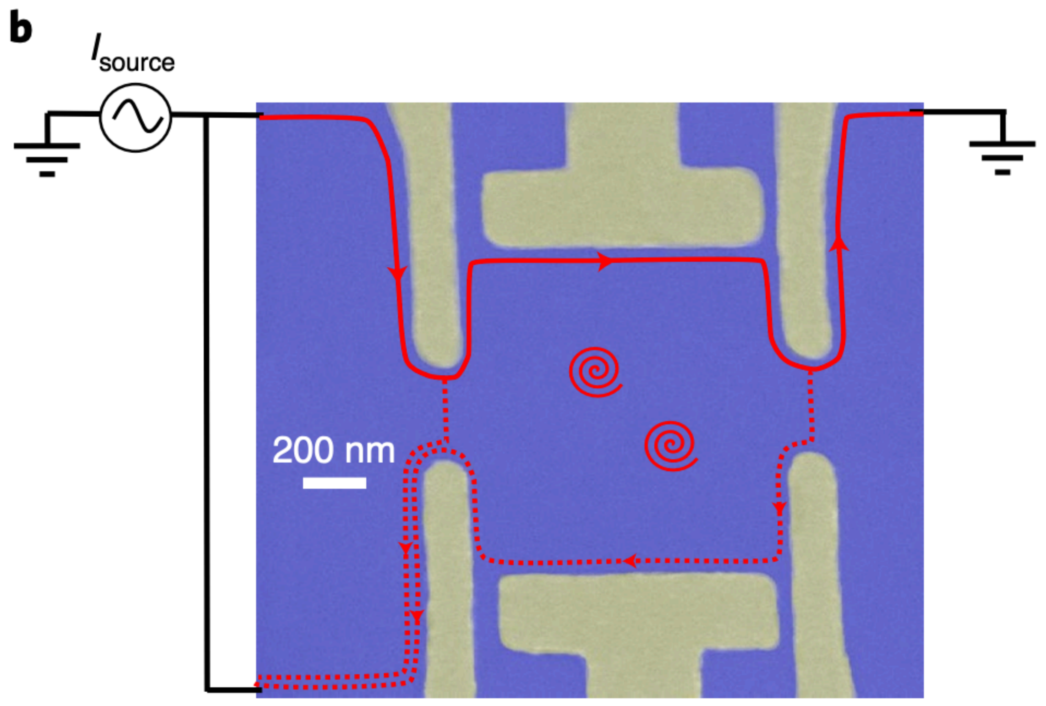


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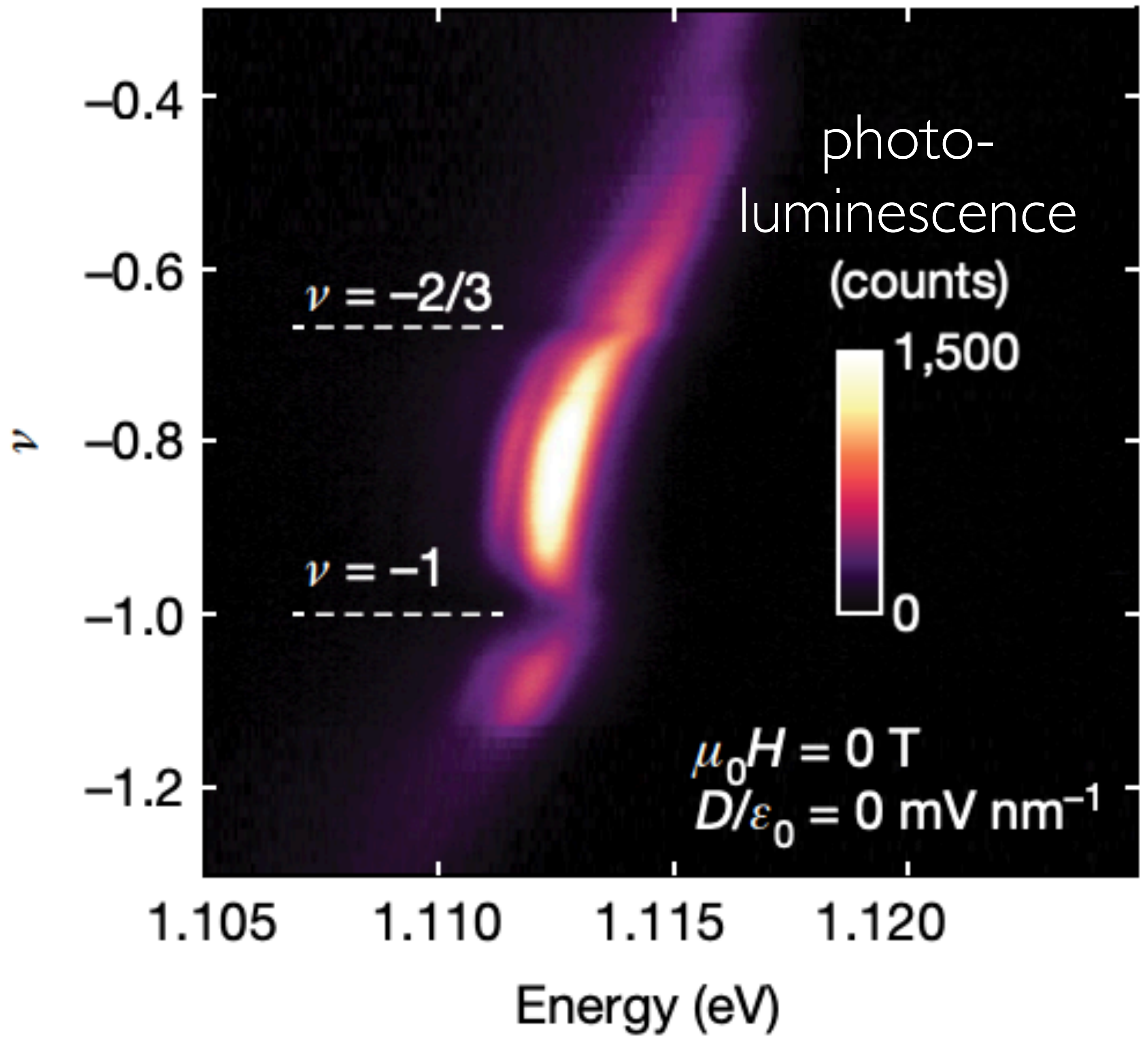


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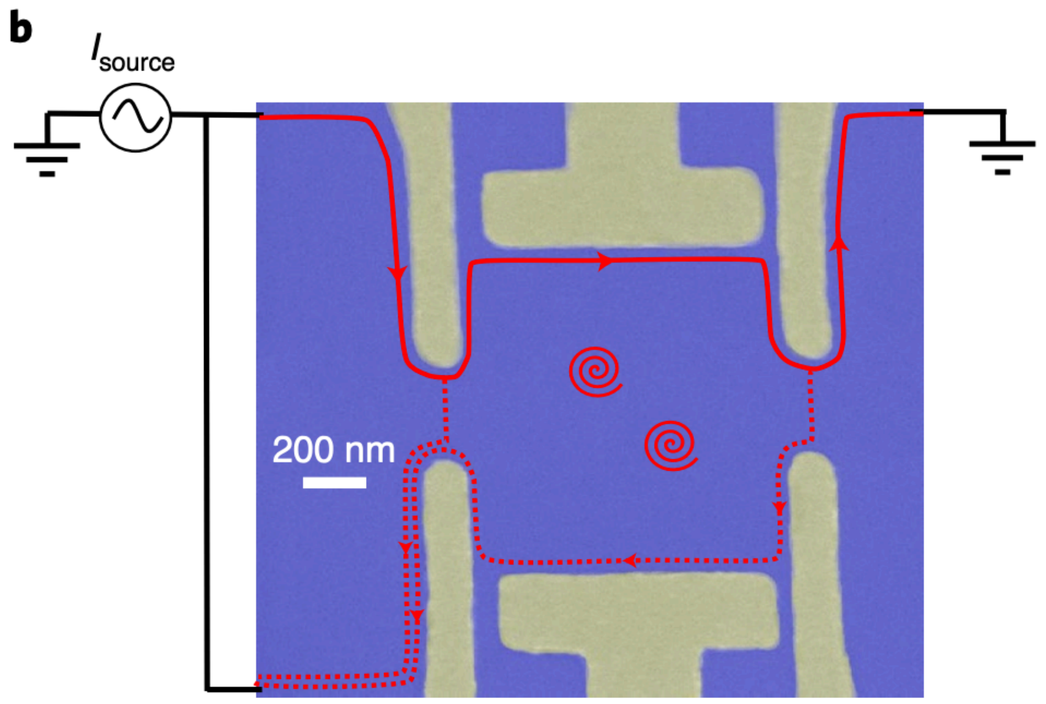


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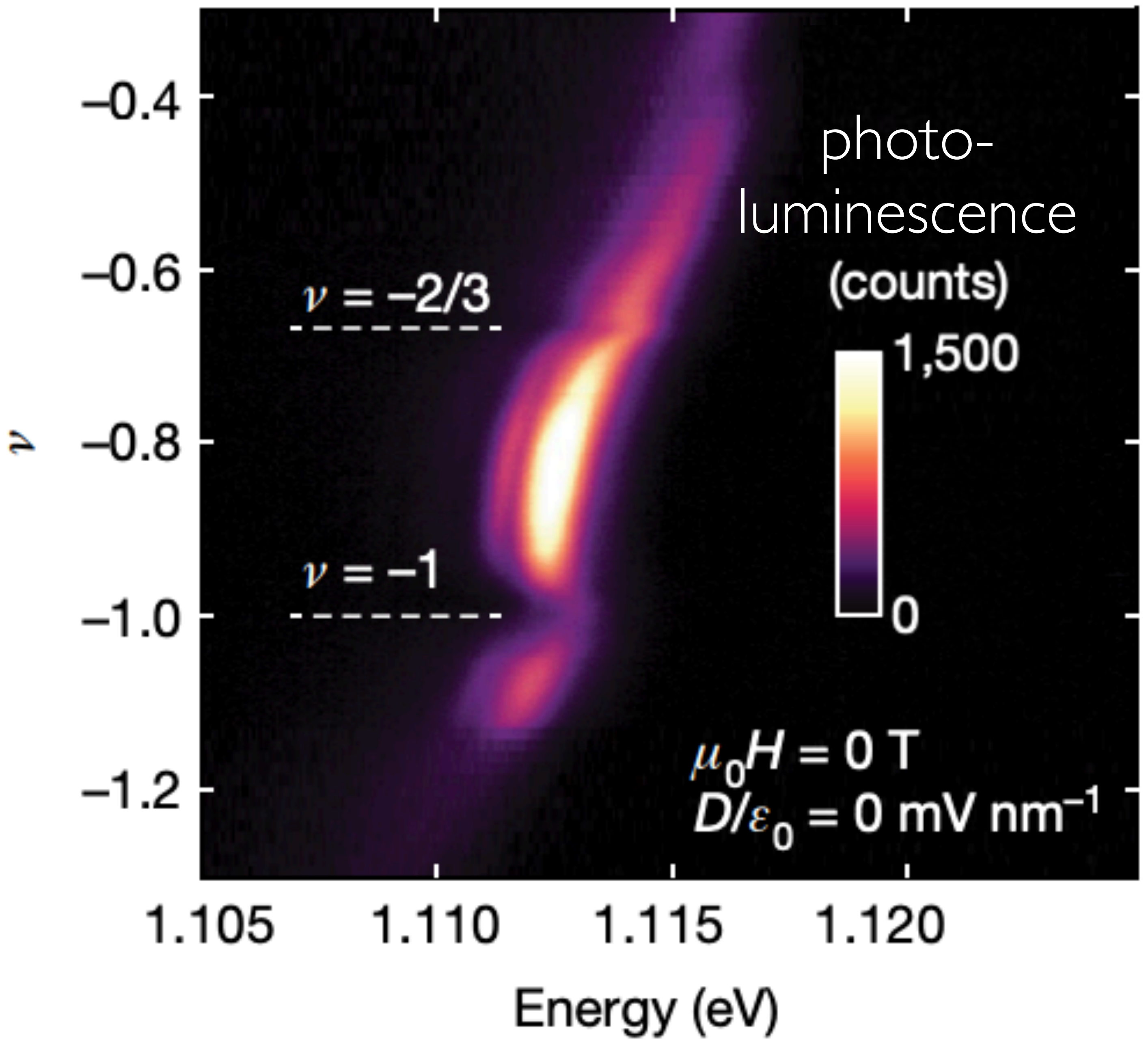
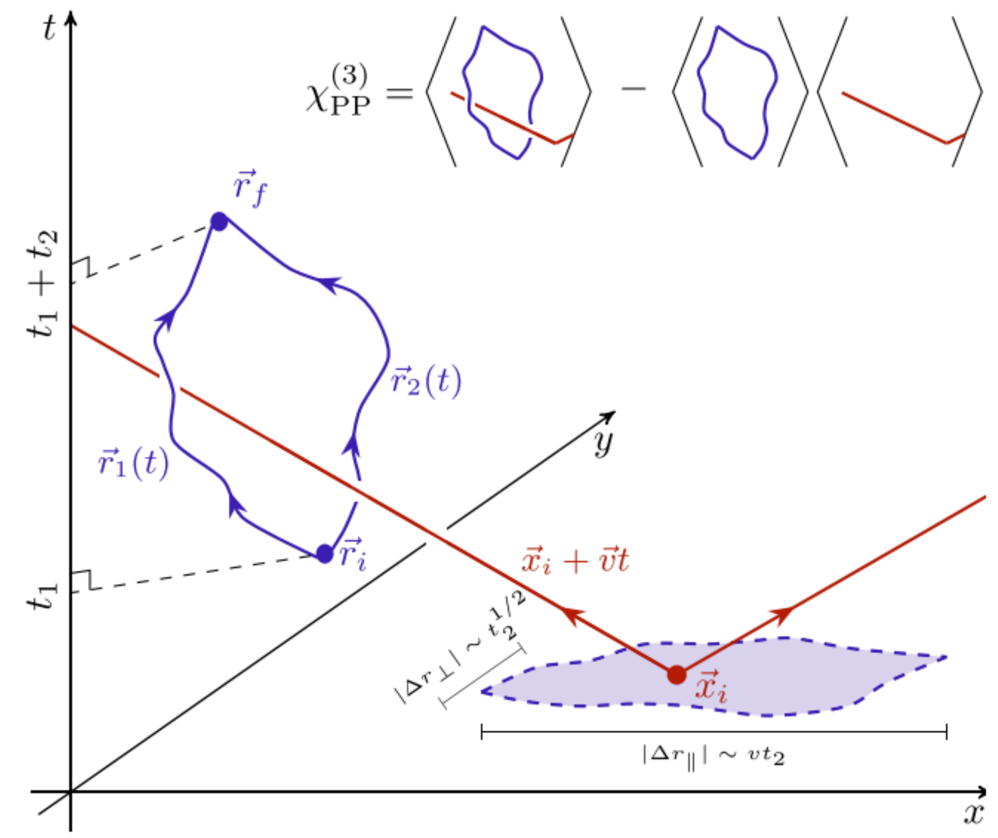



[Cai et al Nature '23]

**Idea:** use "pump-probe" spectroscopy?
(nonlinear optical response)

"linking" of QP trajectories sensitive to statistics




[McGinley-Fava-SP PRL+PRB '24]

… also applies to quantum spin liquids in magnets?

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**topology** **correlations** **tunability**

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**Thanks for listening!**