UNIVERSITY OF OXFORD

# POSSIBLE SOURCES FOR THE GRAVITATIONAL-WAVE BACKGROUND

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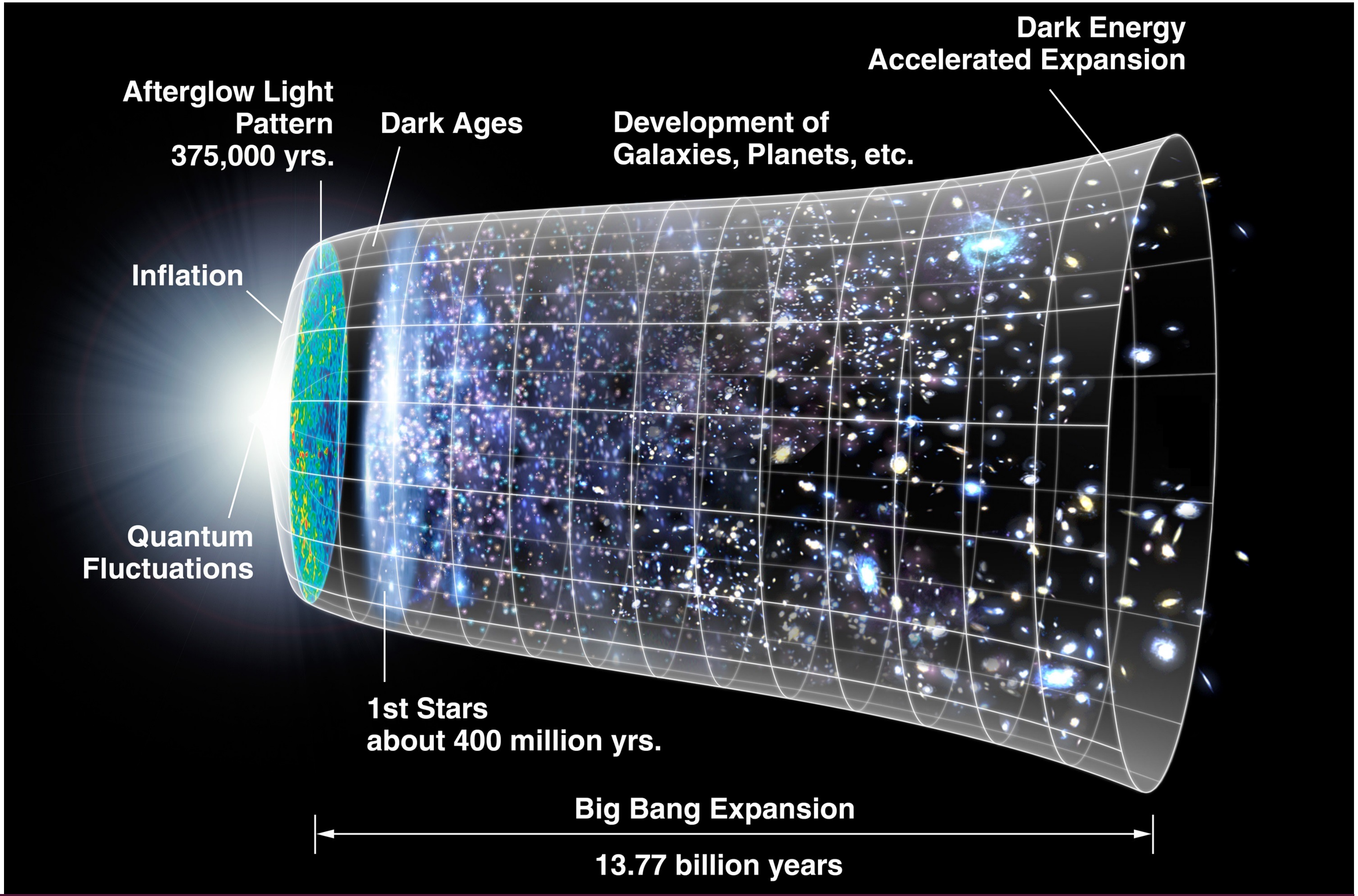
Saturday morning of Theoretical Physics, Oxford, 28.10.23

Image credit:
NASA/WMAP

Distance $\propto a(t)$



# GW ENERGY DENSITY

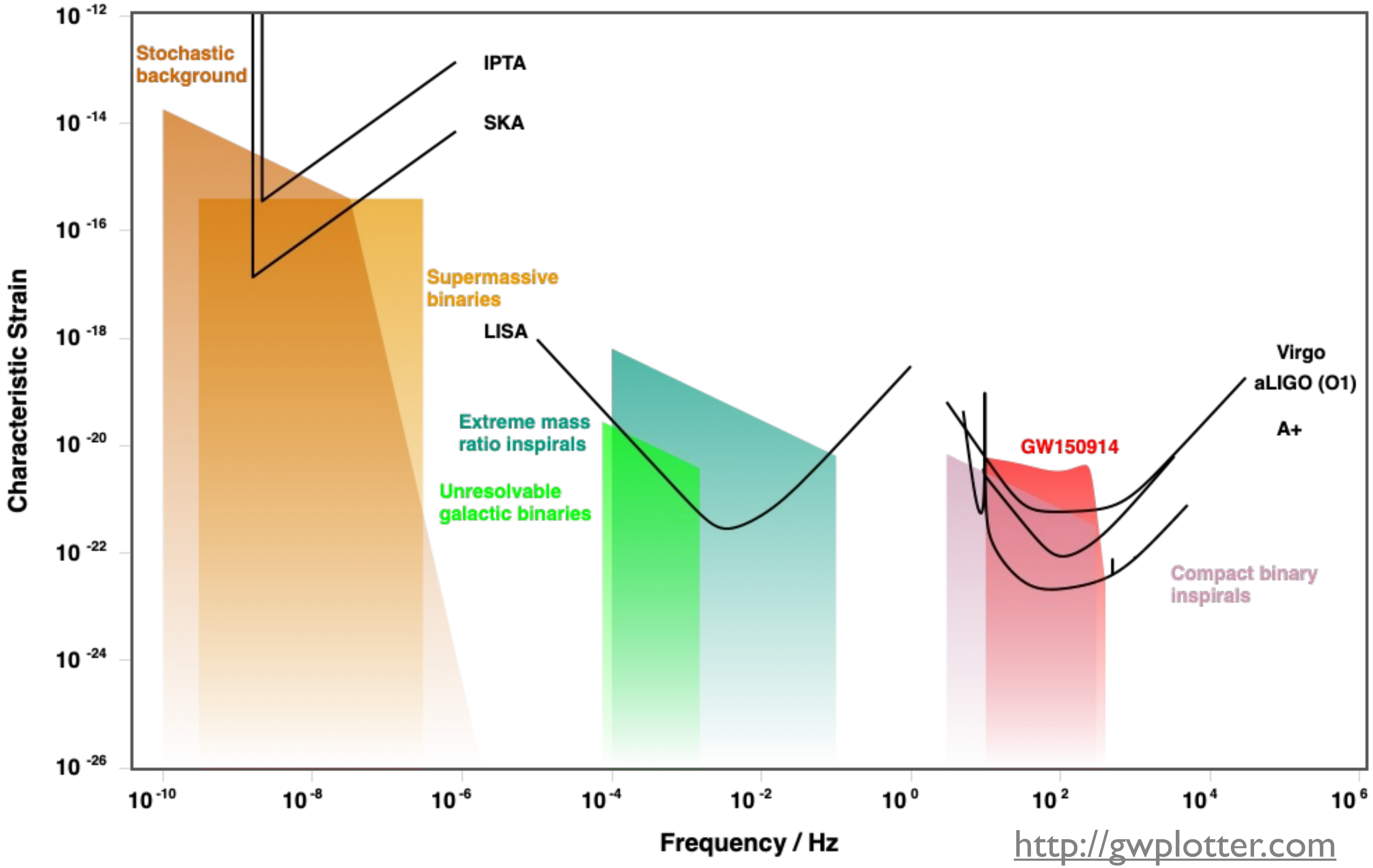
- In this talk: we measure speeds in units of the speed of light, $c = 1$.
- A gravitational-wave carries energy. The energy density is

$$\rho_{gw} = \frac{1}{32\pi G}\langle \dot{h}_{ij}\dot{h}^{ij}\rangle$$

- Measure in units of the critical density, and resolve freuqencies:

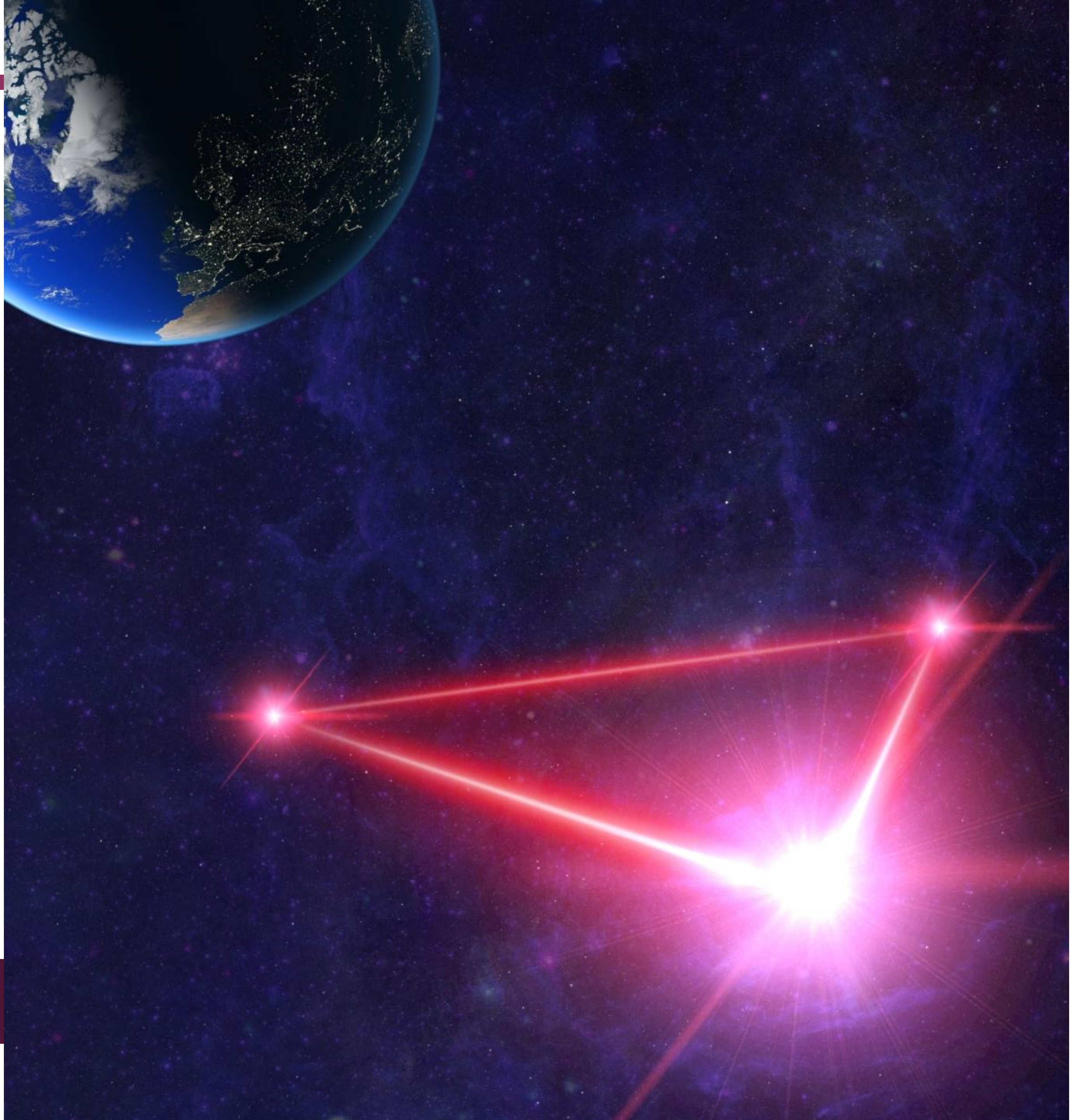
$$\Omega_{gw}(f) = \frac{8\pi G}{3H_0^2}\frac{d\rho_{gw}}{d\ln f} = \frac{2\pi^2 f^3 S_h(f)}{3H_0^2} \equiv \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f)$$

$$\Omega_{gw}(f) \equiv \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f)$$



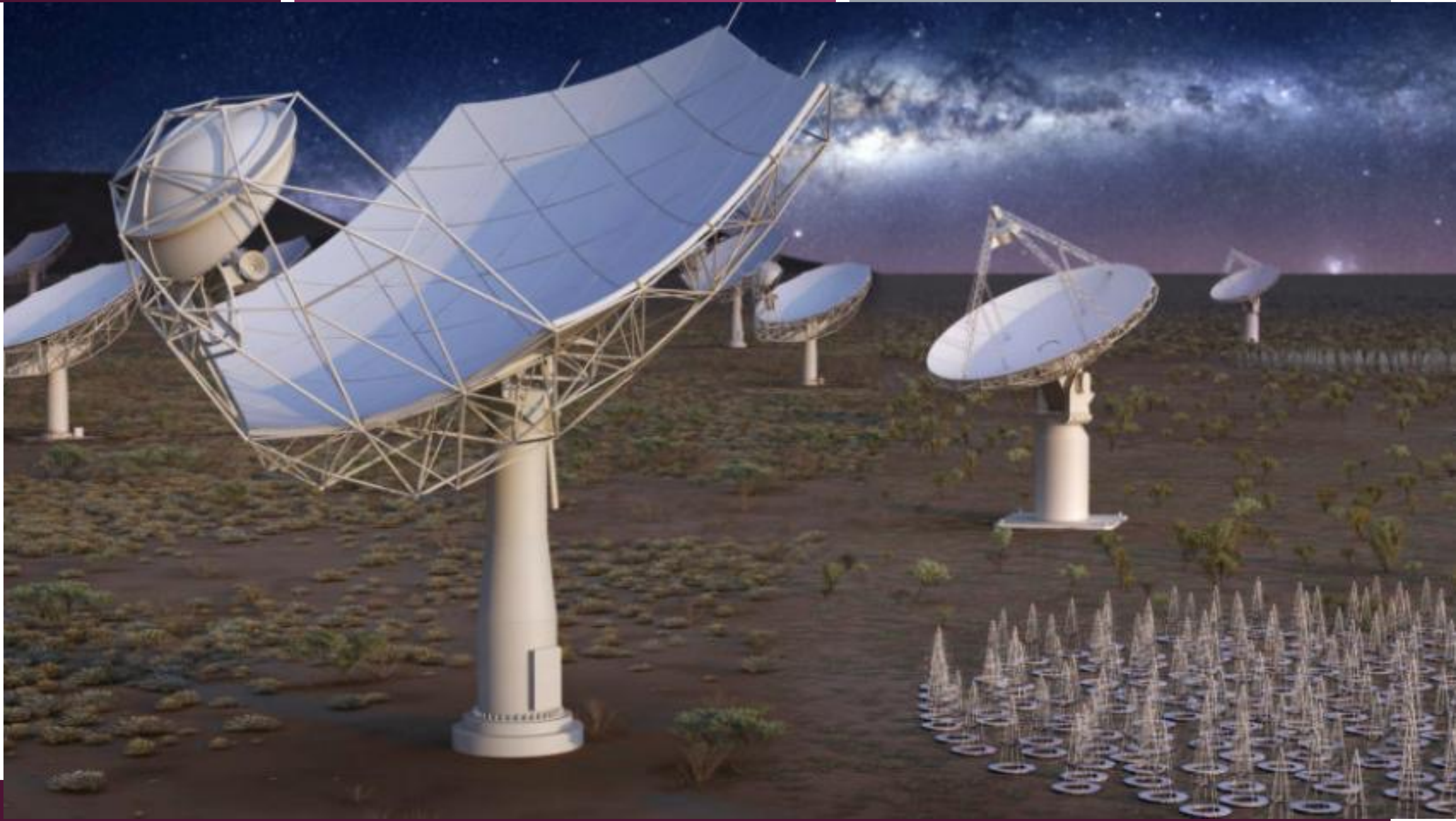

http://gwplotter.com


Copyright: LIGO & Caltech https://www.ligo.caltech.edu https://www.elisascience.org/multimedia/image/lisa-astro2020



Oxford is heavily involved in the design an development of SKA, see:

https://www.physics.ox.ac.uk/research/group/square-kilometre-array-ska

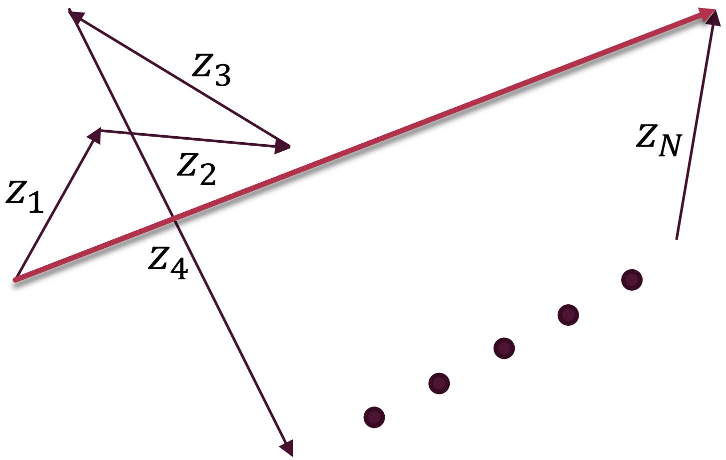


# HIGH FREQUENCIES – COMPACT BINARY IN-SPIRALS

- Gravitational waves from all over the universe constantly bathe our detectors, forming a background.
- Strongest signals are resolved individual events.
- The rest form an essentially stochastic background.
- Think of noise in restaurant.

# GRAVITATIONAL WAVES ADD LIKE A RANDOM WALK

- A wave is $\mathrm{z} = he^{\mathrm{i}\phi}$.

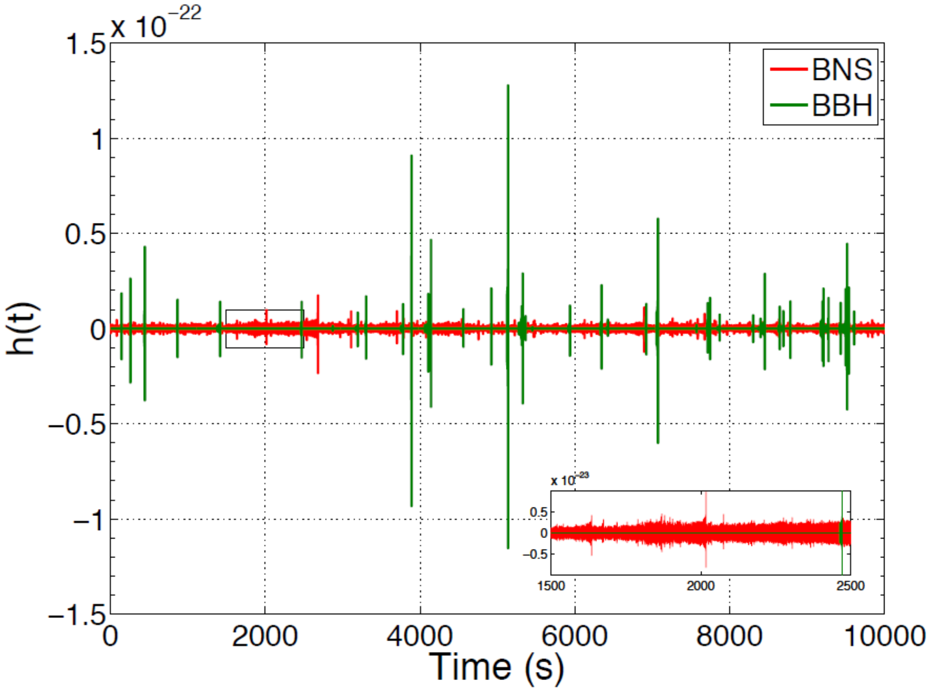



- Total strain from $N(t)$ sources is position of random walker after $N(t)$ steps.

# TOY MODEL FOR SOURCES

$$h(t) = h_+ F_+ + h_\times F_\times = b\frac{d}{r}\cos(\omega t + \varphi)$$

- Source at distance $r$ with constant frequency $\omega$ and amplitude $bd$.
- Assume difference sources are i.i.d.
- Homogeneous & isotropic distribution.



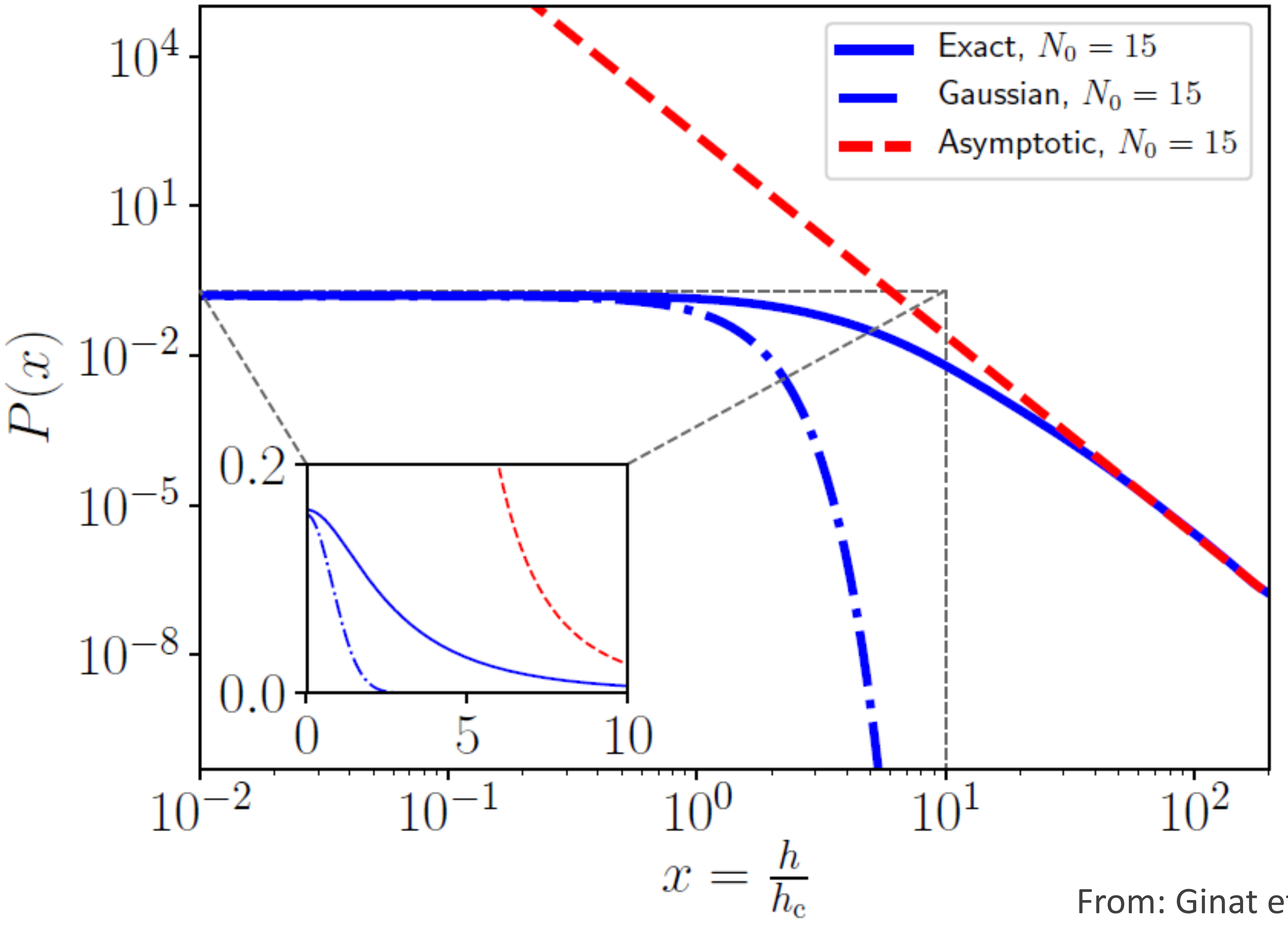

From: LIGO & Virgo 2018

# FLUCTUATION PROBABILITY DISTRIBUTION

- What is the probability $P(\sum_k h_k = h)$ ?
- At $h \to 0$ we expect $P$ to go to some constant:
  - Low strain likely due to destructive interference of many sources
  - $P(h) = P(-h)$ as toy model is symmetric
- Can achieve large $h$ if either
  - Lots of sources interfere constructively (exponentially low probability)
  - One source is strong. Probability of this is $P(h)dh = P\left(r = \frac{bd}{h}\right) dr \propto \left(\frac{bd}{h}\right)^2 dr$
  - So $P(h) \propto h^{-2} \frac{dr}{dh} \propto h^{-4}$

Simple idea is true even for realistic sources.

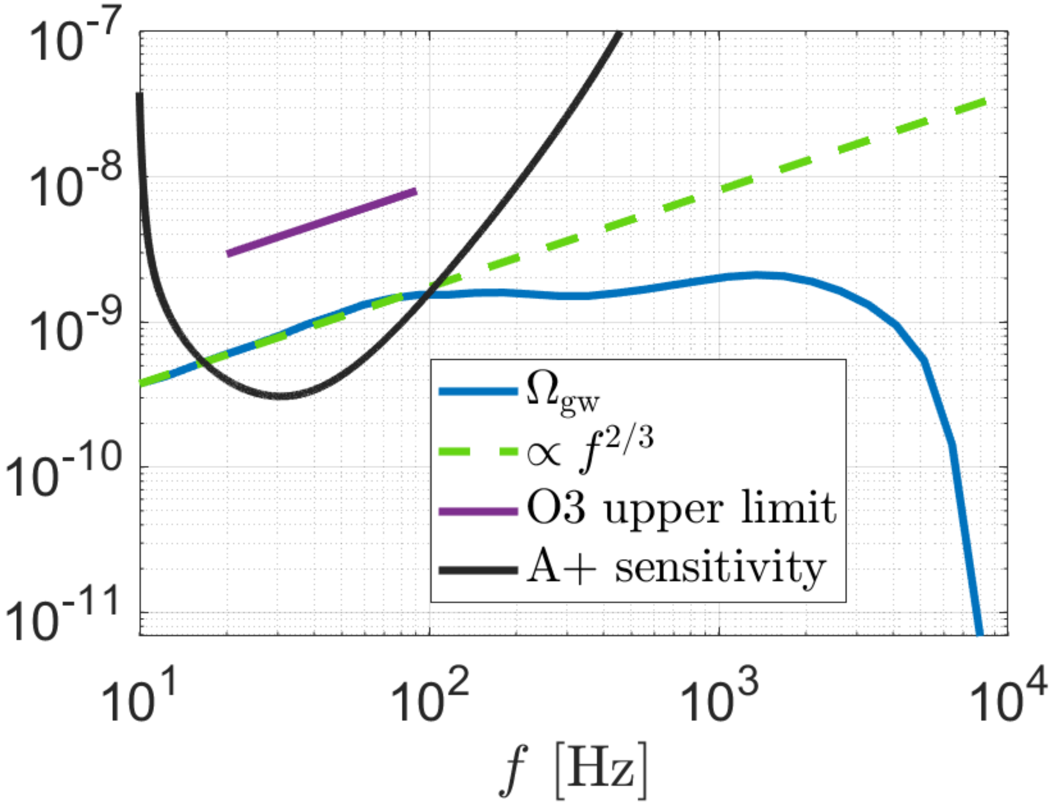
Define $h_c$ as expected value of $|h|$ for one source.




From: Ginat et al. (2020)

# PROBABILITY FOR STRAIN AT GIVEN FREQUENCY

$$\Omega_{gw}(f) \equiv \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f)$$




From: Ginat et al. (2023)

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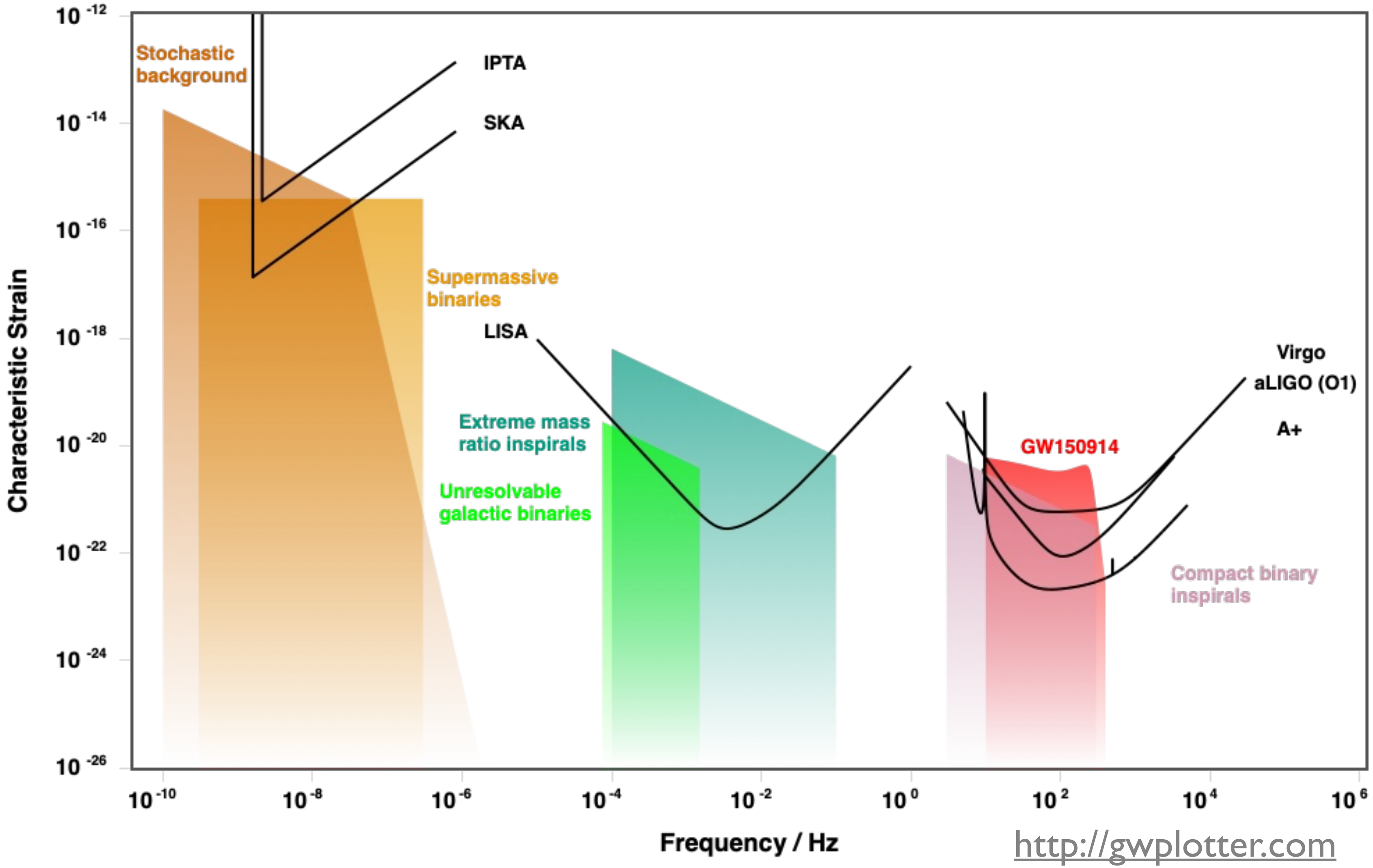



http://gwplotter.com

# LISA – WHITE DWARF BINARIES

Same type of system as black holes/ neutron stars, but:

- Frequencies are much lower – longer periods $\sim$ hours.
- Most sources contributing to SGWB are from the Milky Way.
- NOT homogeneous or isotropic (they follow the Galactic density profile).
- Less sources are active at given moment in time, i.e. emit at LISA's frequency range during an observation window of length $T$.

# GRAVITATIONAL WAVES FROM THE EARLY UNIVERSE

Beyond the standard model physics, e.g.:

- 1st order phase-transitions in the early universe
- https://www.ptplot.org/ptplot/
- Cosmic string collisions
- GWs emitted during inflation

All rise to a peak and then decay.

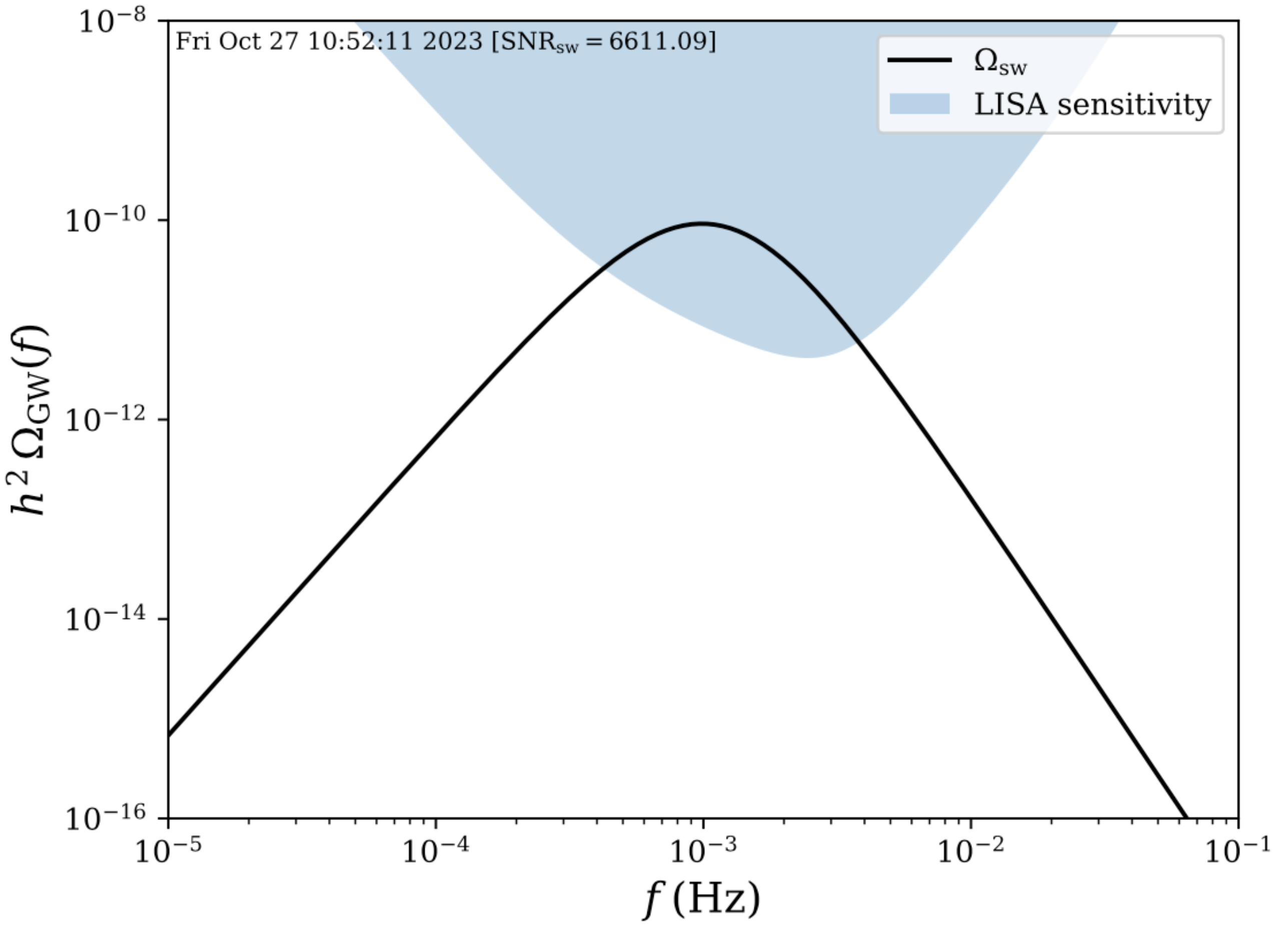
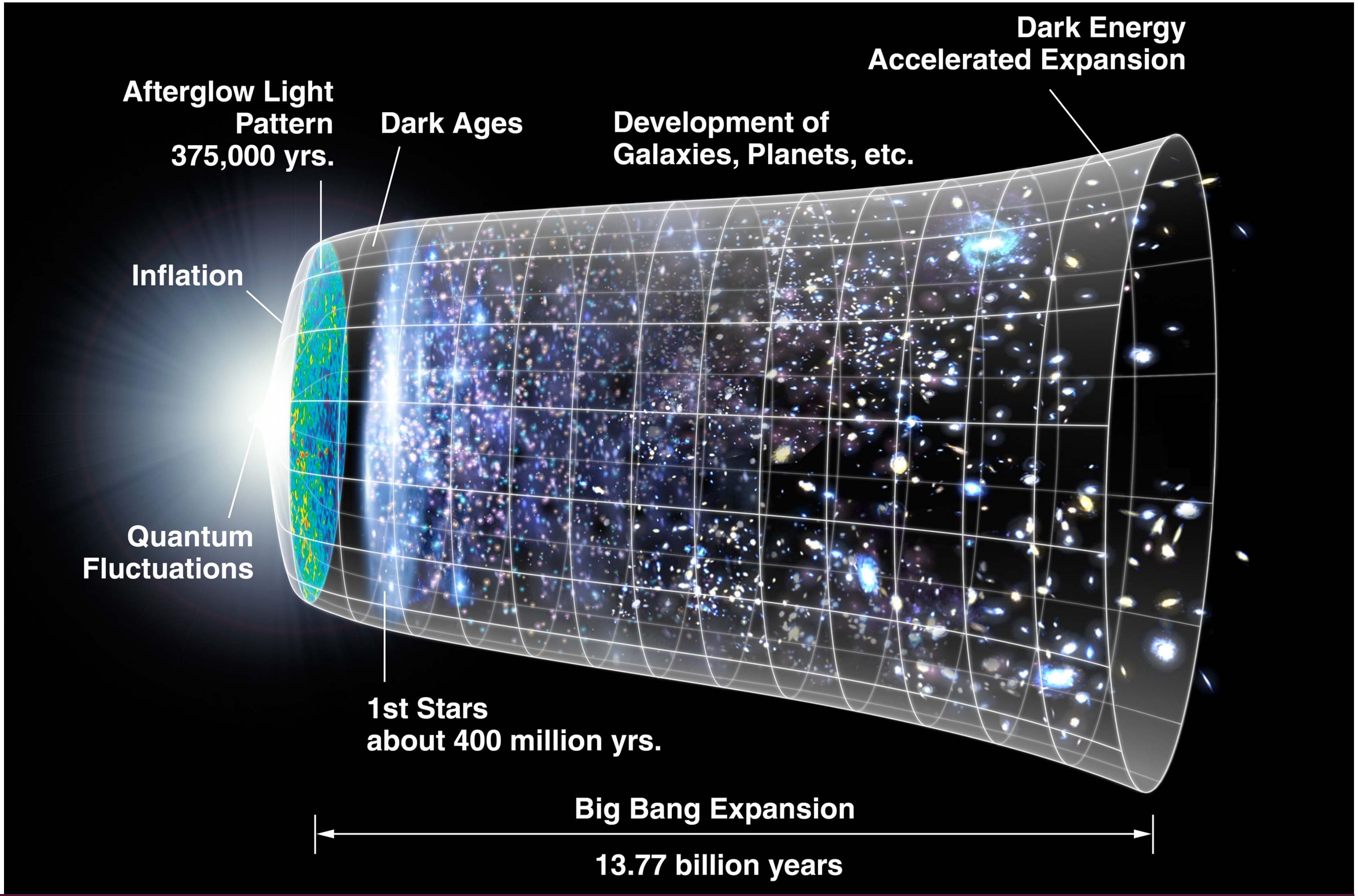


Image credit:
NASA/WMAP

Distance $\propto a(t)$



# GW EMISSION DURING RADIATION DOMINATION

- Metric of expanding universe: $ds^2 = -c^2 dt^2 + a(t)^2 d\boldsymbol{x}^2$
- Define conformal time $a d\eta = dt$, so that the metric is
$$ds^2 = a^2(-d\eta^2 + d\boldsymbol{x}^2)$$
- Einstein field equations give ($h \equiv a\chi$), for source $\sigma$
$$\chi'' + k^2\chi = 16\pi G a^3 \sigma$$
- Solved by
$$\chi(\eta_f, \boldsymbol{k}) = \frac{16\pi G}{k} \int_{\eta_{in}}^{\eta_f} d\eta\, a^3(\eta) \sigma(\eta, \boldsymbol{k}) \cos\left(k(\eta_f - \eta)\right)$$

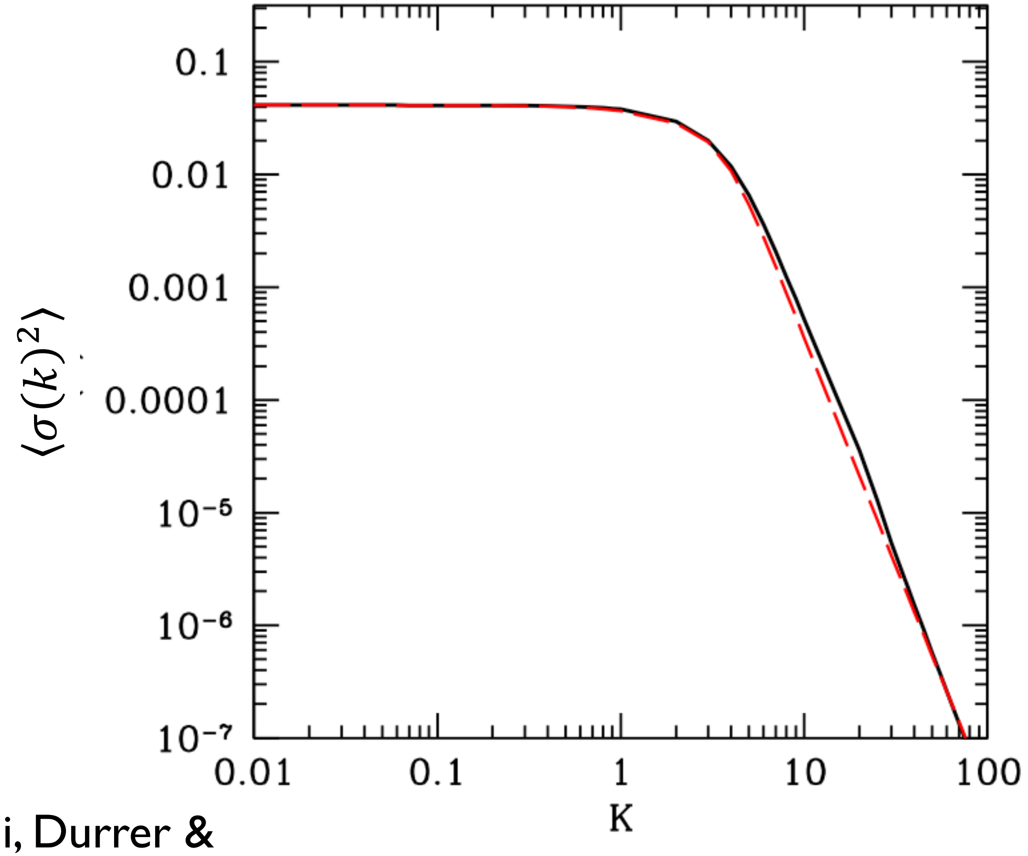
# CAUSAL SOURCES – LOW FREQUENCIES

- For GW the dispersion relation is simple: $k = \frac{2\pi f}{c}$.
- For $k\ L_{source} \ll 1$, we will show $\Omega_{gw}(f) \sim f^3$.
- Causal sources separated by $\lambda \gg L_{source}$ cannot be correlated.
- At scales much longer than the typical source size, emission must be uncorrelated, so $\langle \sigma^2(k) \rangle = const.$
- Why?

# SOURCE CORRELATION FUNCTION

At scales much longer than the typical source size, emission must be uncorrelated, so

$$\langle \sigma^2(k) \rangle = const$$




From: Caprini, Durrer & Servant (2008)

# CAUSAL SOURCES – LOW FREQUENCIES

- From solution $\langle h^2(k,\eta)\rangle \propto \langle \sigma^2(k)\rangle\, k^{-2}$. So

$$\langle h^2(\boldsymbol{x},\eta)\rangle \propto k^3\, \langle \sigma^2(k)\rangle\, k^{-2}$$

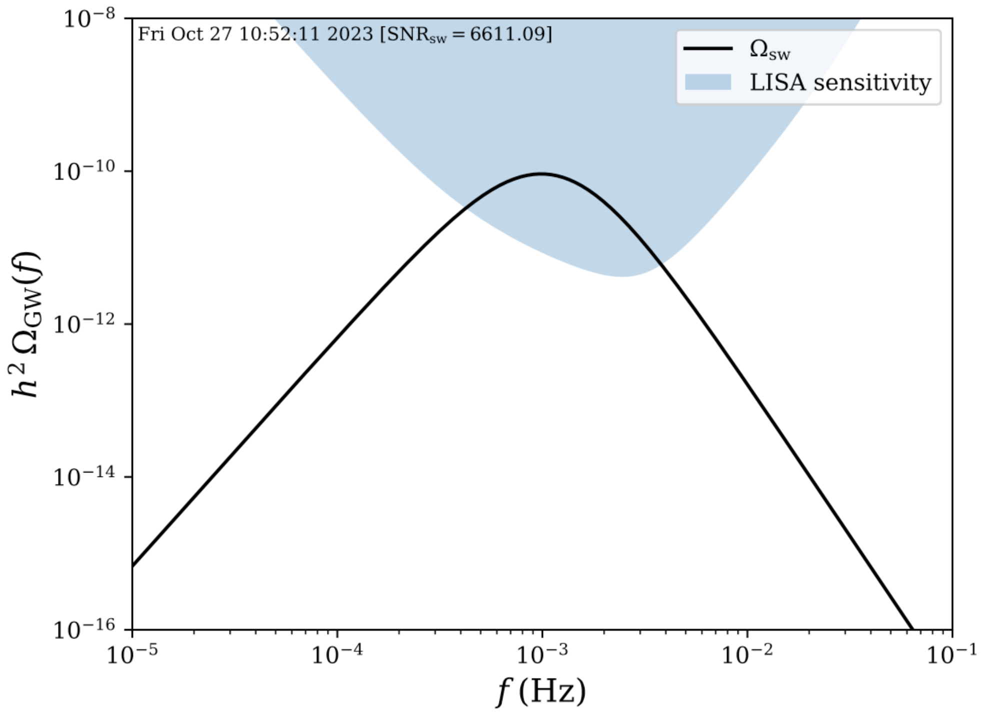
$$\Omega_{gw}(f) \propto f^2 \langle h^2(\boldsymbol{x},\eta)\rangle$$

- So with $k \propto f$, we get

$$\Omega_{gw} \propto f^3$$

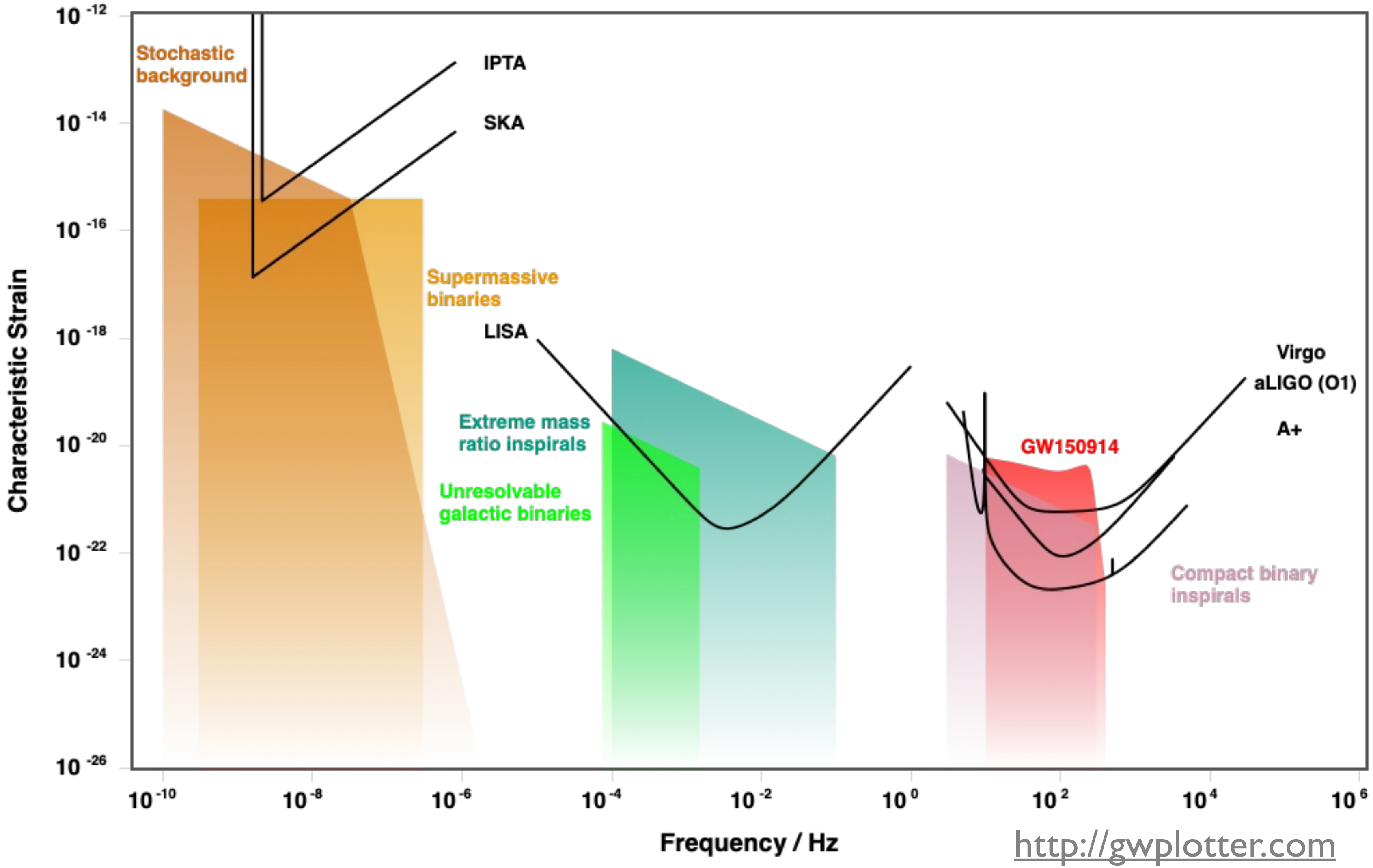
# CAUSAL SOURCES – HIGH FREQUENCIES

- At large $kL_{source} \gg 1$, we must have $\langle\sigma^2(k)\rangle$ decaying with $k$, hence $\Omega_{gw}(f)$ decays with $f$.
- So the general shape rise as $f^3$ up to a peak $f_0$ and then decays.
- $f_0$ determined by the physics of the GW emission process.




https://www.ptplot.org/ptplot/

$$\Omega_{gw}(f) \equiv \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f)$$

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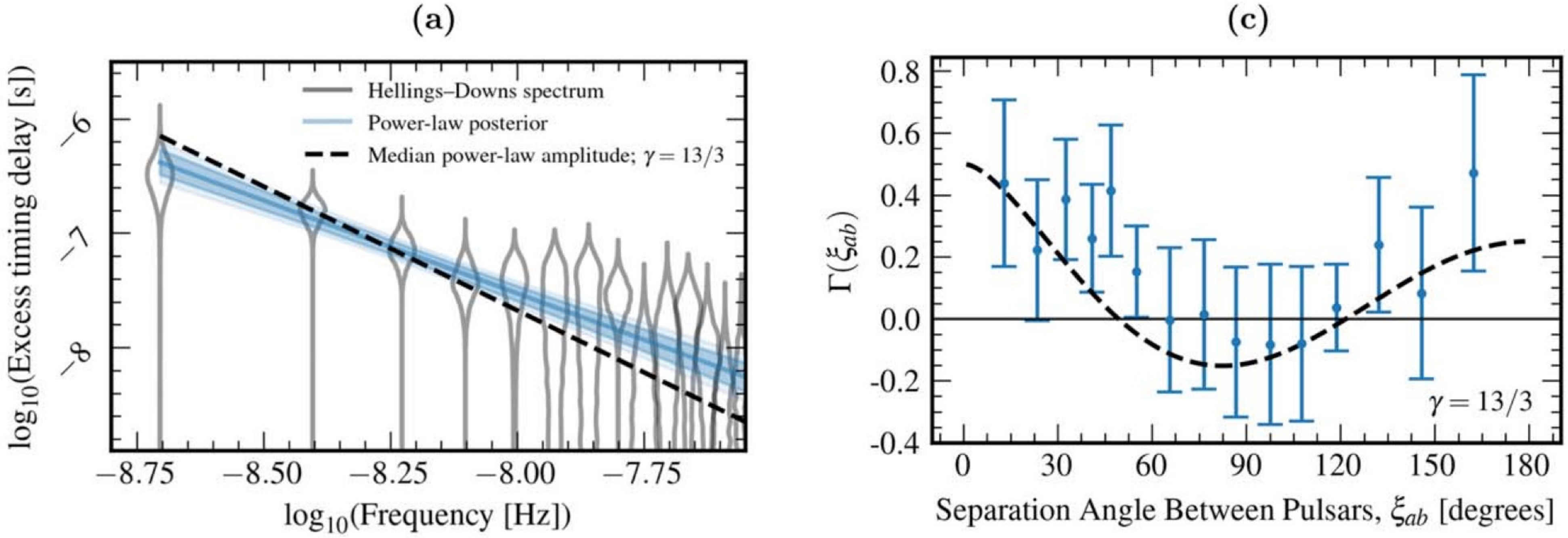
# NANOGRAV: POSSIBLE DETECTION

The Astrophysical Journal Letters, 951:L8 (24pp), 2023 July 1 Agazie et al.

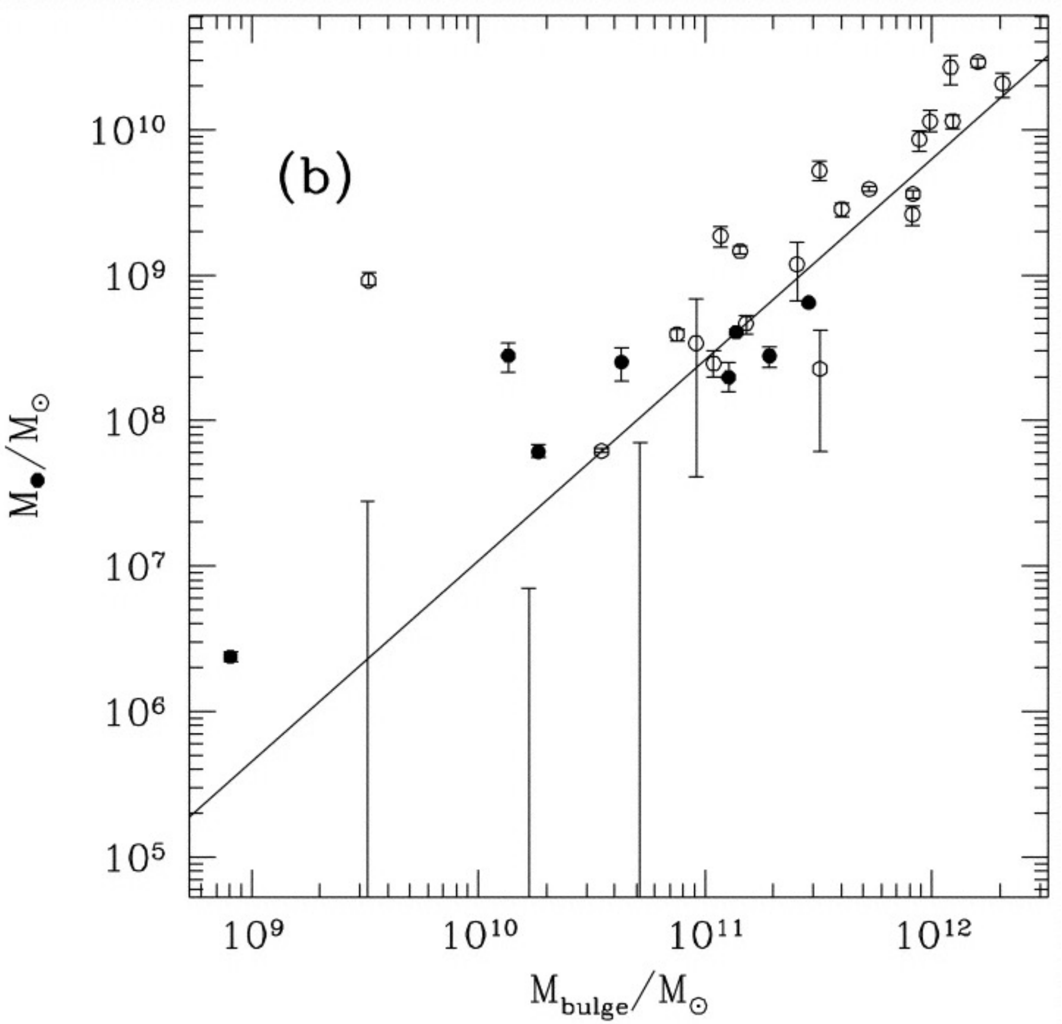


# PULSAR TIMING ARRAYS – SMBH BINARIES?

Same as stellar-mass binaries, but

- Frequencies are much lower
- Individual mergers resolved by LISA (period $\sim$ days-hours)
- Background stems from mergers of galaxies (period $\sim$ decades)

# $M_{BH}$-$M_{gal}$ ("MAGORRIAN") RELATION

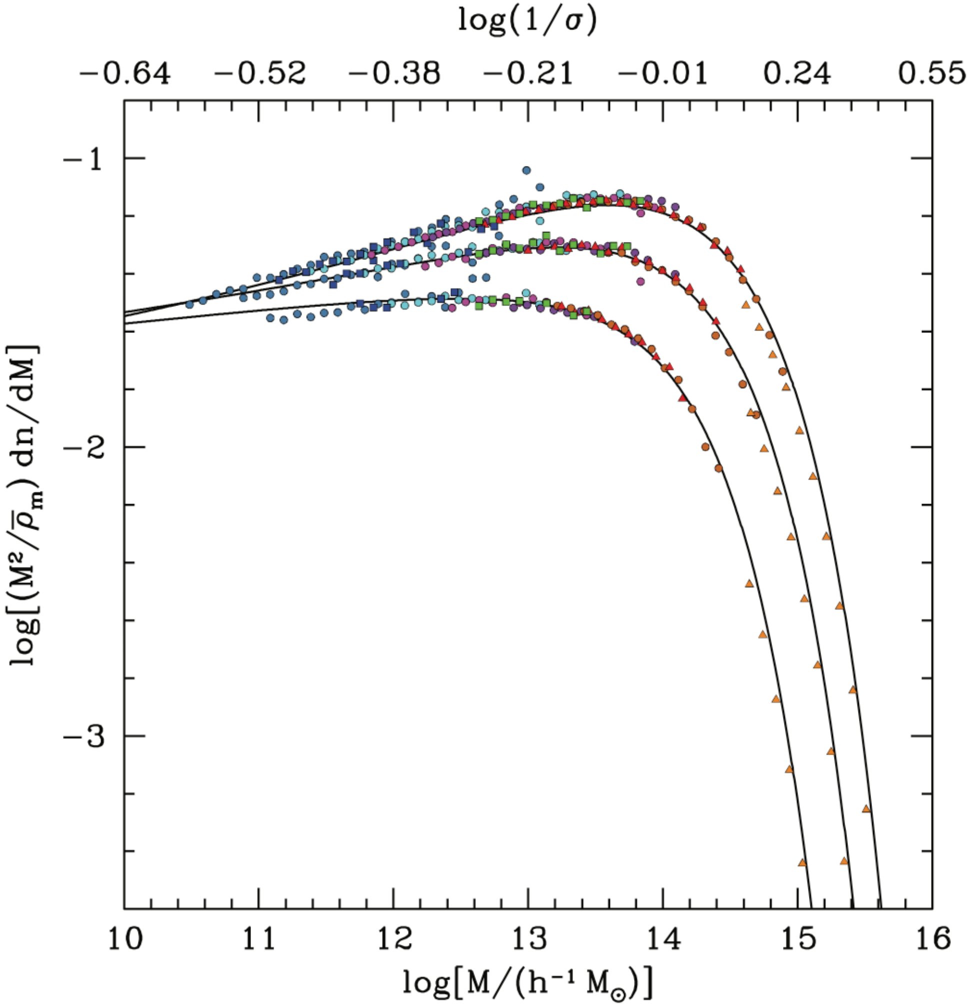



From: Magorrian et al. (1998)

# HALO MASS FUNCTION

$$\frac{dn}{dM} = f(\sigma)\frac{\bar{\rho}_m}{M}\frac{d\ln\sigma^{-1}}{dM}$$

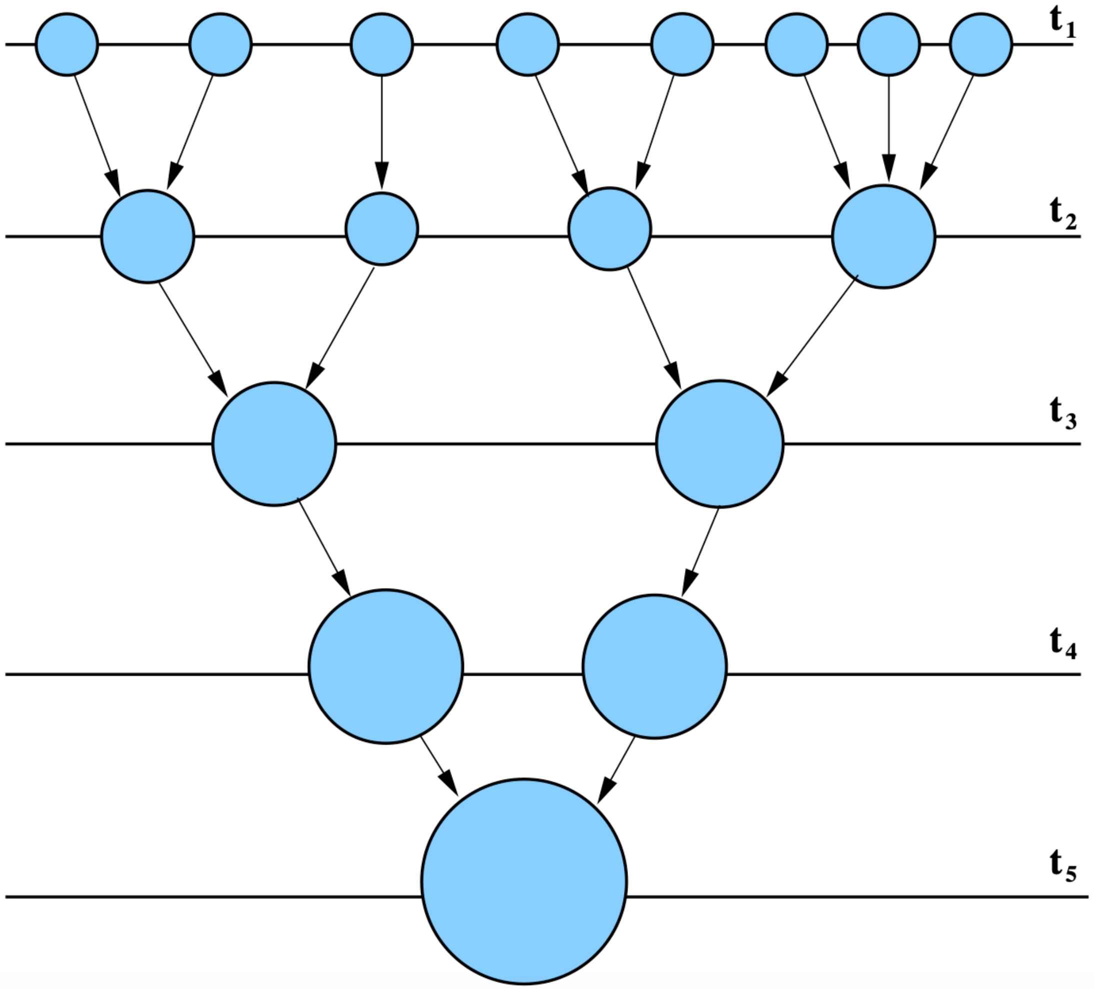
Press & Schechter (1974), Tinker et al. (2008)




From: Tinker et al. (2008)

# GALAXIES MERGE HIERARCHICALLY

From: Baugh (2006)



# COMBINING TO GET RATE

- Rate

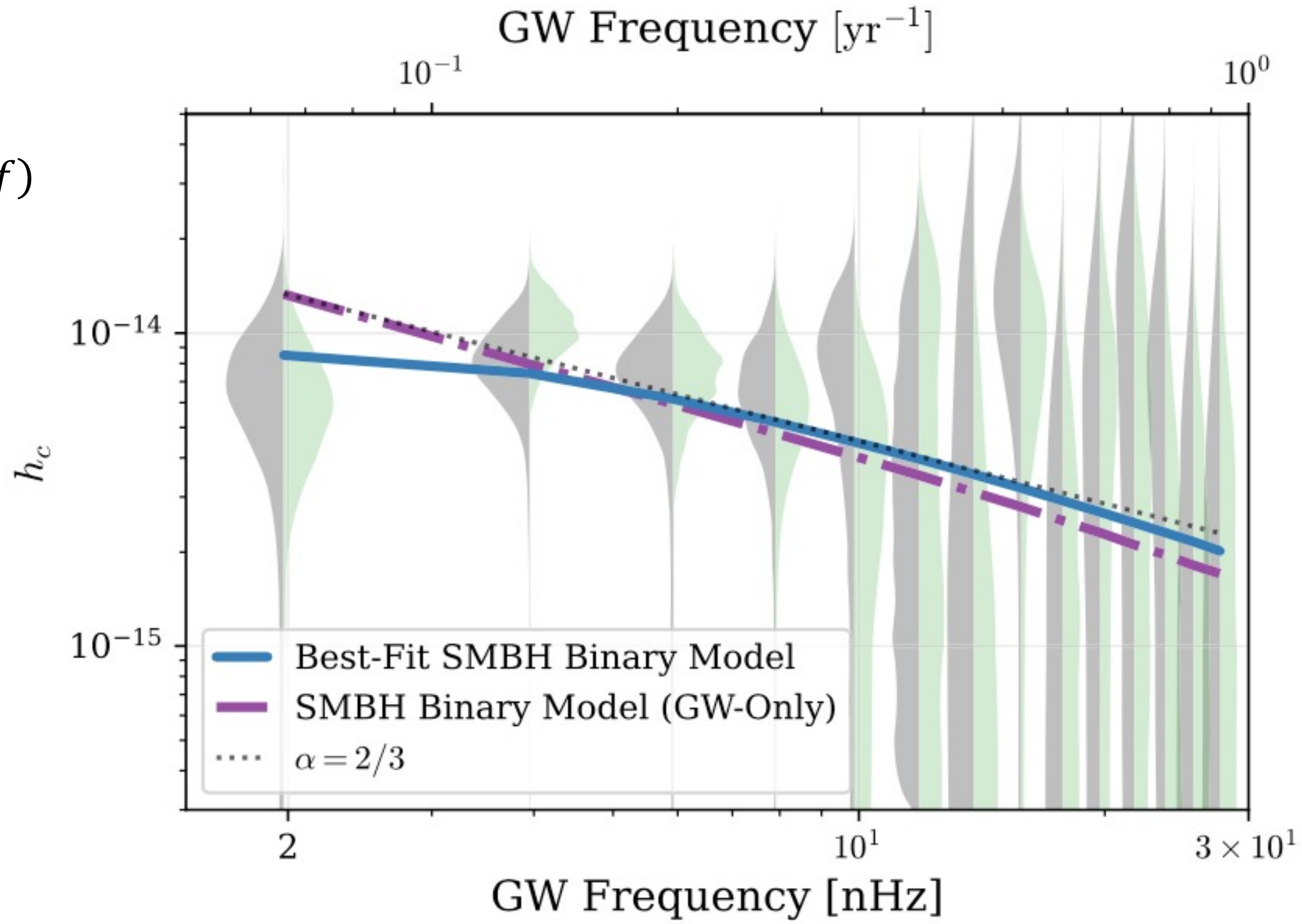
$$\Phi = \frac{dn_{gal}}{dM_1}\frac{dn_{merg}}{dt}\frac{dn_{gal}}{dM_2}$$

- $\frac{dn_{merg}}{dt}$ = rate of galaxy mergers

$$\Omega_{gw}(f) \propto \int dM_1 dM_2 dt\, \Phi \frac{dE_{gw}}{dt}\frac{dt}{df} \propto f^{\frac{2}{3}} \int dM_1 dM_2\, \Phi$$

# NANOGRAV: BINARY SMBH BACKGROUND CONSISTENT

$$f^{\frac{2}{3}} \sim \Omega_{gw}(f) \equiv \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f)$$




From: Agazie et al. (2023)

# CONCLUSIONS AND OUTLOOK

- The stochastic gravitational-wave background can allow us to observe many physical phenomena hitherto unseen.
- Signal already observed consistent with SMBH binary GWB.
- Up-coming detectors should observe it.