The Miracle of Quantum Error Correction

Hillary 2024 Morning of Theoretical Physics

Benedikt Placke

benedikt.placke@physics.ox.ac.uk

UNIVERSITY OF
OXFORD

Quantum vs. Classical Information

Classical Information

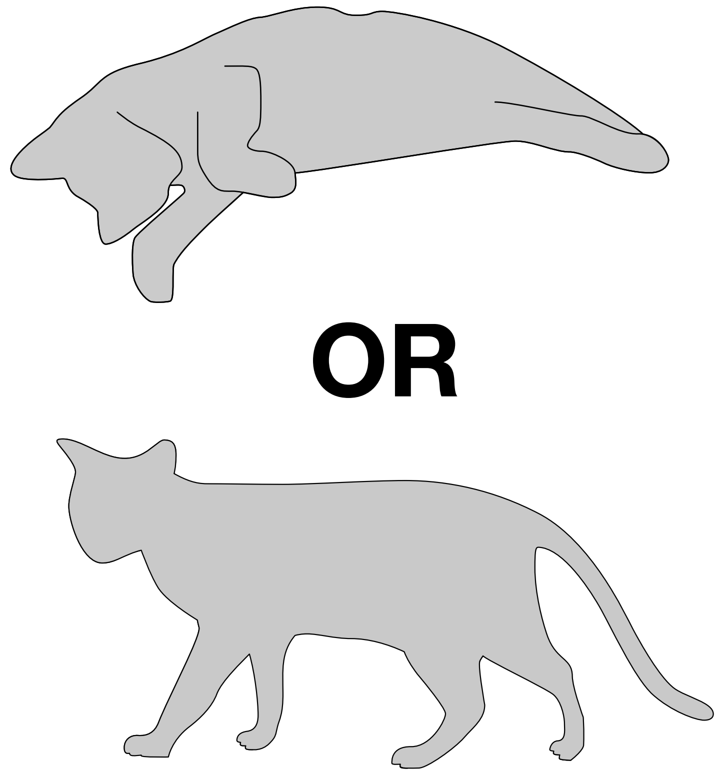
The “classical” bit is the fundamental unit of information.

Classical Bit

$$\sigma \in \{0,1\}$$

It is either

- 0 or 1
- Yes or No
- Dead of Alive



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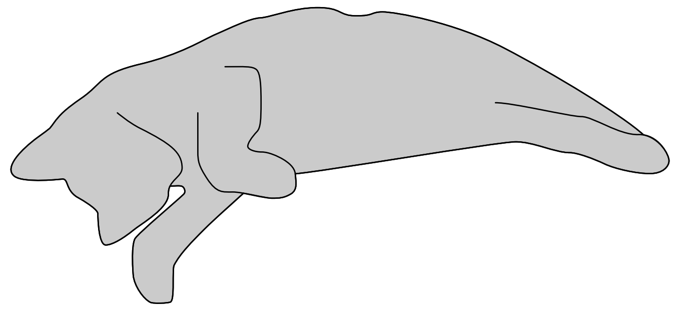
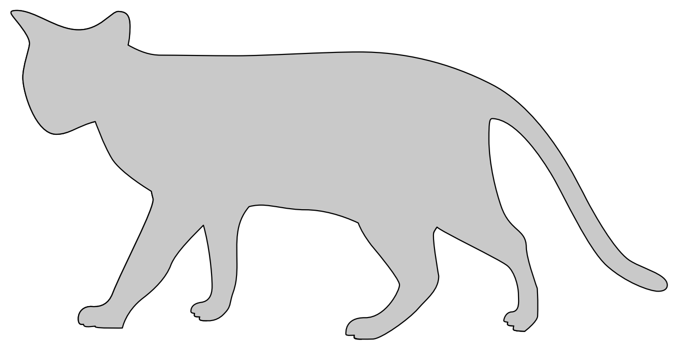


OR



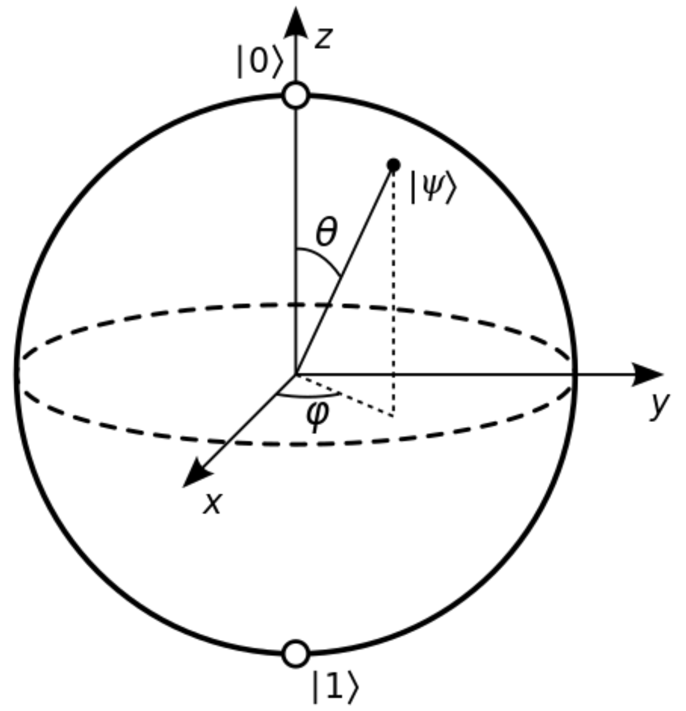
Quantum Information

Information needed to describe "minimal" quantum system

Qubit

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\alpha, \beta \in \mathbb{C},\ |\alpha|^2 + |\beta|^2 = 1$$



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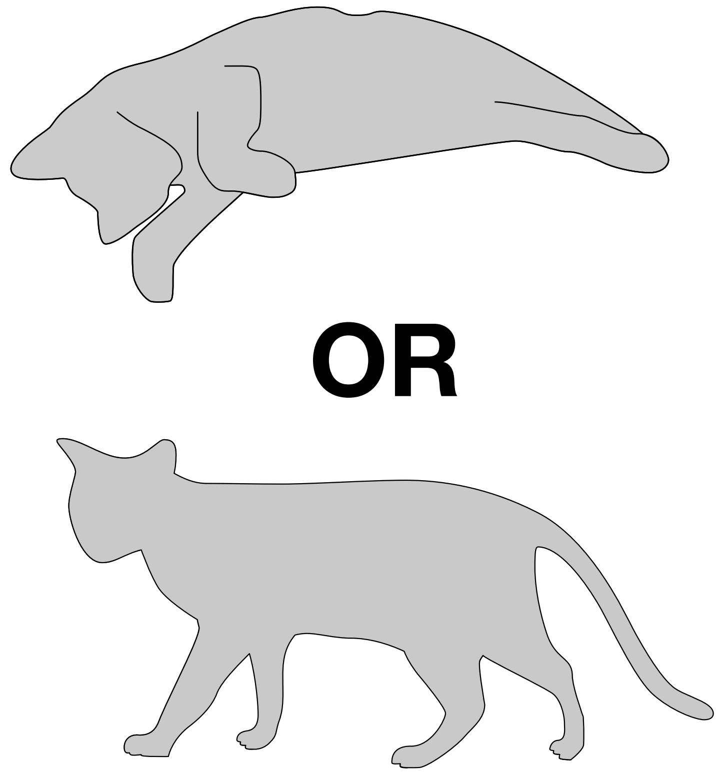

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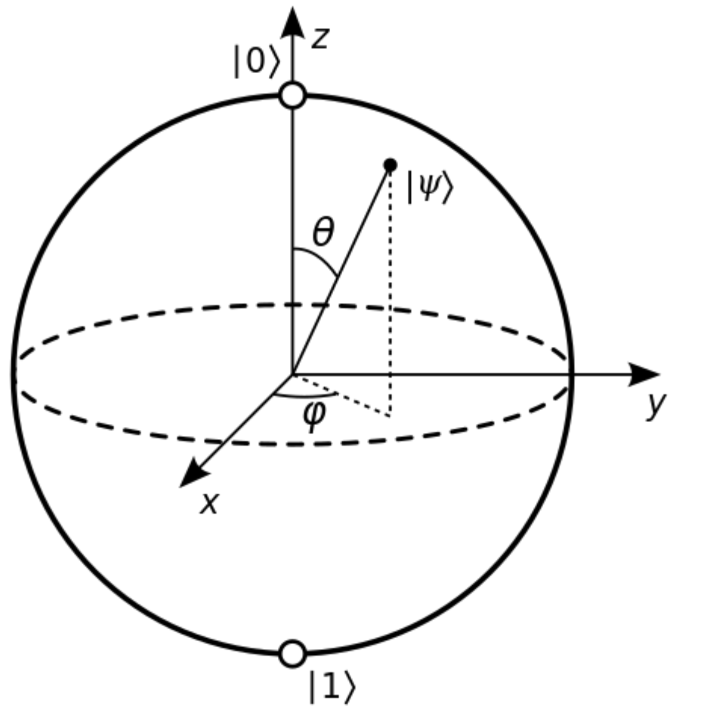

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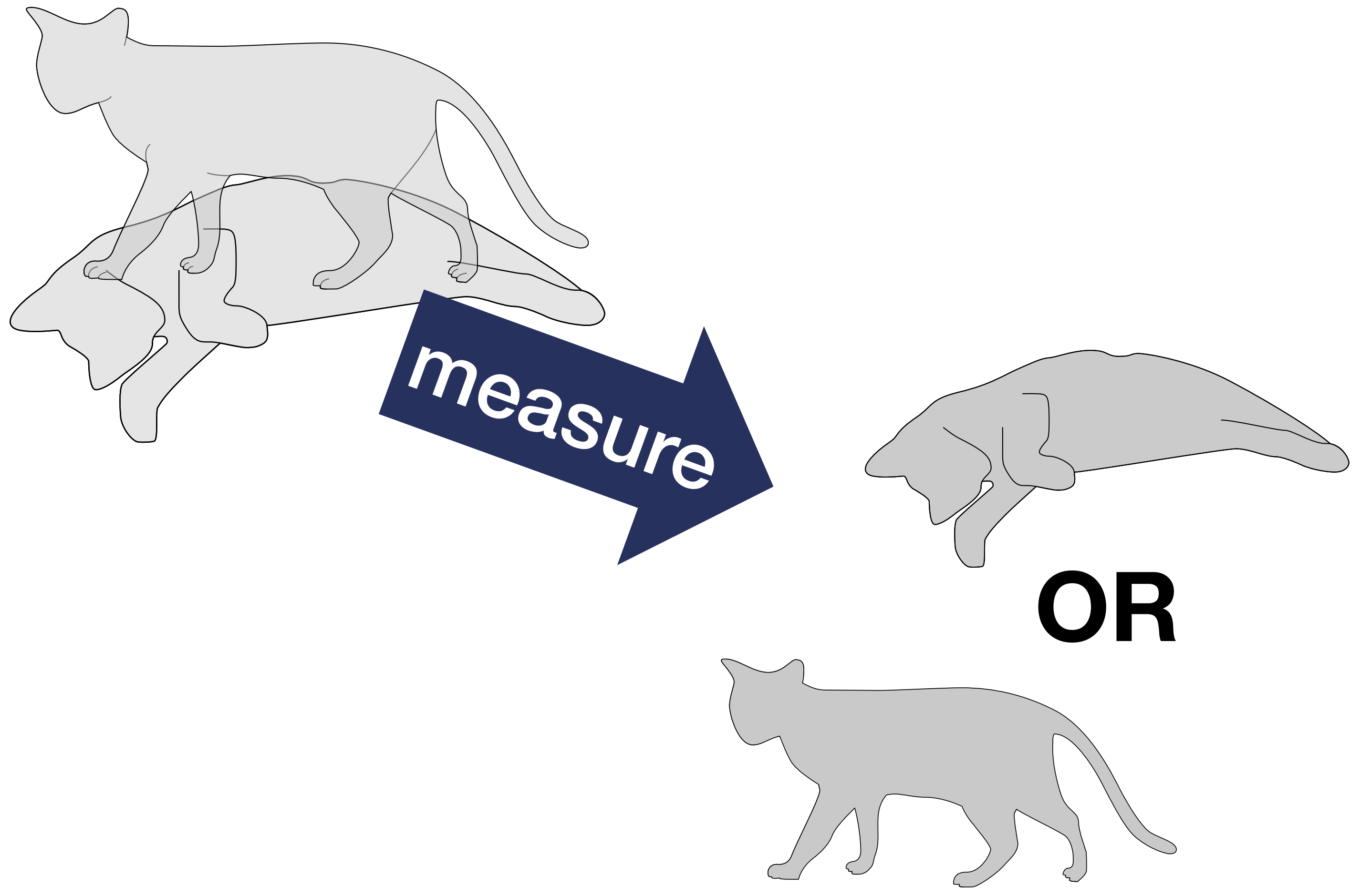
with probability $|\alpha|^2$ and $|\beta|^2$



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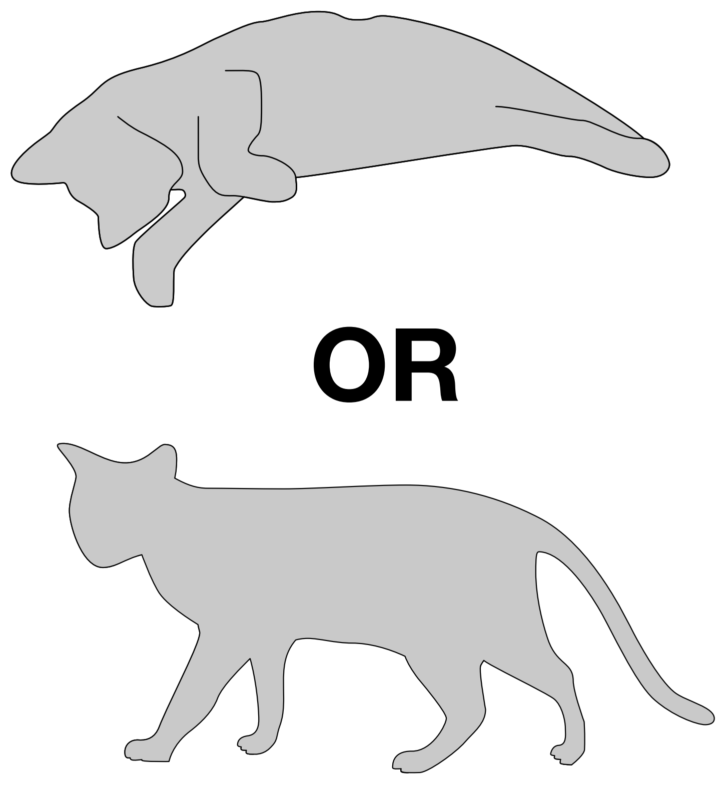

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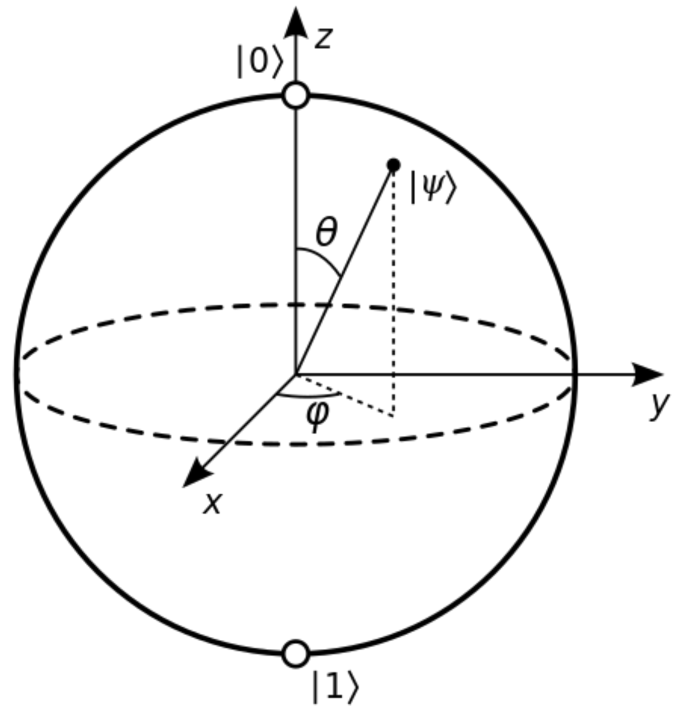

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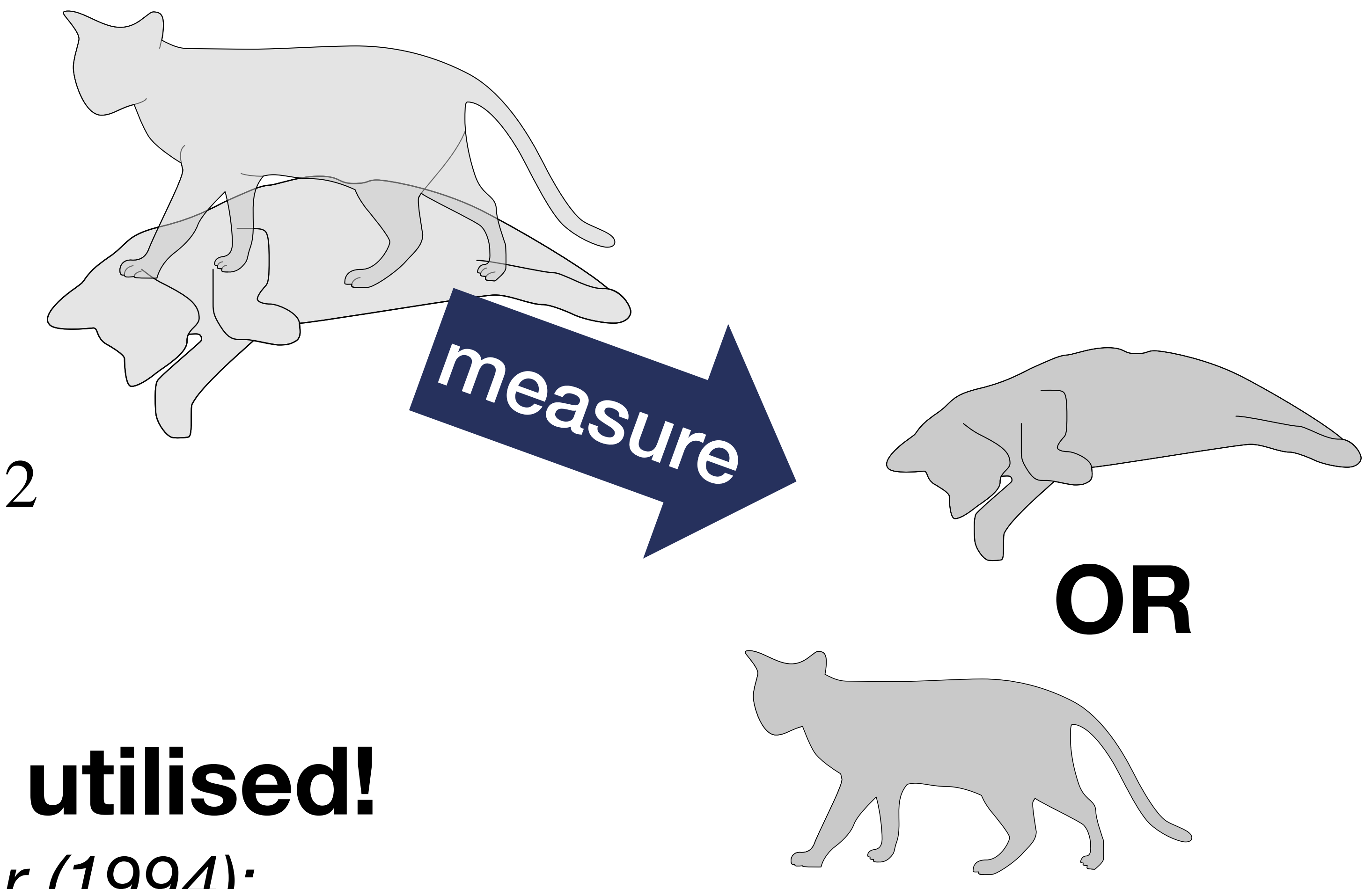

This weirdness can be utilised!

Deutsch & Jozsa (1992); Shor (1994); Ekert (1991)

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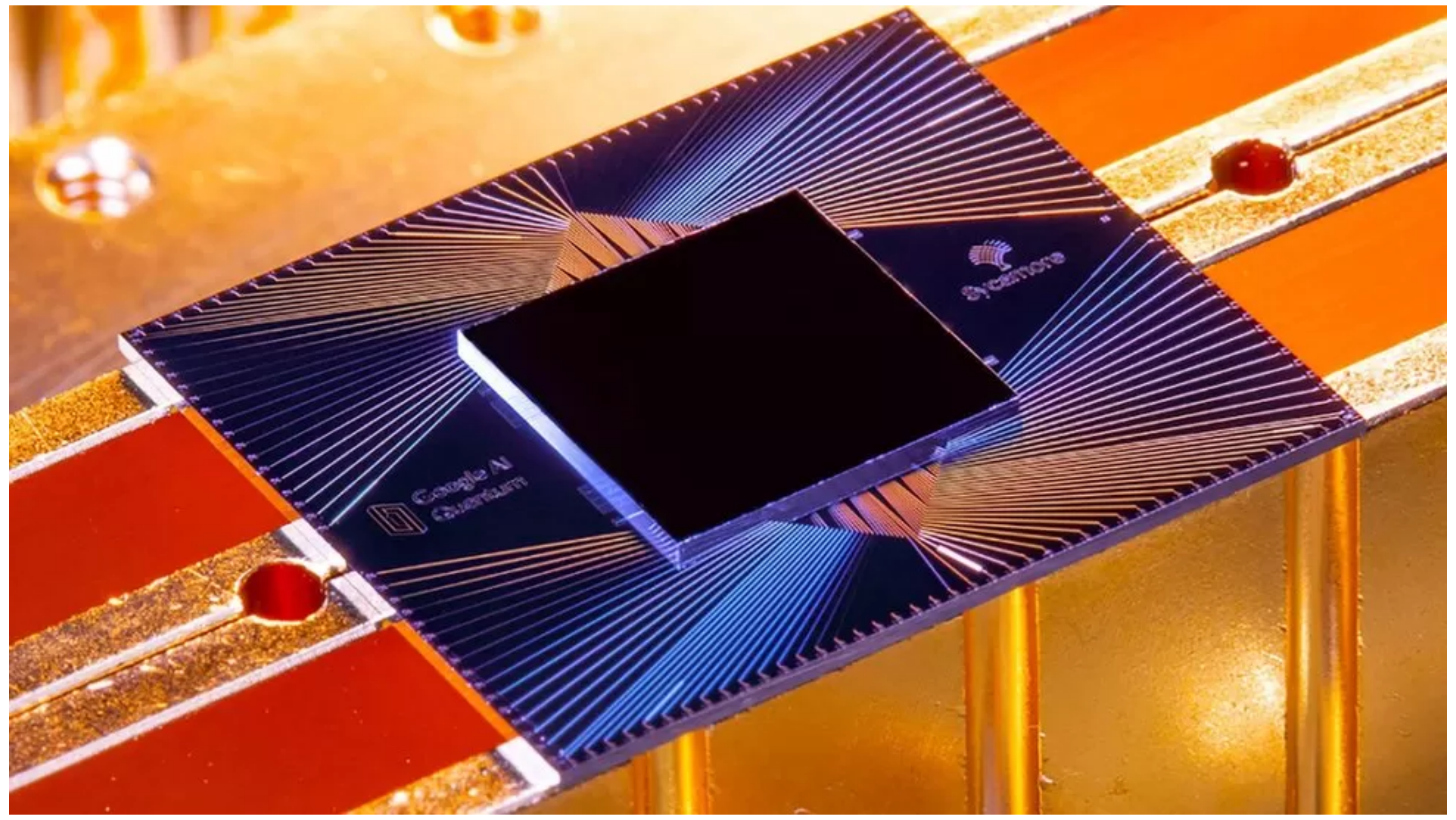
Present-Day Quantum Computers Come in Many Forms

Superconducting Qubits



Google Quantum AI

Oxford: Peter Leek

Google Quantum AI

YQ

IBM Quantum

QUDEV

ALICE & BOB

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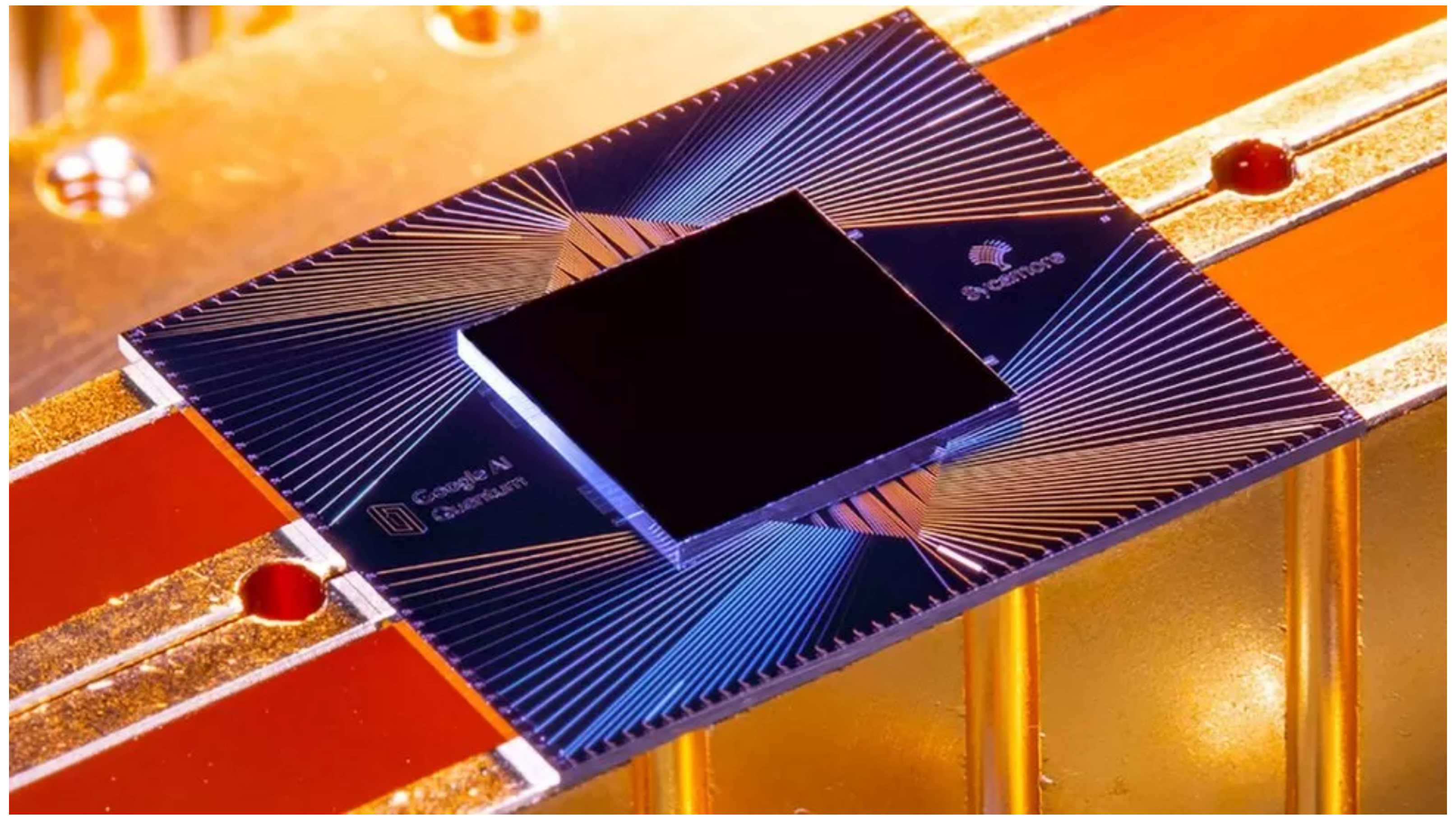

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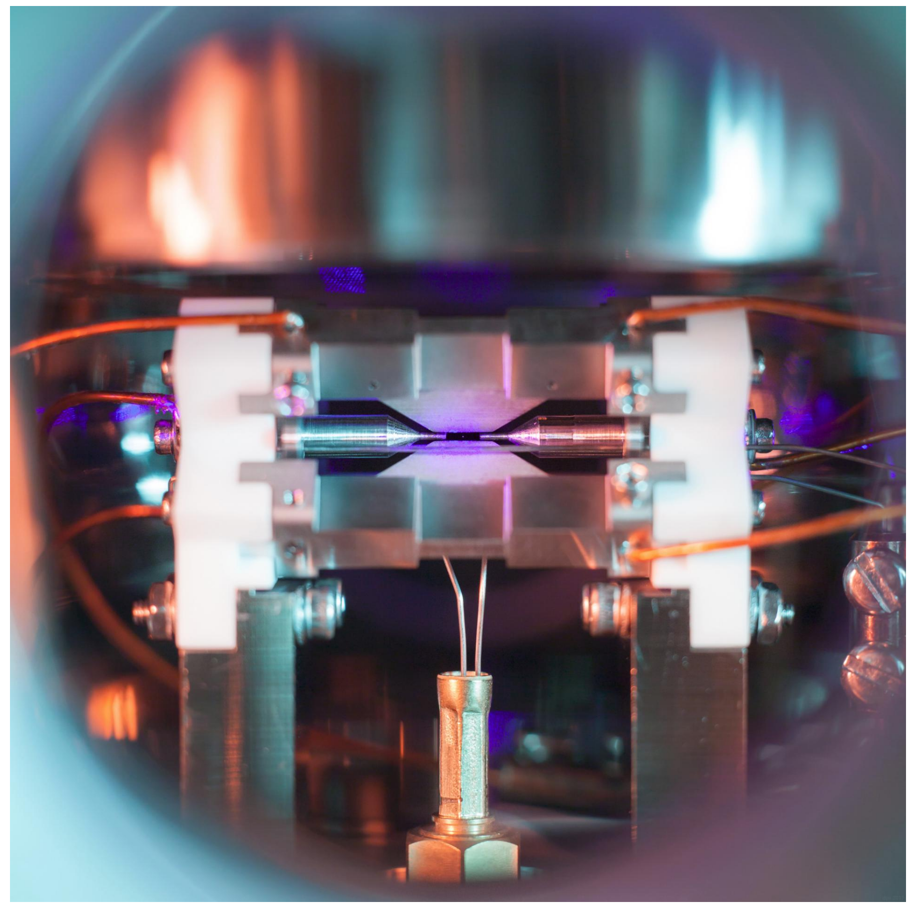
Trapped Ions



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NIST

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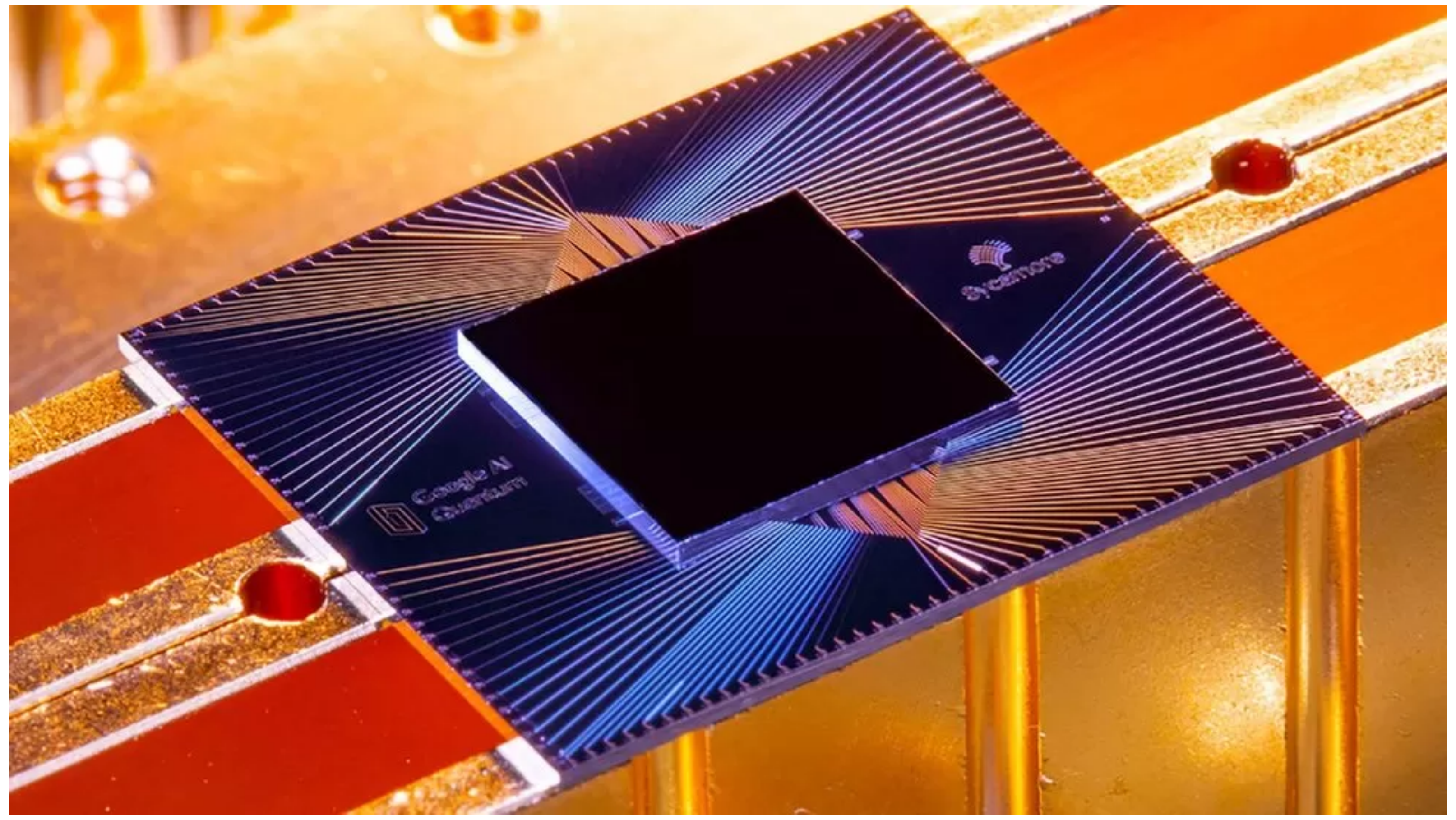

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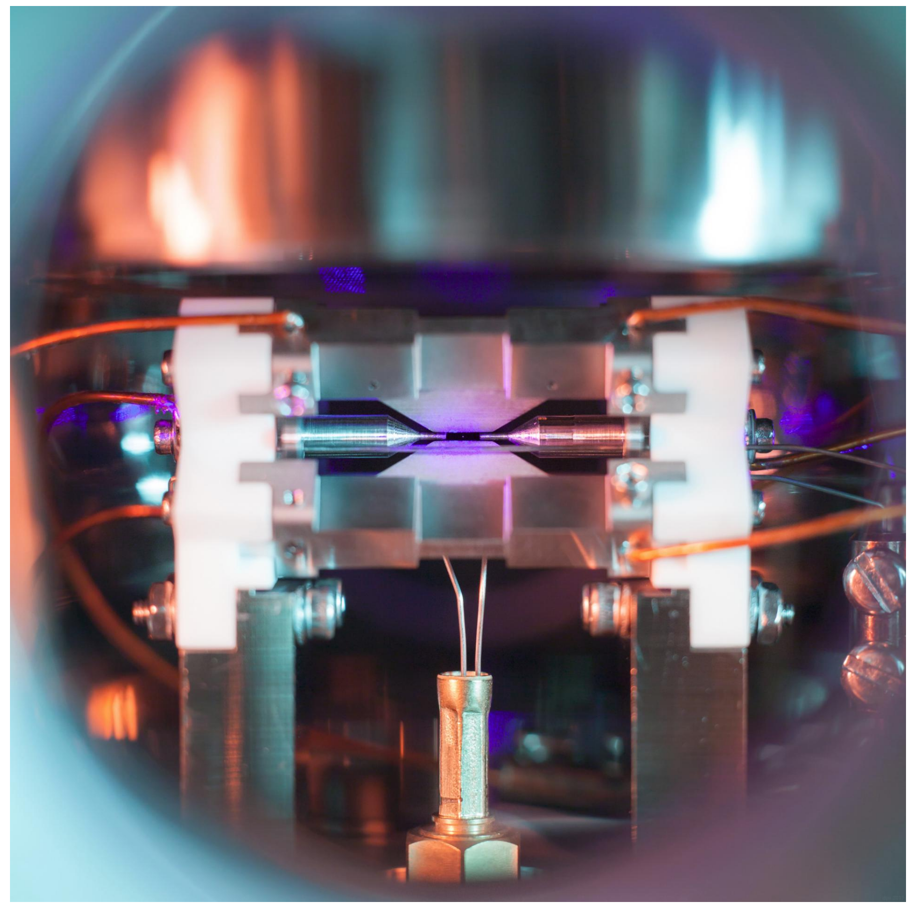

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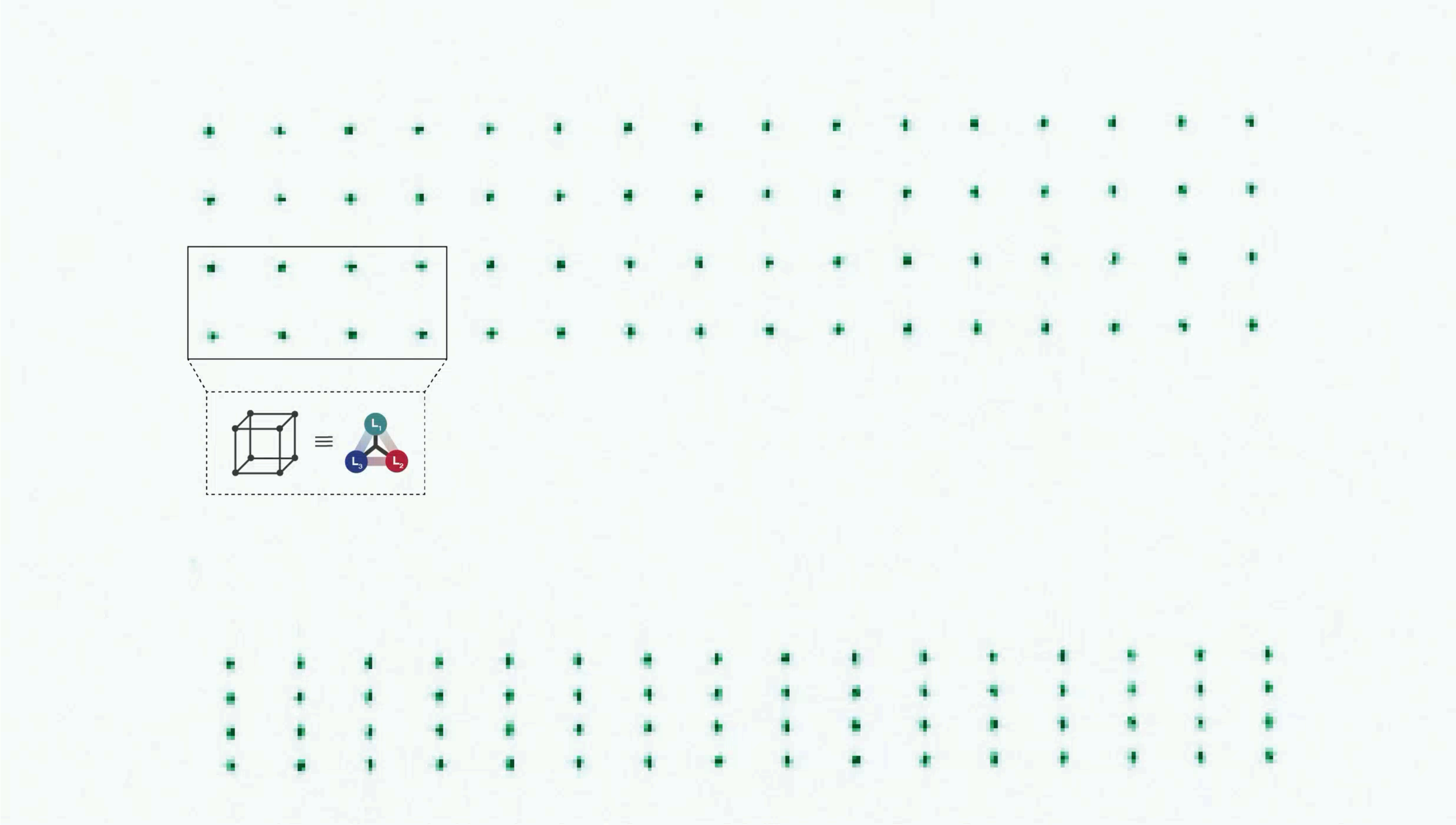
Reconfigurable Atom Arrays



Harvard / QuEra

|QuEra> Computing Inc.

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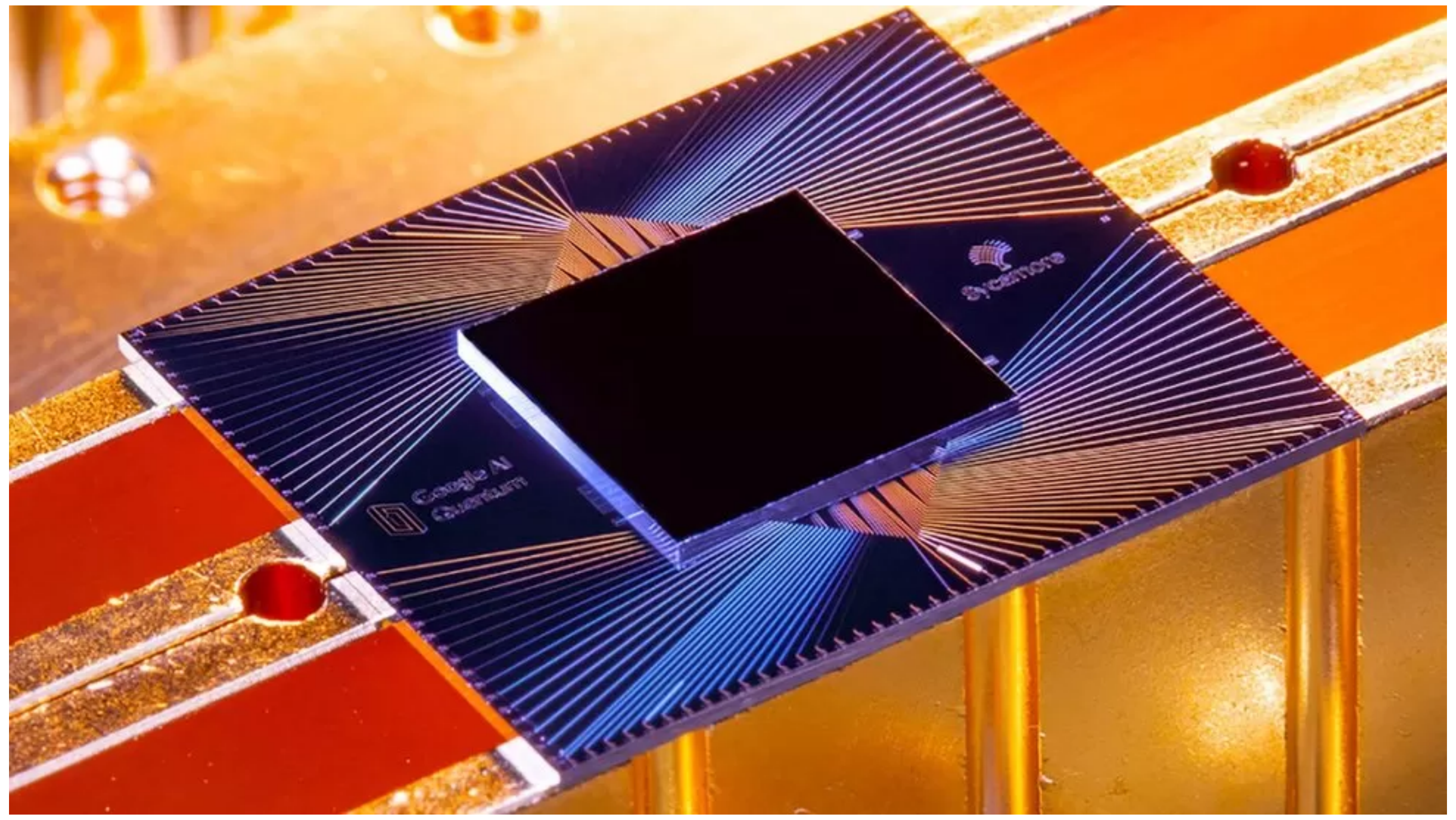

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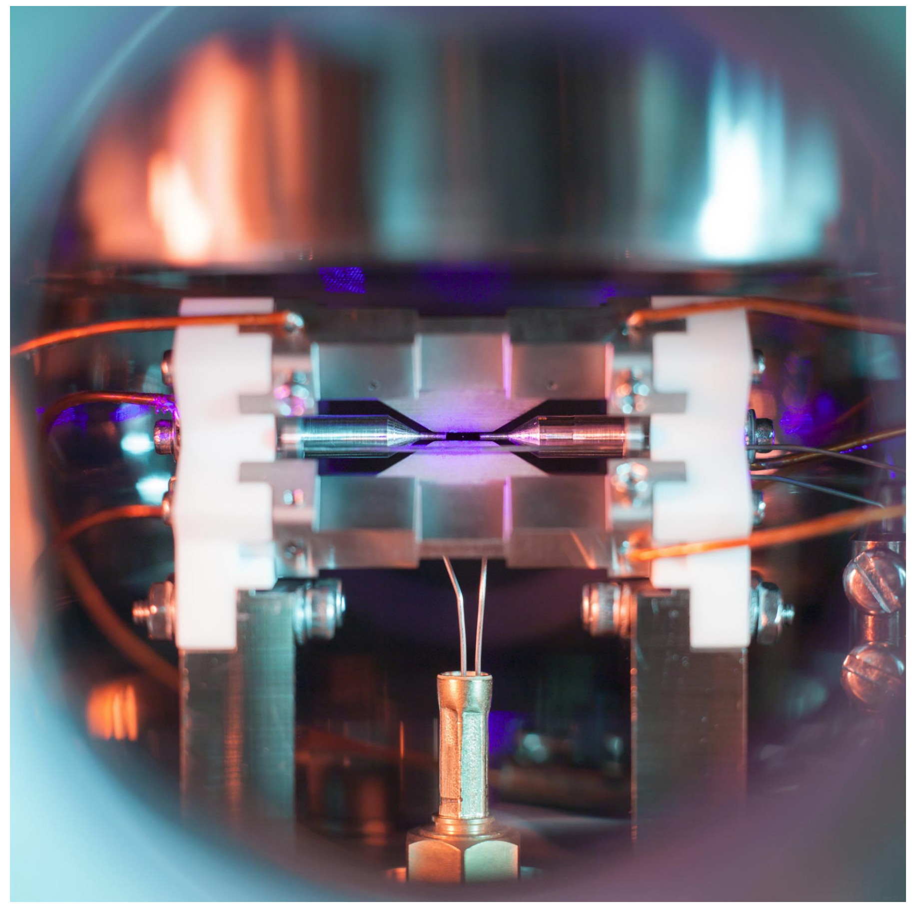

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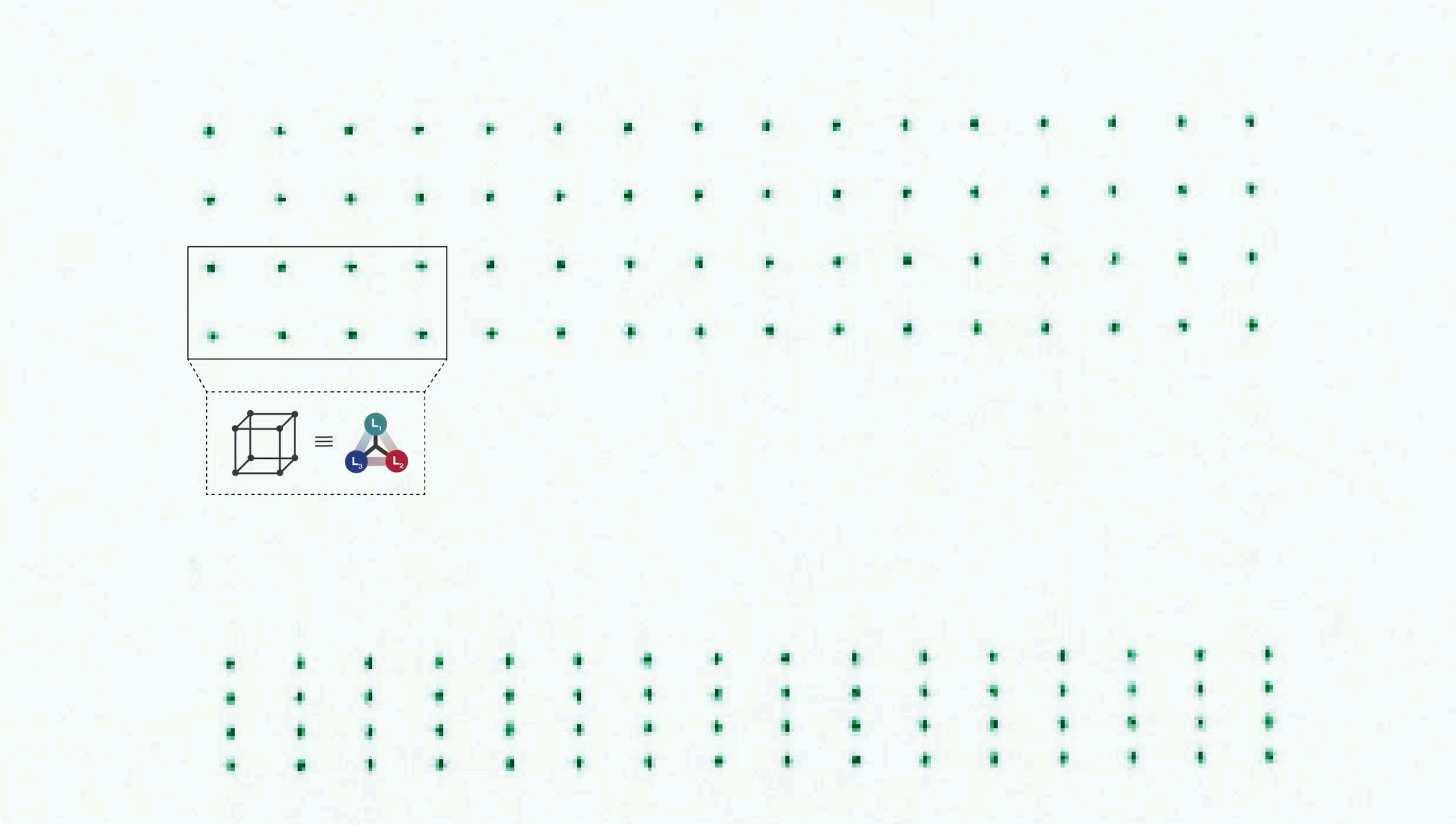

Harvard / QuEra

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Computing Inc.

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And many other platforms ...

PsiQuantum	Photonic
D-Wave	Quantum Annealers
QUANTUM MOTION	Spin Qubits

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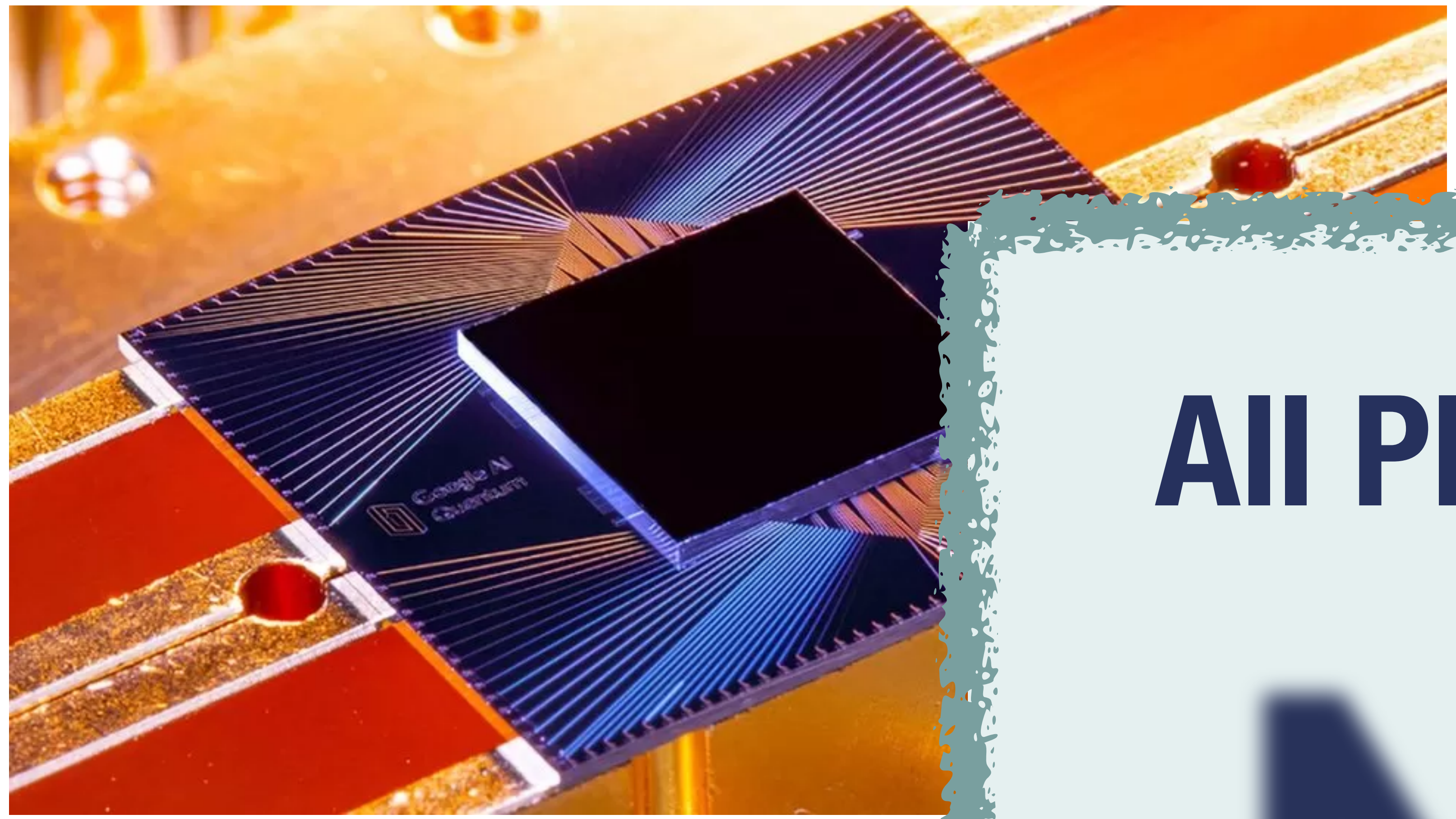

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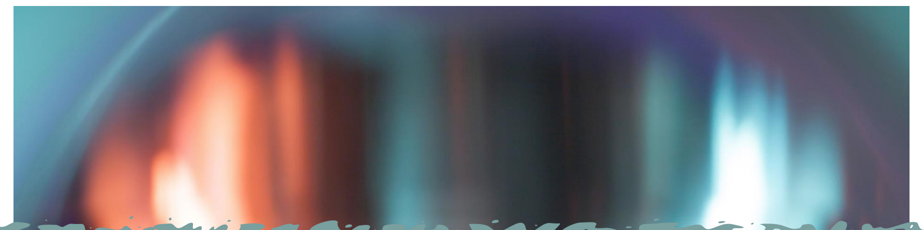

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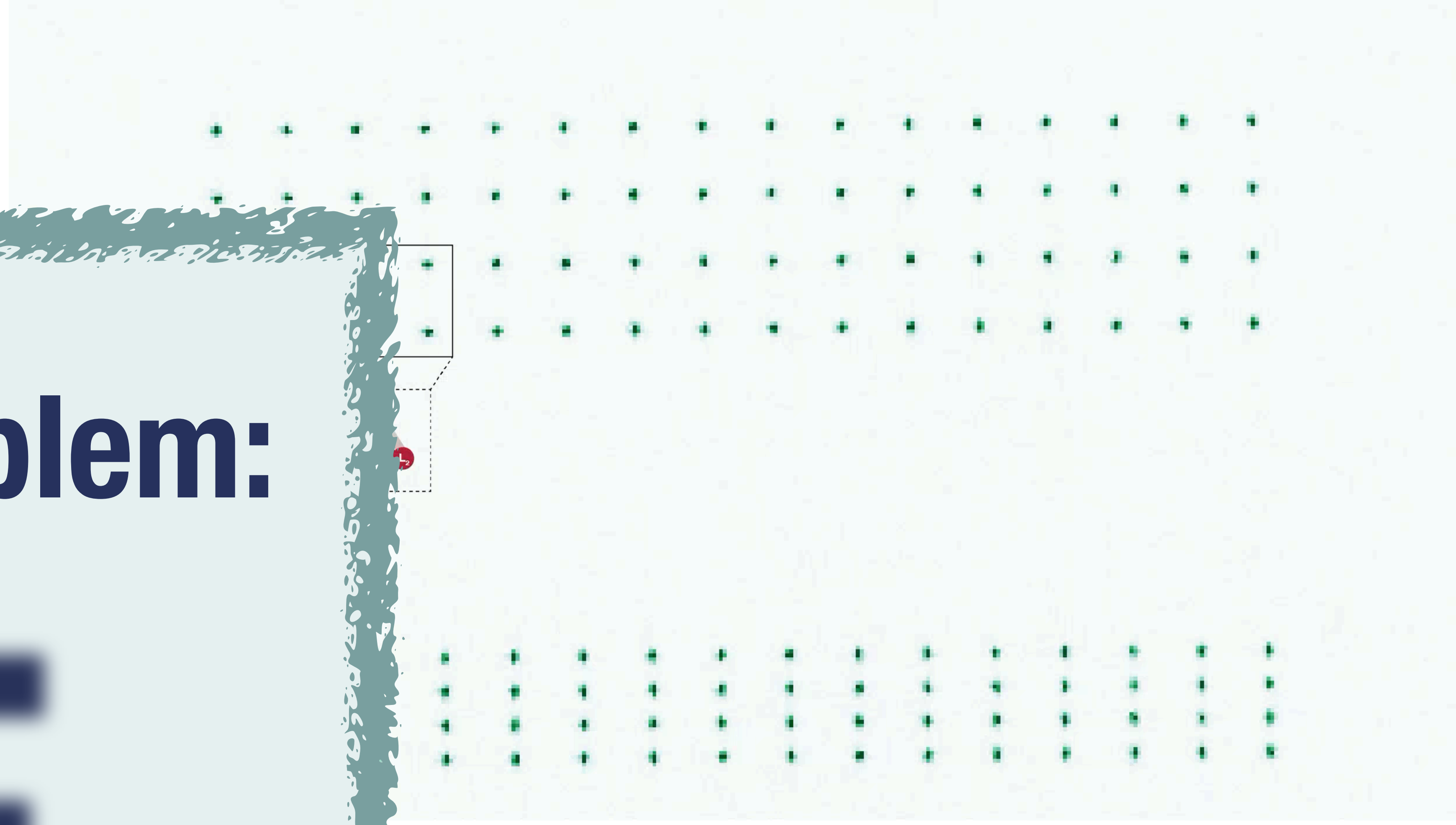

Harvard / QuEra

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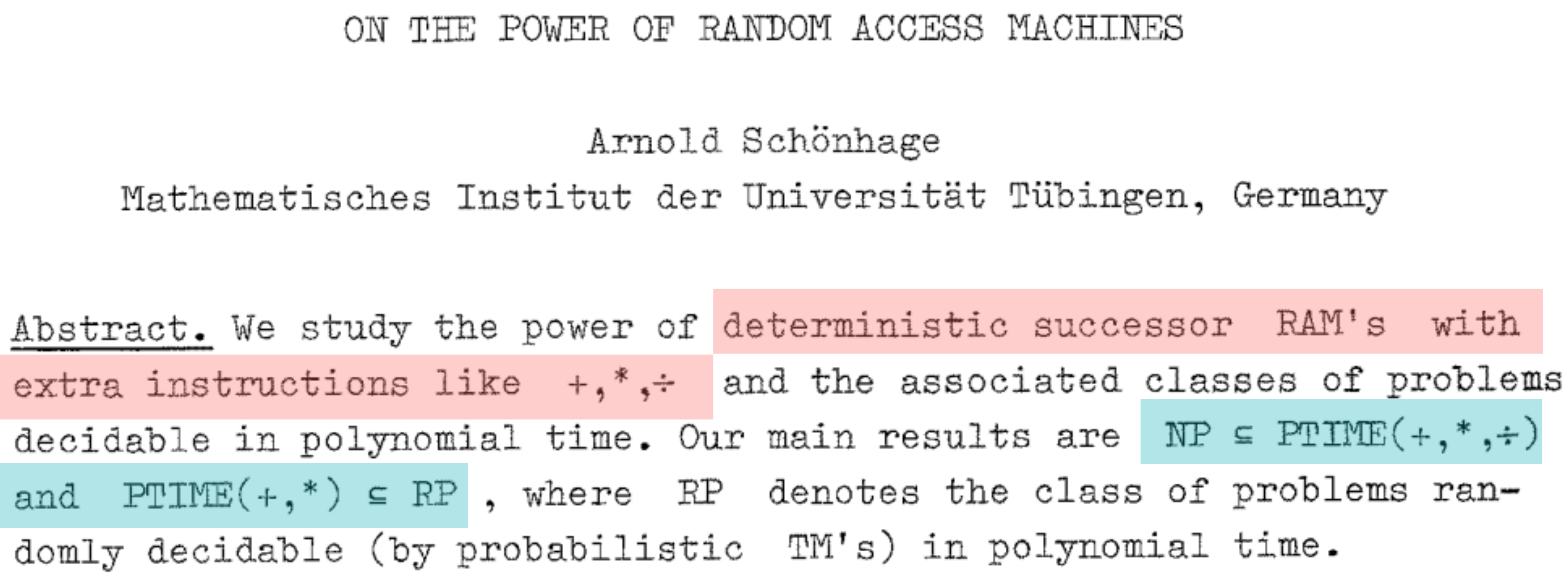
All Platforms Share One Problem:

NOISE

Quantum Computing is not the first “alternative” Idea…



ON THE POWER OF RANDOM ACCESS MACHINES

Arnold Schönhage
Mathematisches Institut der Universität Tübingen, Germany

Abstract. We study the power of deterministic successor RAM's with extra instructions like +,*,÷ and the associated classes of problems decidable in polynomial time. Our main results are $NP \subseteq PTIME(+,*,\div)$ and $PTIME(+,*) \subseteq RP$, where RP denotes the class of problems randomly decidable (by probabilistic TM's) in polynomial time.

Schönhagen (1979); https://doi.org/10.1007/3-540-09510-1_42

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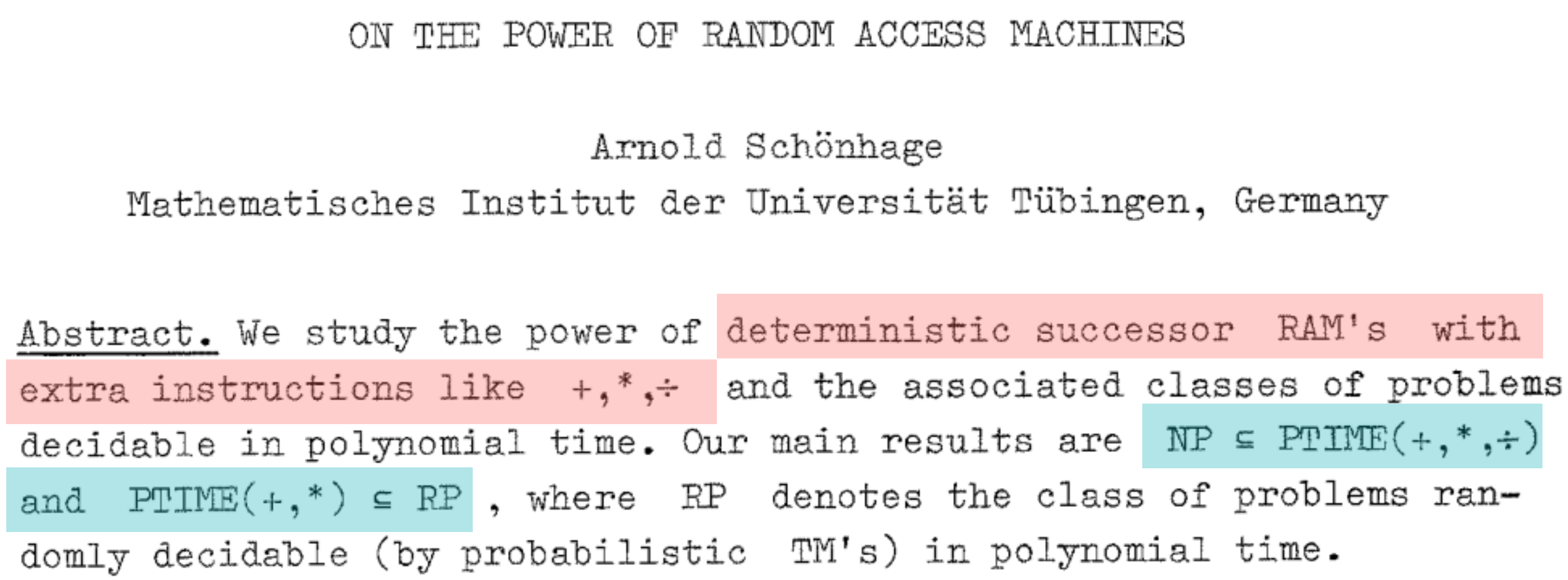

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Suppose we had access an
Random Access Machine

- Memory can store **floating point numbers** $\vec{\sigma} = (f_1, f_2, \ldots, f_n)$
- Can perform deterministic, **arbitrarily precise arithmetic** $(+ , * , \div)$ on $\vec{\sigma}$

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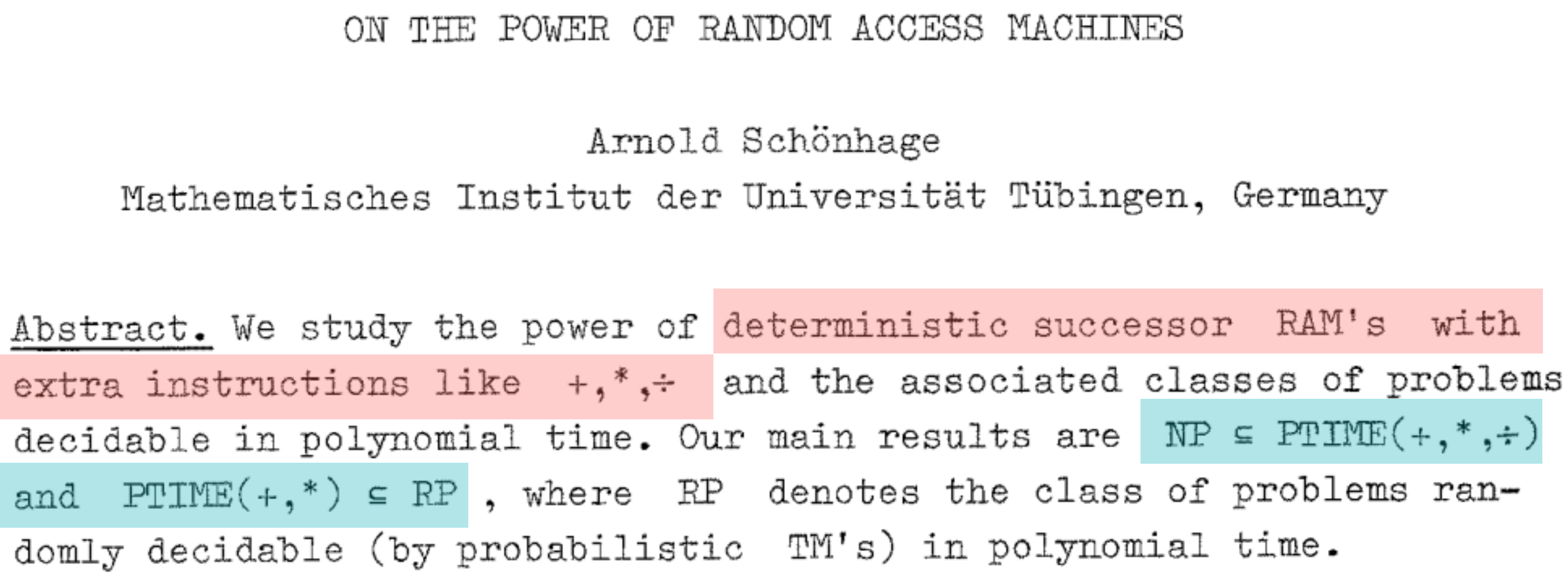

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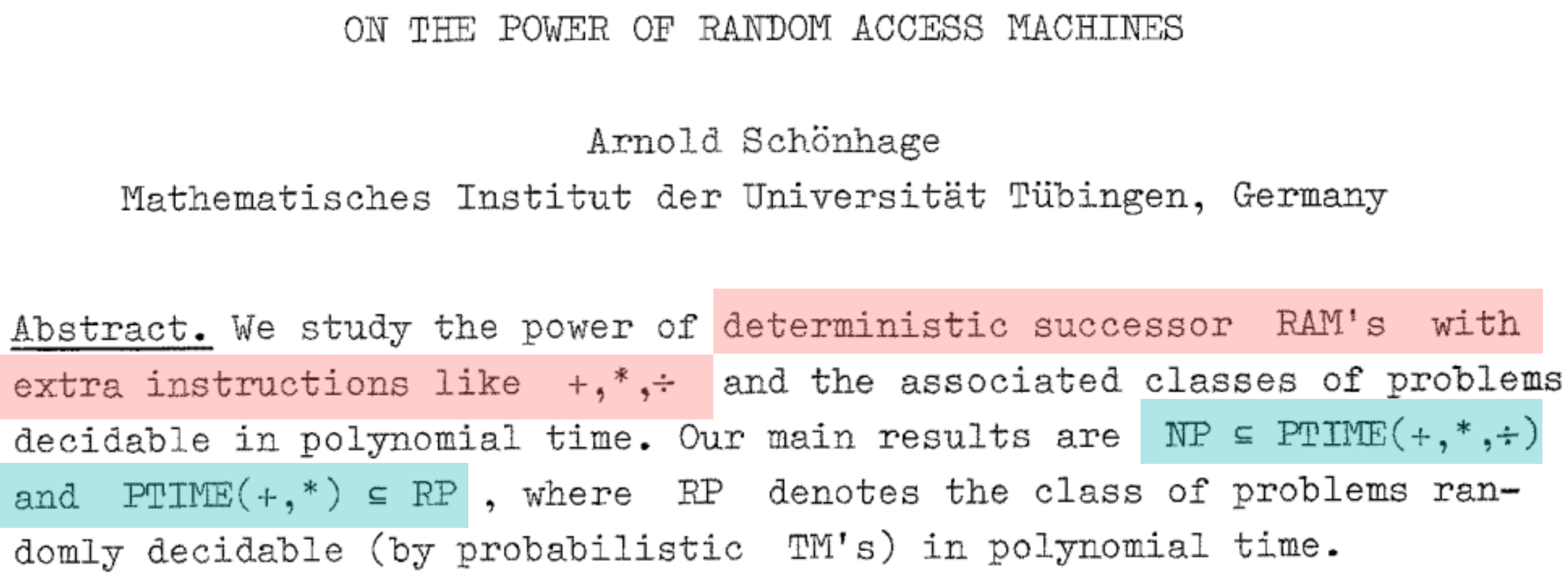

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but… **NOISE**

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No known way to avoid accumulation of errors in floating point arithmetic

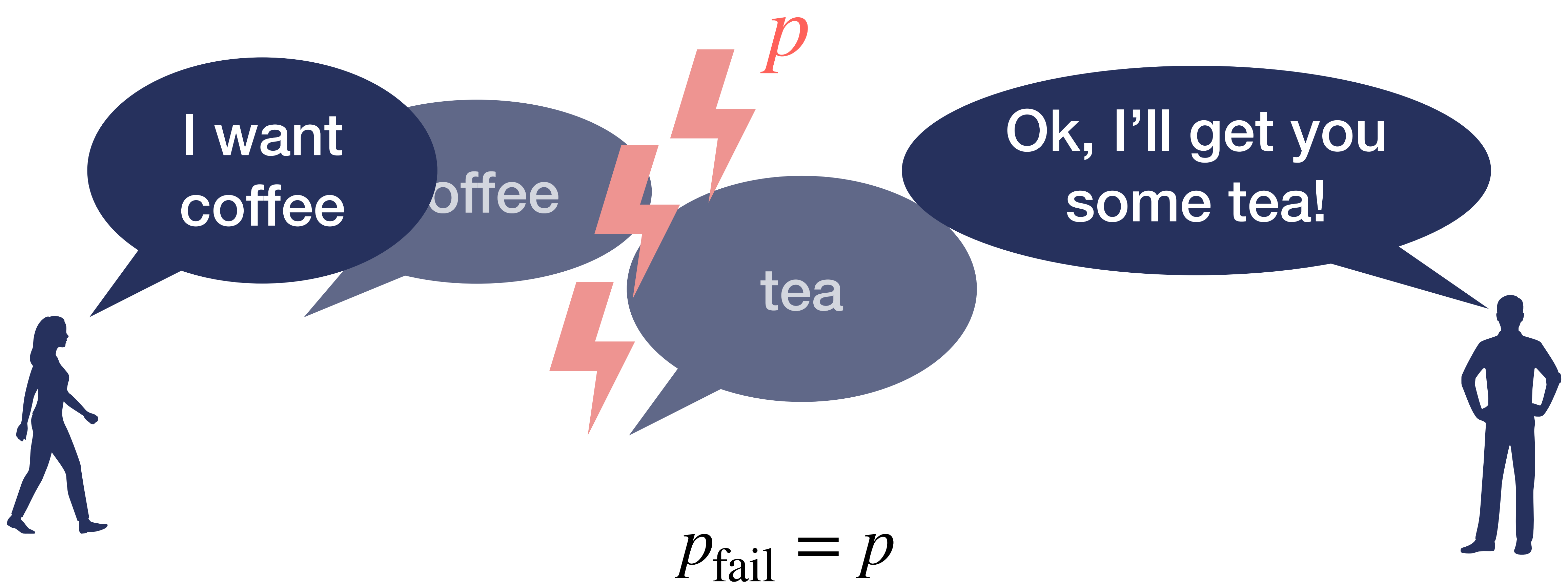
No known fault-tolerant implementation of RAMs 🙁

So what does work, anyway?

Classical, Discrete Error Correction is Simple!

Imagine sending a single bit through “noise”

$$\sigma = 0 \quad \xrightarrow{\text{Noise}} \quad \sigma = \begin{cases} 1 & \text{with prob}.\ p \\ 0 & \text{with prob}.\ 1-p \end{cases}$$



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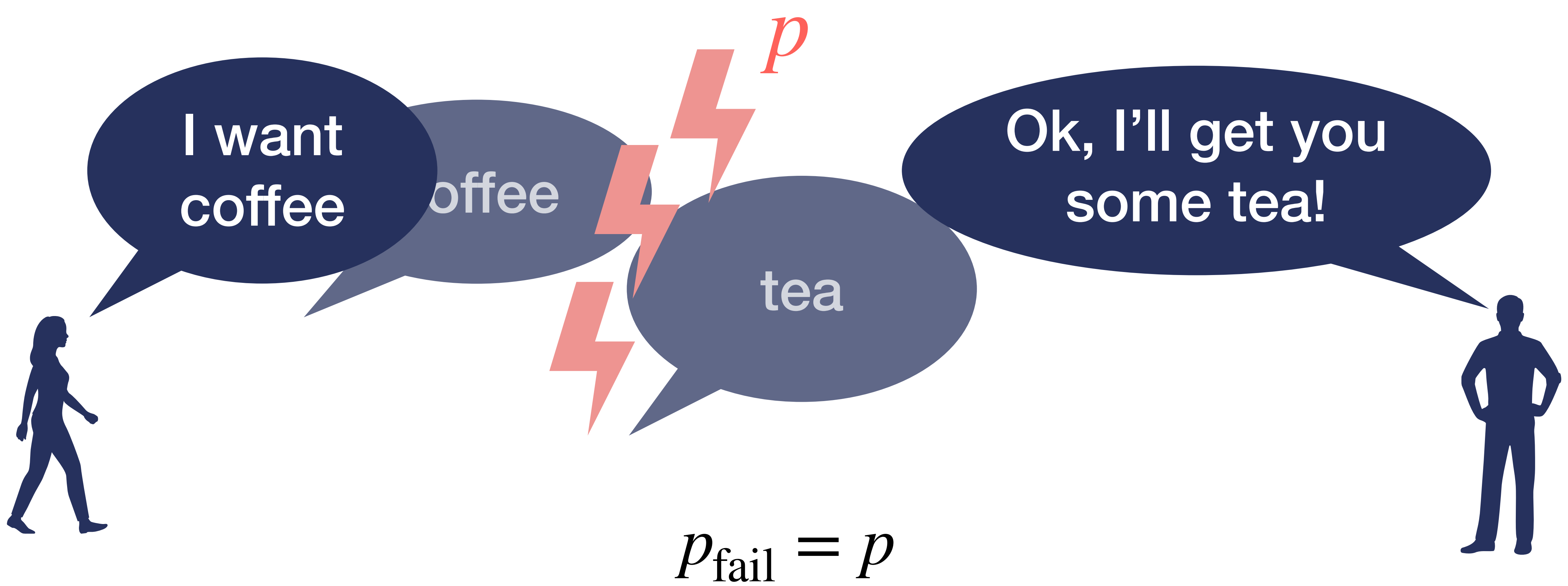
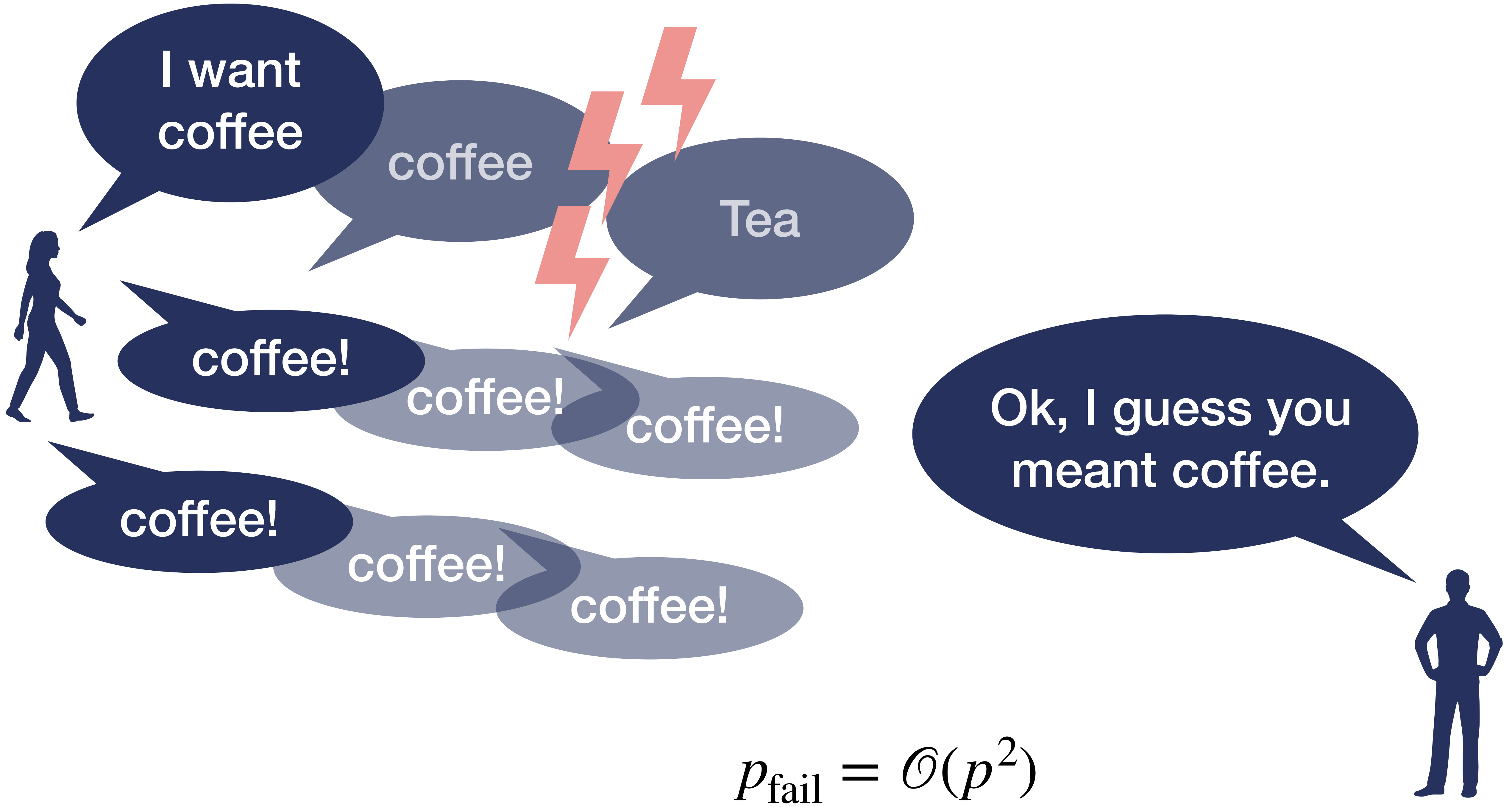


$$p_{\text{fail}} = p$$

We ourselves do simple error correction intuitively:
the repetition code

$$\bar{0} = \underbrace{0\ldots0}_{n\text{ times}} \qquad \bar{1} = \underbrace{1\ldots1}_{n\text{ times}}$$

If noise flips less than half the bits ($p \ll 1$), we can recover the original state by majority voting.



$$p_{\text{fail}} = \mathcal{O}(p^2)$$

But Quantum Error correction !?

No cloning theorem

We cannot “copy” arbitrary quantum information!

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Proof. Imagine there exists an operation U

$U|\psi\rangle|e\rangle = |\psi\rangle|\psi\rangle$ for all states ψ and some auxiliary state e

Then $\langle\psi|\phi\rangle\underbrace{\langle e|e\rangle}_{=1} = \langle\psi|\langle e|\underbrace{U^\dagger U}_{=1}|\phi\rangle|e\rangle = \langle\psi|\phi\rangle^2$

$\Rightarrow \langle\psi|\phi\rangle \in \{0,1\}$ So ψ and ϕ are identical or orthogonal!

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We cannot do “majority voting” since measurements are projective

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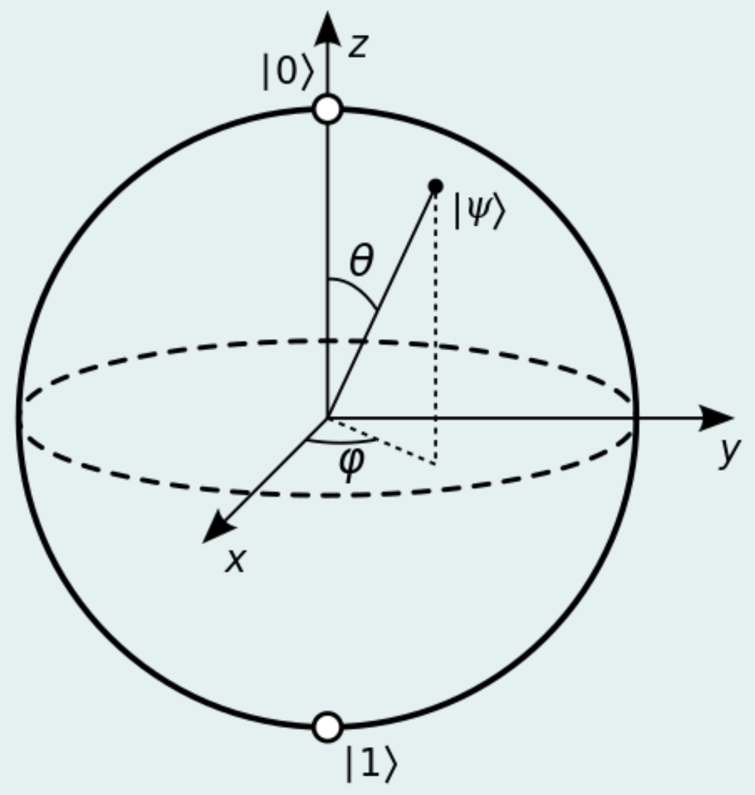
Errors on Bloch sphere are continuous



- Errors can be arbitrarily small rotations on Bloch Sphere
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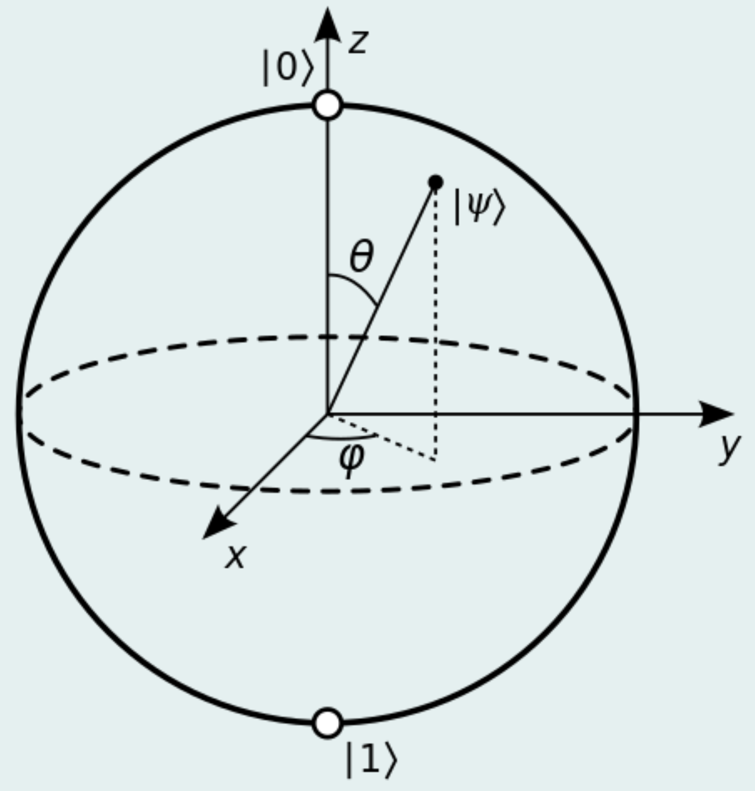

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Surprisingly:

Shor (1995) and Steane (1996)

Quantum Error Correction is Possible!

But we need all the weirdness (and beauty) of Quantum Mechanics to make it work

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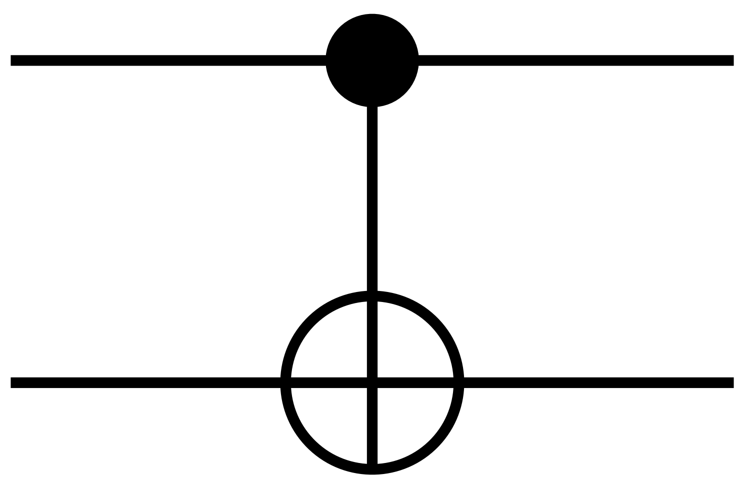
The Quantum Repetition Code (1/2)

Cloning any state is not necessary!



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$\alpha|00\rangle + \beta|11\rangle$

CNOT gate

- Quantum CNOT clones states $|0\rangle$ and $|1\rangle$ (which are orthogonal)
- Produces redundancy by <u>entanglement</u>

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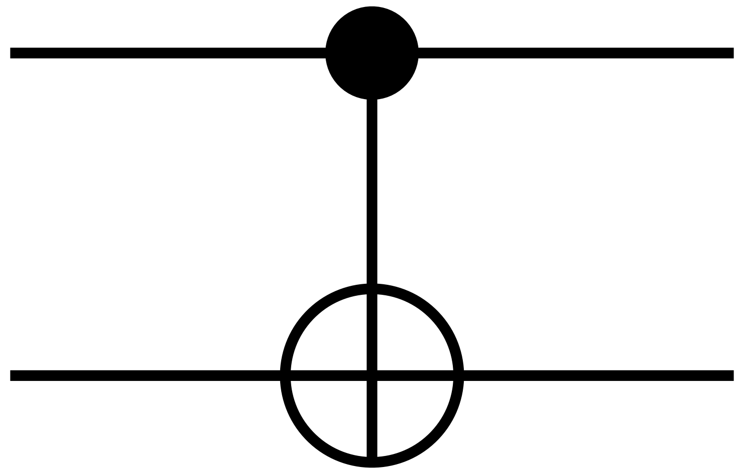

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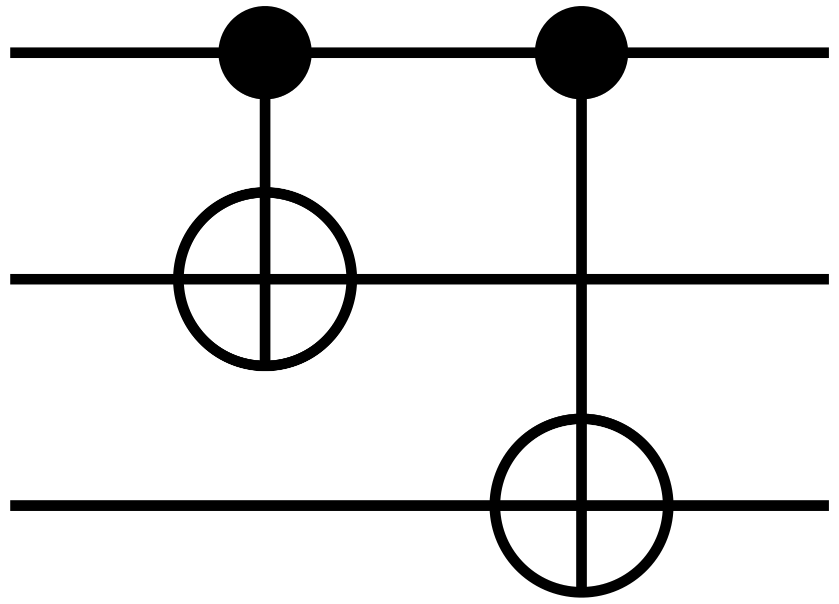
The quantum version of our redundant coffee/tea order:



$|\psi\rangle$

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$$\underbrace{\alpha|000\rangle + \beta|111\rangle}_{= |\bar{\psi}\rangle}$$

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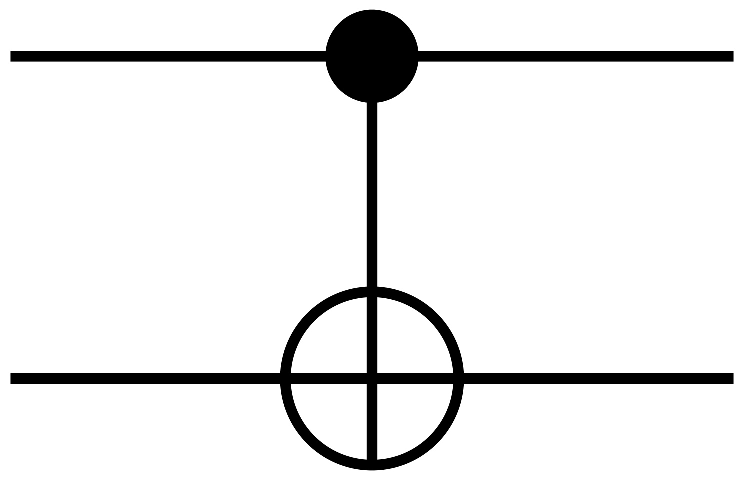

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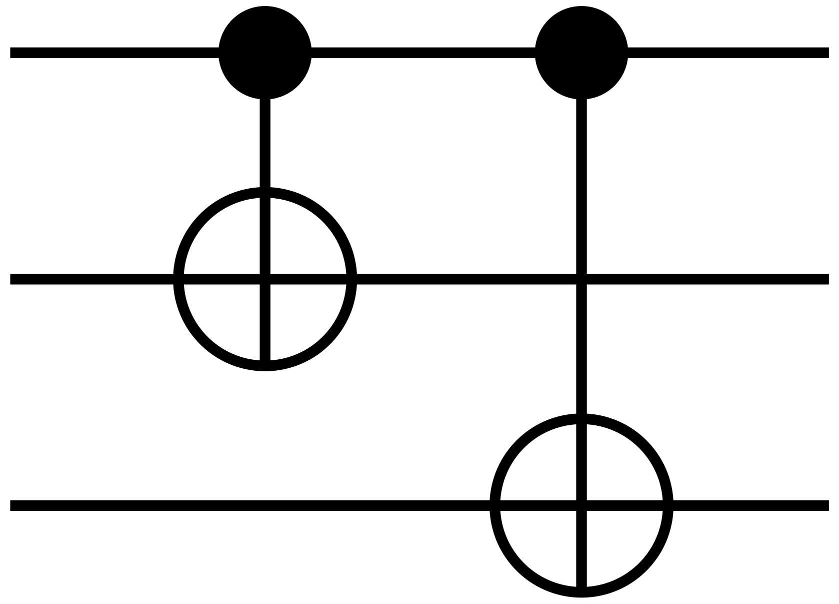

$|\psi\rangle$

$|0\rangle$

$|0\rangle$

$$\underbrace{\alpha|000\rangle + \beta|111\rangle}_{= |\bar{\psi}\rangle}$$

What about measurements?

$$|\bar{\psi}\rangle = \alpha|000\rangle + \beta|111\rangle$$

measure Z_1 “what is the value of the first bit”?

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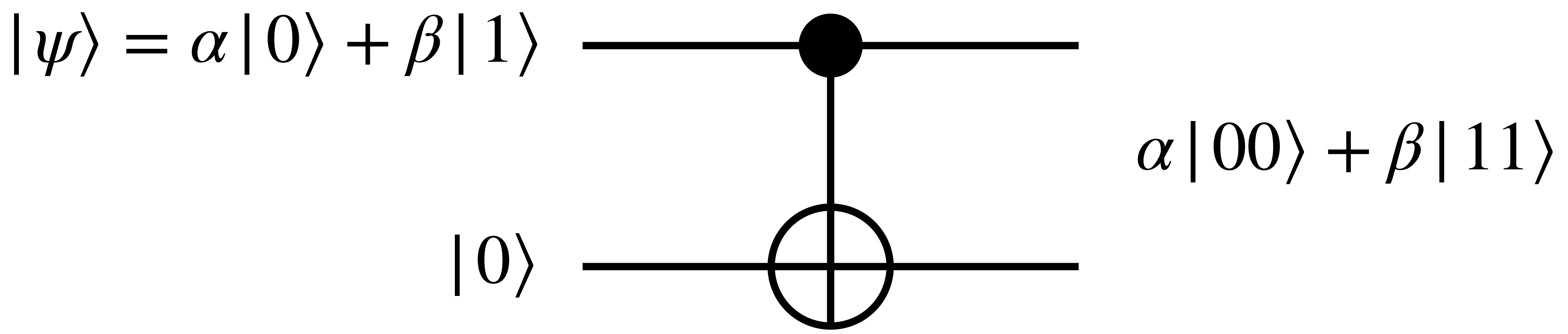
The Quantum Repetition Code (1/2)

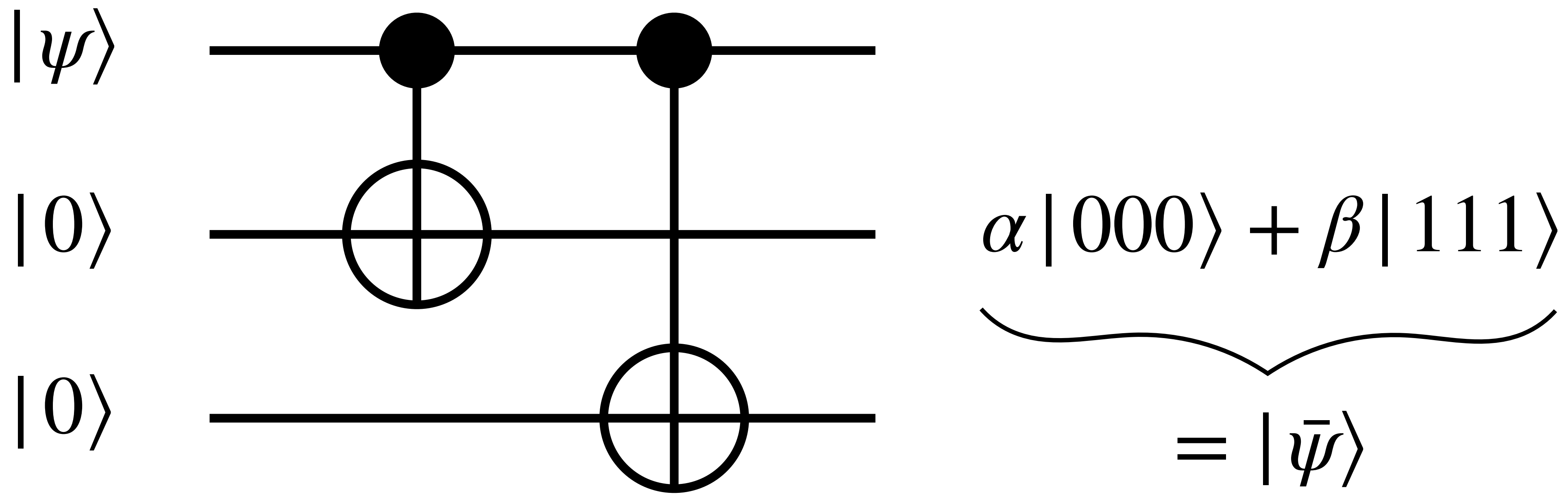
Cloning any state is not necessary!



CNOT gate

- Quantum CNOT clones states $|0\rangle$ and $|1\rangle$ (which are orthogonal)
- Produces redundancy by <u>entanglement</u>

The quantum version of our redundant coffee/tea order:



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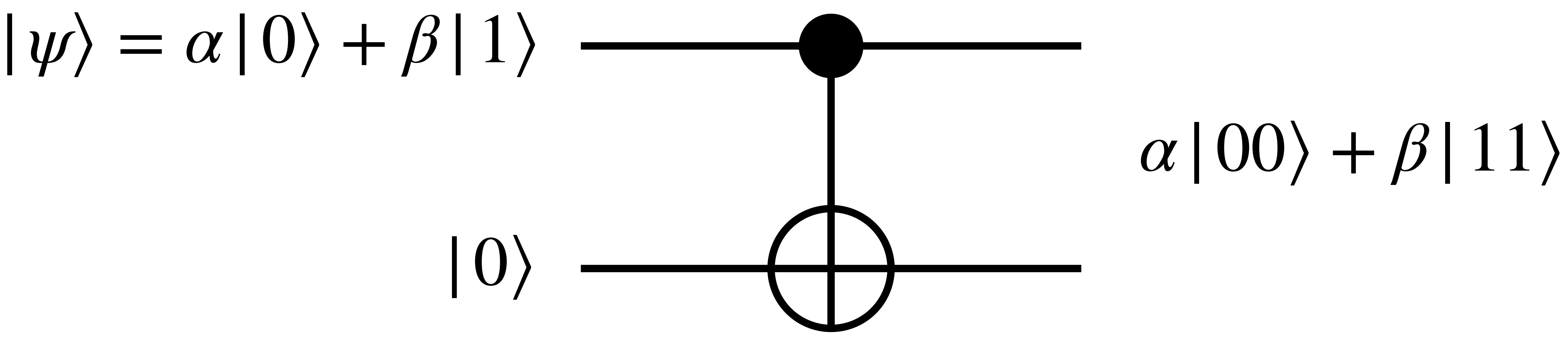

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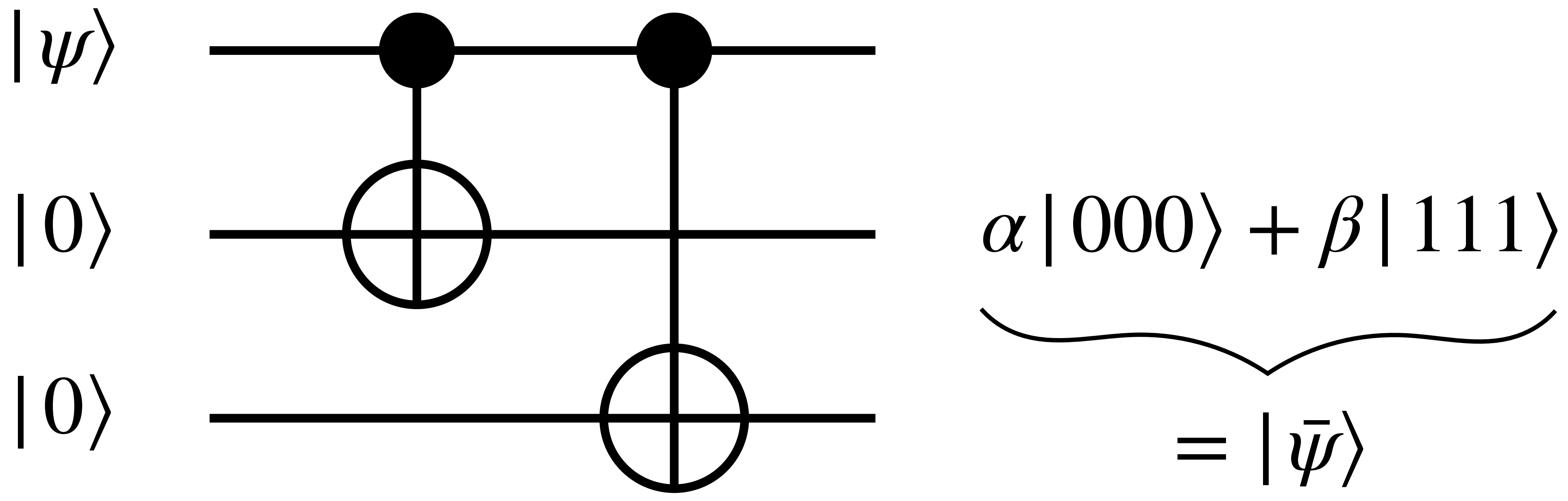

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Other parity Z_2Z_3 also well defined!

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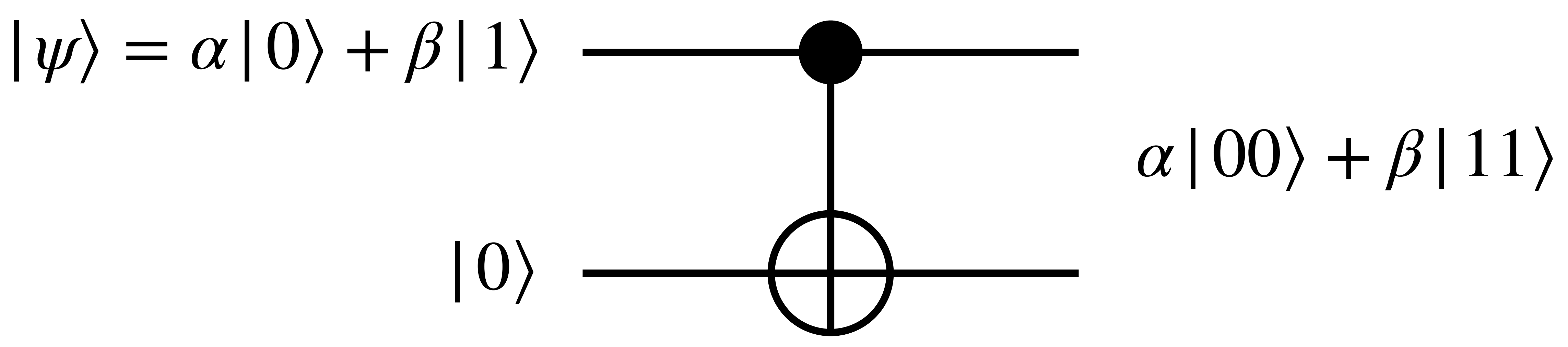

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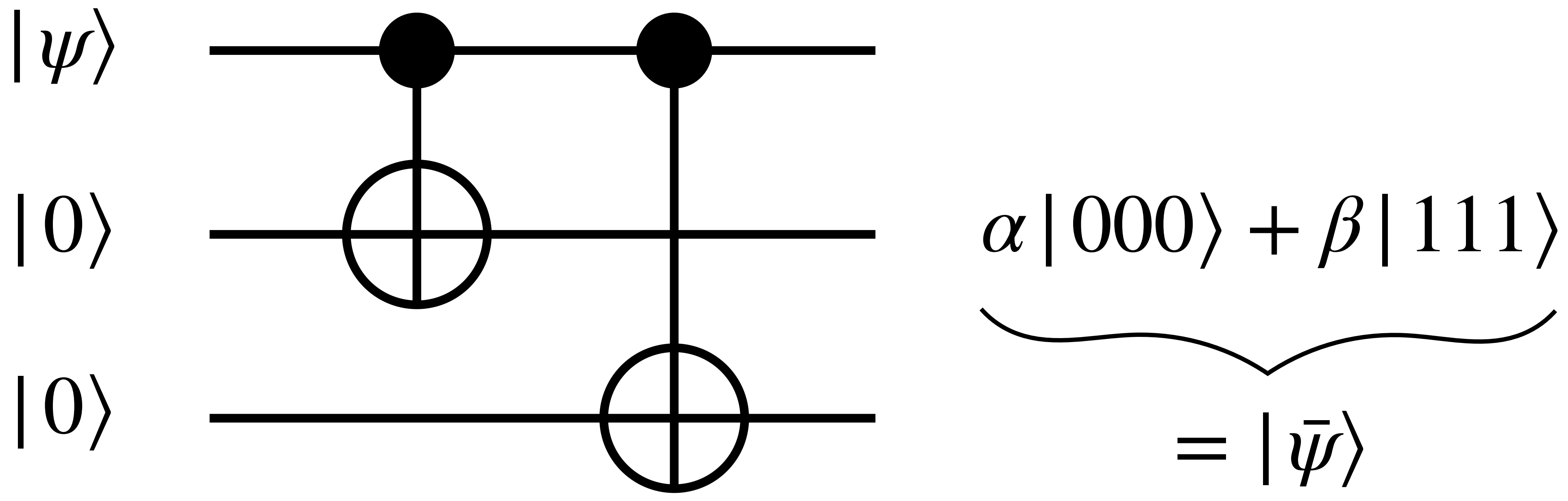

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Measuring either parity in $|\bar{\psi}\rangle$ will

$\Rightarrow$ will yield $+1$ (=) <u>with certainty</u>

$\Rightarrow$ will leave the state <u>invariant</u>

$\Rightarrow$ can be used to diagnose errors! (next slide)

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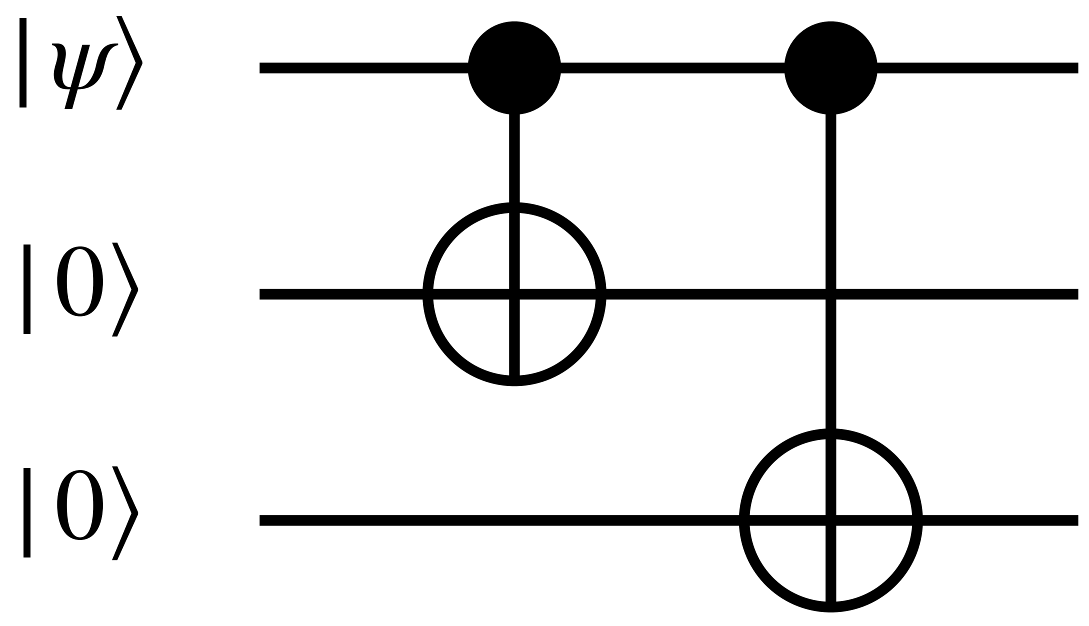
The Quantum Repetition Code (2/2)



Encoded State

$$\underbrace{\alpha|000\rangle + \beta|111\rangle}_{= |\bar{\psi}\rangle}$$

Parity Measurements

$Z_1Z_2 \quad Z_2Z_3$ "are first/last two bits equal?"

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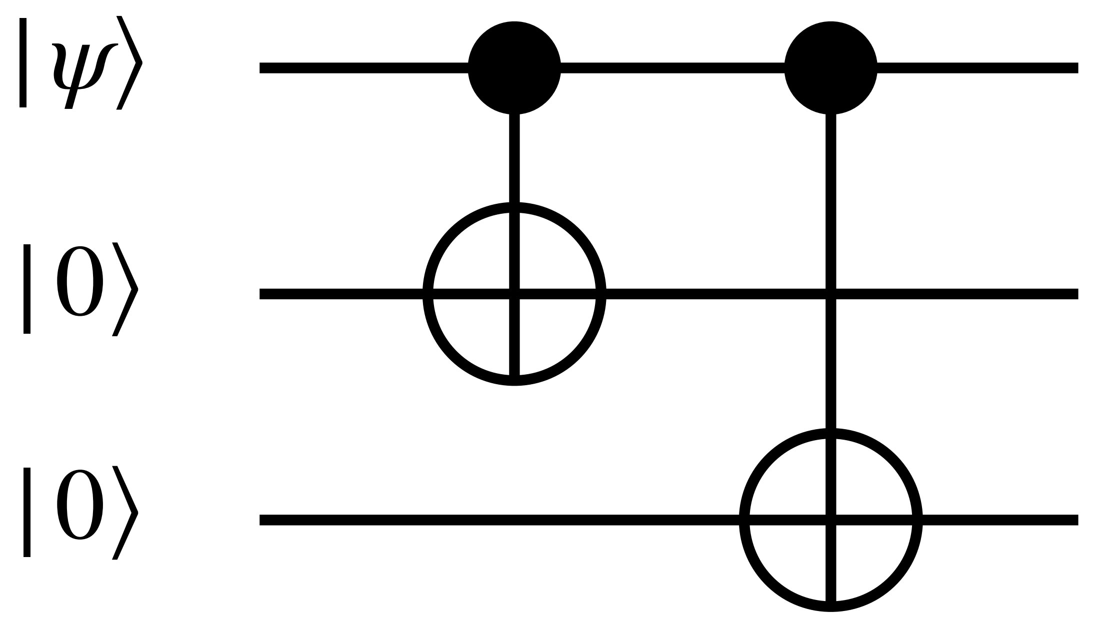

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$$X|0\rangle = |1\rangle$$
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“bit-flips” / Pauli-X

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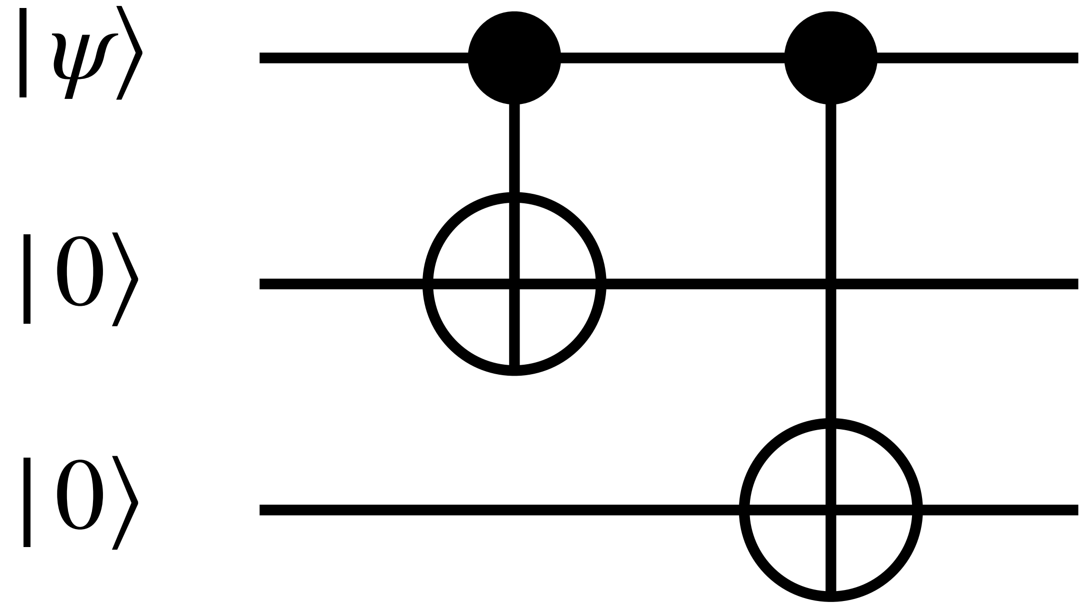

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So what happens to the encoded state?

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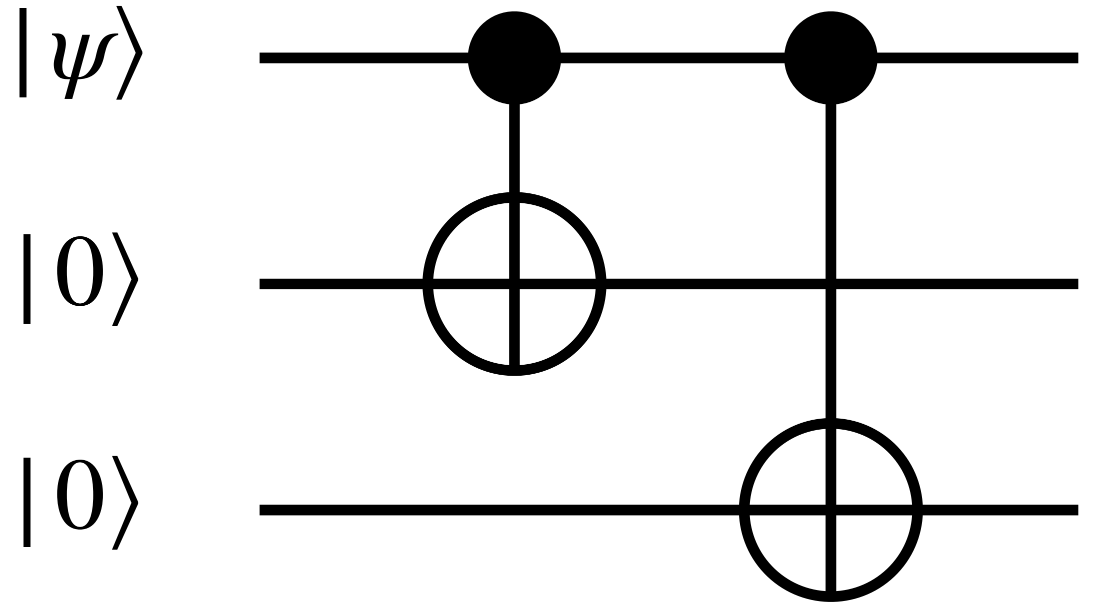

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	Z_1Z_2	Z_2Z_3
$X_1|\bar{\psi}\rangle = \alpha|100\rangle + \beta|011\rangle$		

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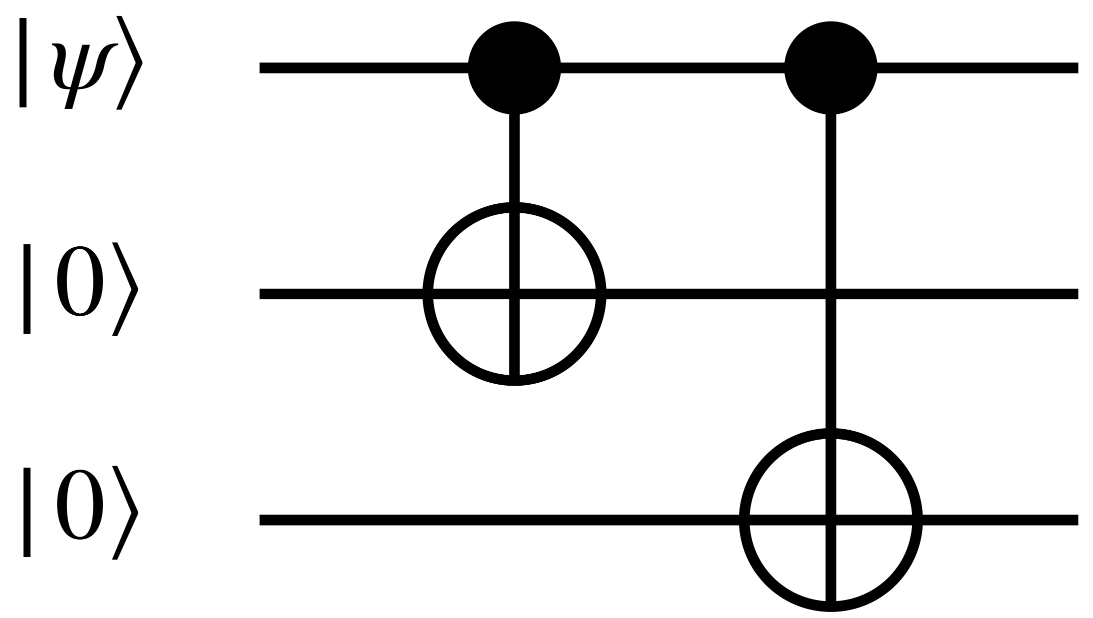
The Quantum Repetition Code (2/2)



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	Z_1Z_2	Z_2Z_3
$X_1|\bar{\psi}\rangle = \alpha|100\rangle + \beta|011\rangle$	$\neq$	$=$

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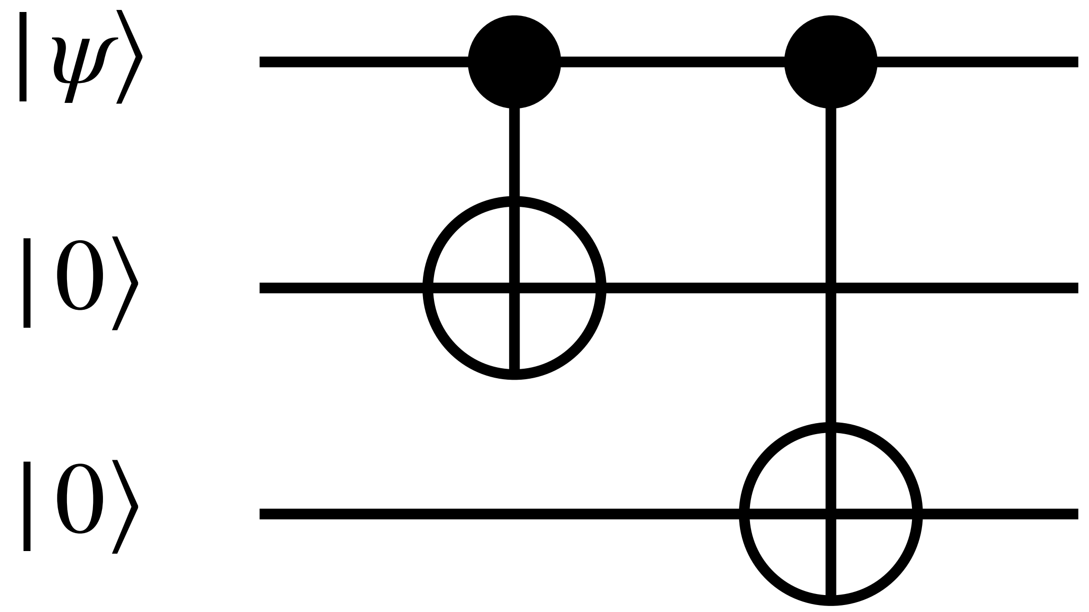

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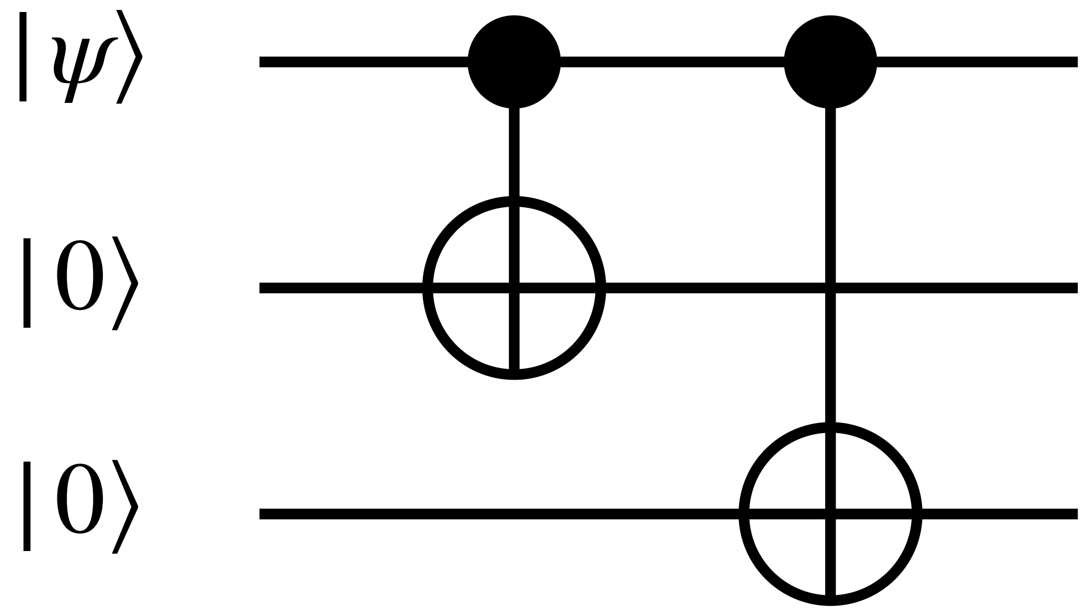

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$X_3|\bar{\psi}\rangle = \alpha|001\rangle + \beta|110\rangle$	$=$	$\neq$

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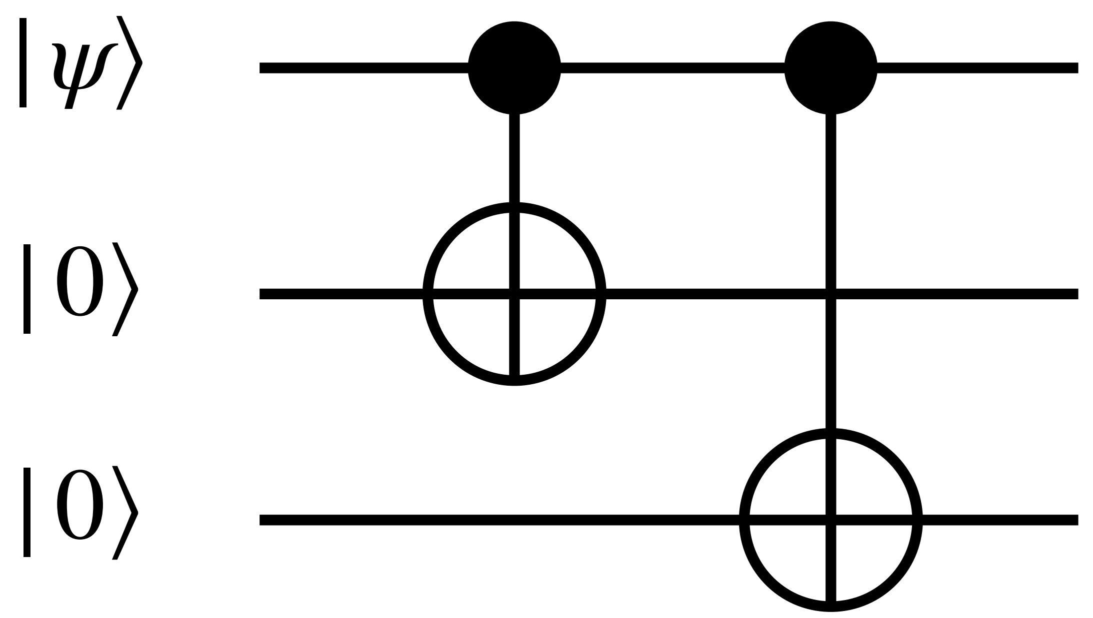
The Quantum Repetition Code (2/2)



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$X_3|\bar{\psi}\rangle = \alpha|001\rangle + \beta|110\rangle$	$=$	$\neq$

Unique outcome of parity measurements in each case : “syndrome”

We can correct (single) bit flips without learning anything about the stored quantum information!

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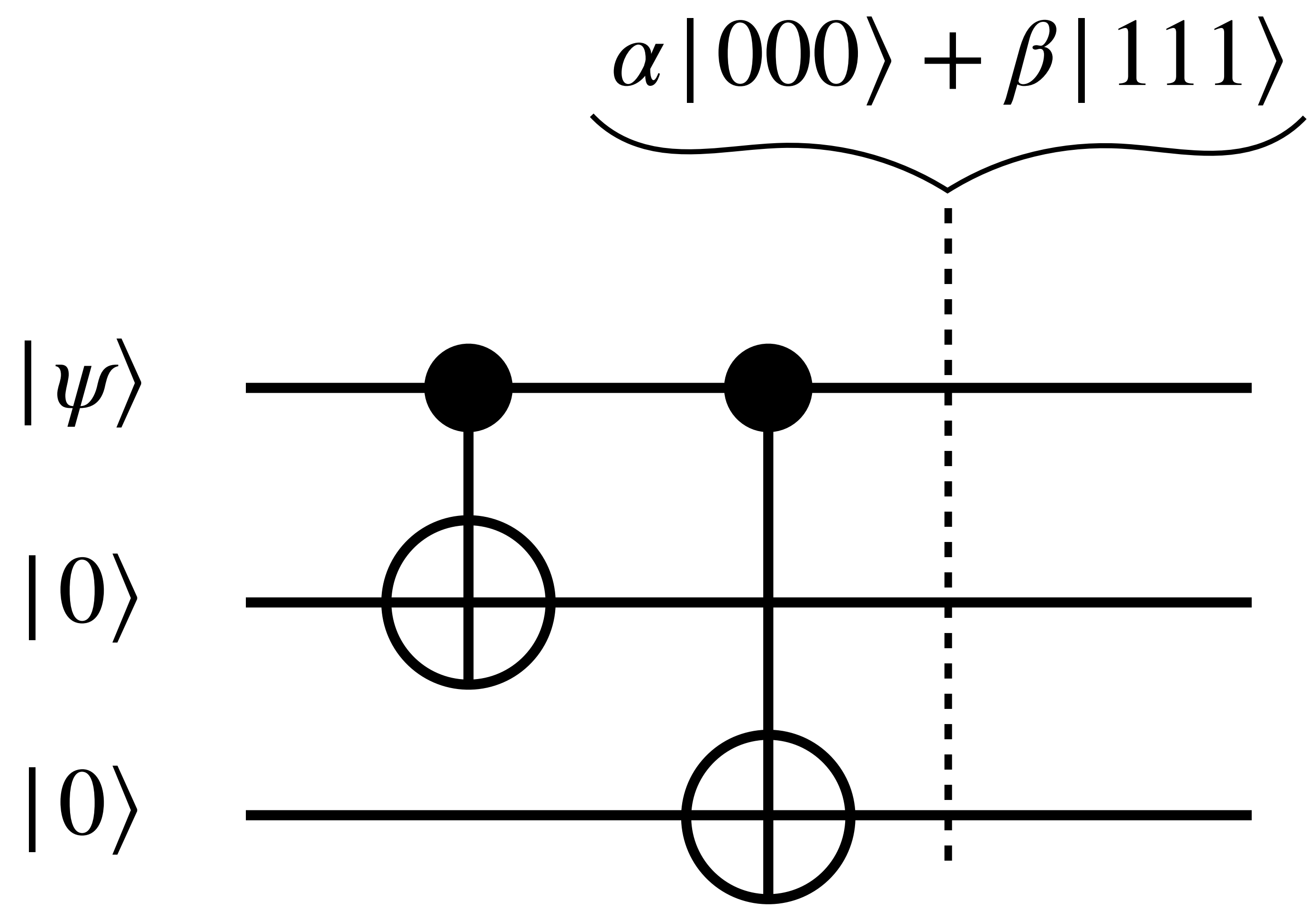
(Just?) Another Repetition Code

Let’s do something more quantum

Bit flip code encoding



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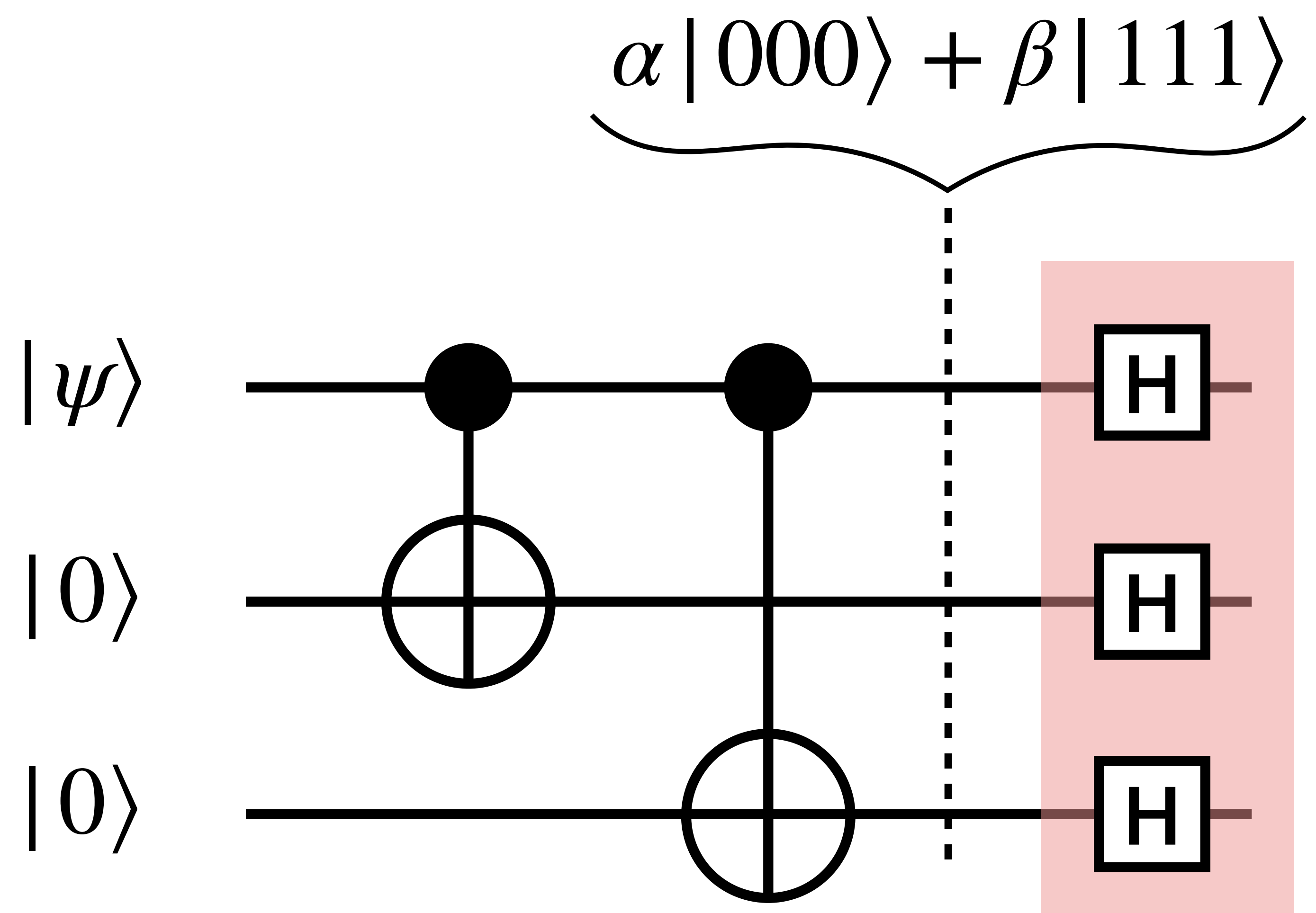
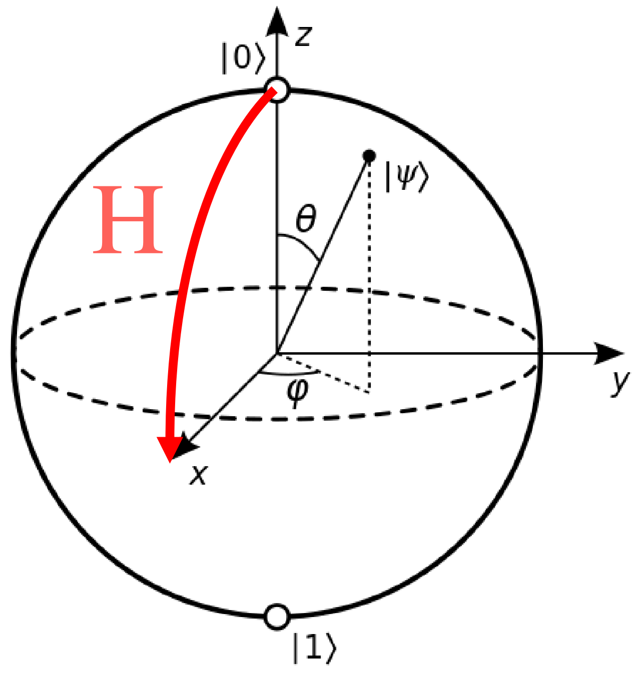
~~Bit~~ Phase flip code encoding



"Hadamard" Gate

$$H|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) =: |+\rangle$$

$$H|1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) =: |-\rangle$$



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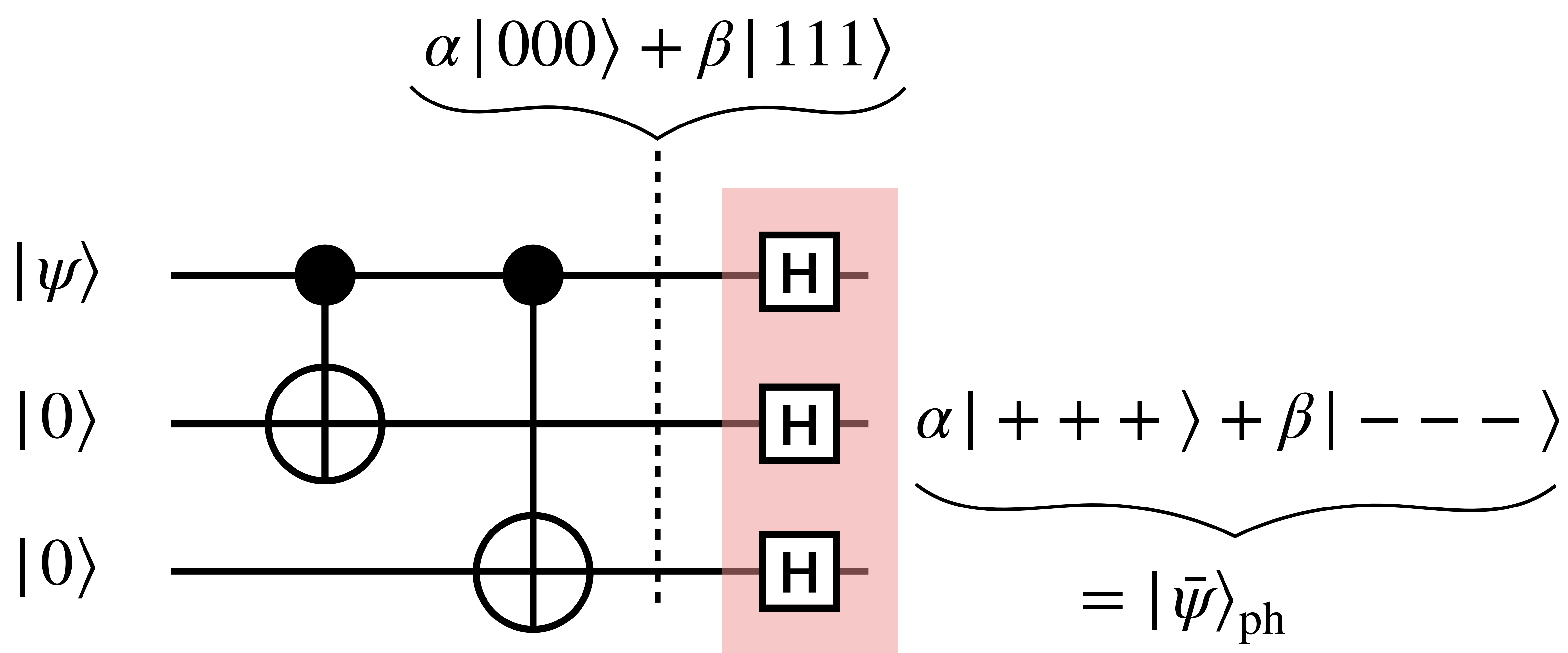

$$\underbrace{\alpha|000\rangle+\beta|111\rangle}$$

$$\underbrace{\alpha|+++\rangle+\beta|---\rangle}_{=|\bar{\psi}\rangle_{\text{ph}}}$$

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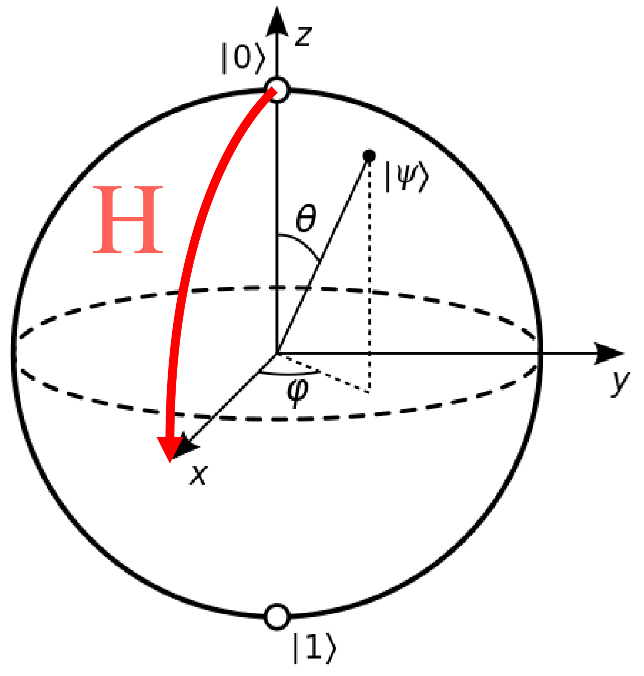

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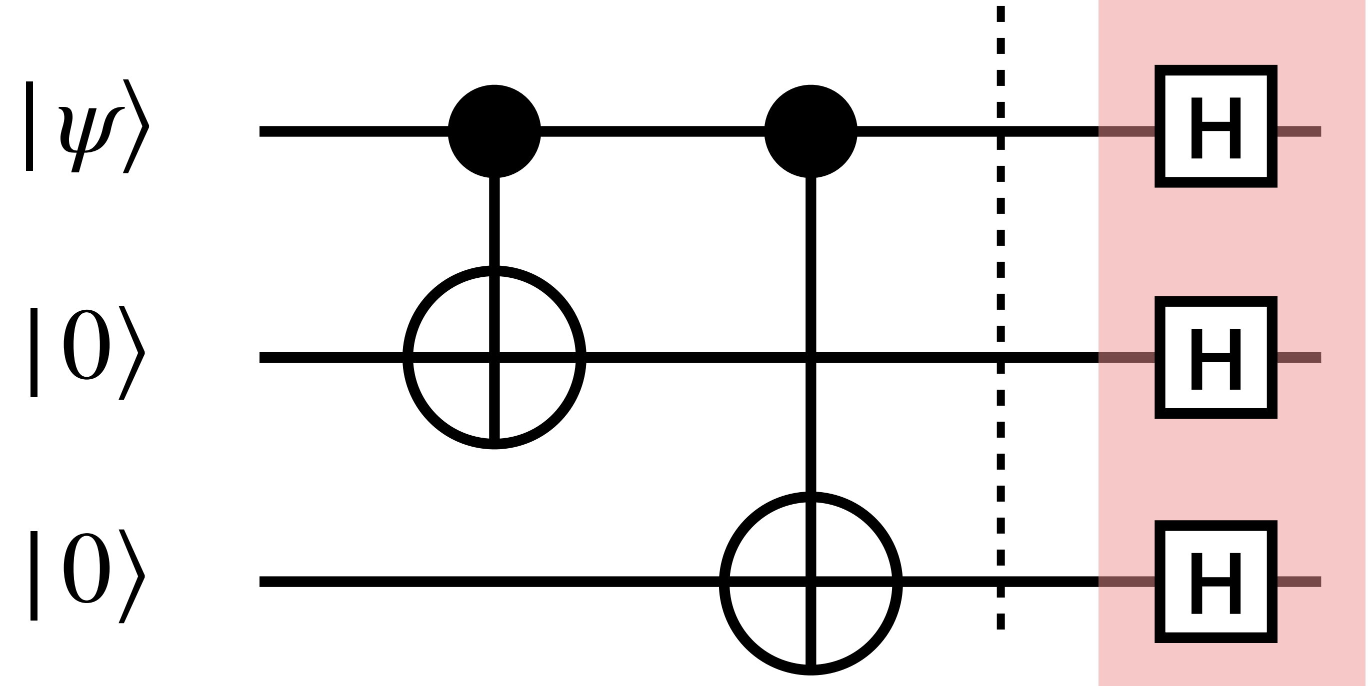

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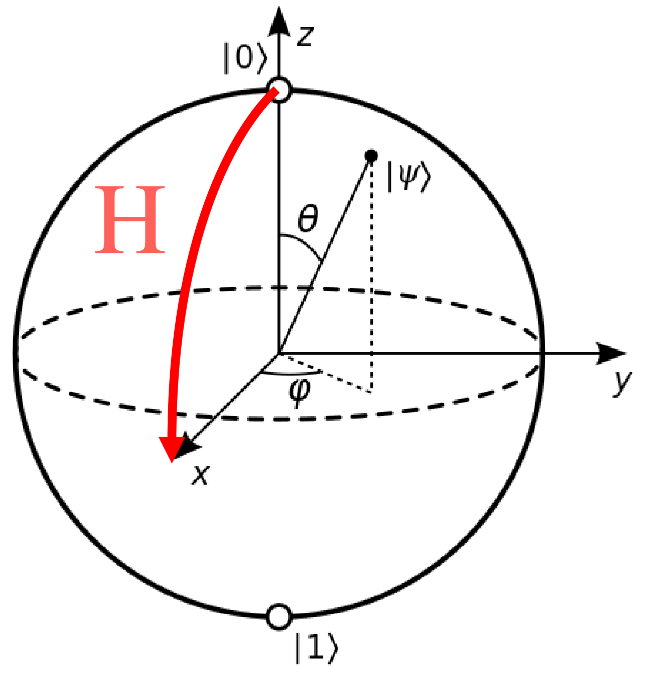

Hadamard Gate implements "basis change"

$$X_i \leftrightarrow Z_i$$

Checks become

$$X_1X_2, \quad X_2X_3$$

This code corrects single <u>phase-flips</u>!

$$Z|+\rangle = |-\rangle$$
$$Z|-\rangle = |+\rangle$$

"phase-flips" / Pauli-Z

remember:

$$Z|0\rangle = |0\rangle$$
$$Z|1\rangle = -|1\rangle$$

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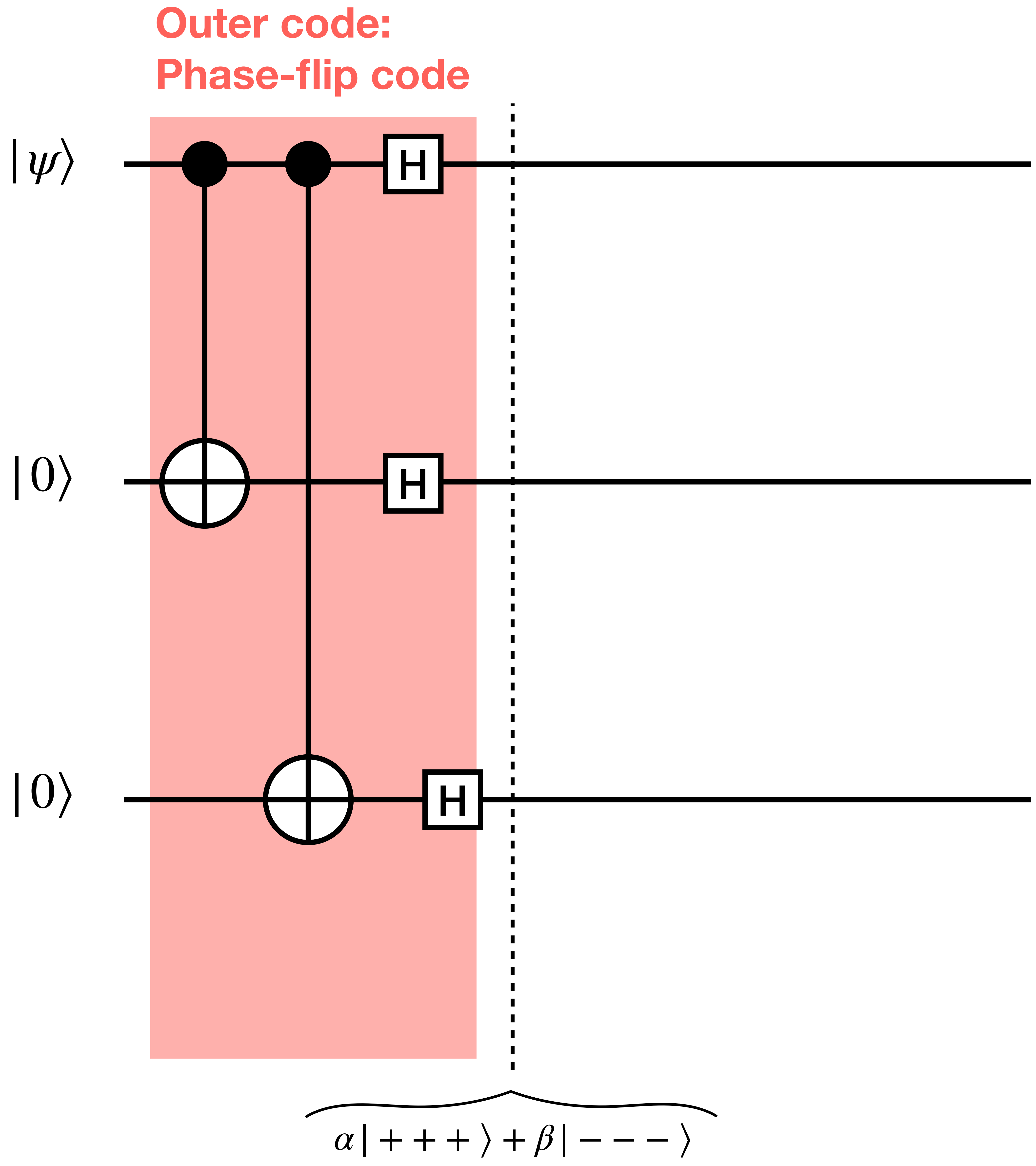
The Shor Code



$$\overbrace{\alpha|+++\rangle+\beta|---\rangle}$$

We now *concatenate* the bit- and phase-flip code.

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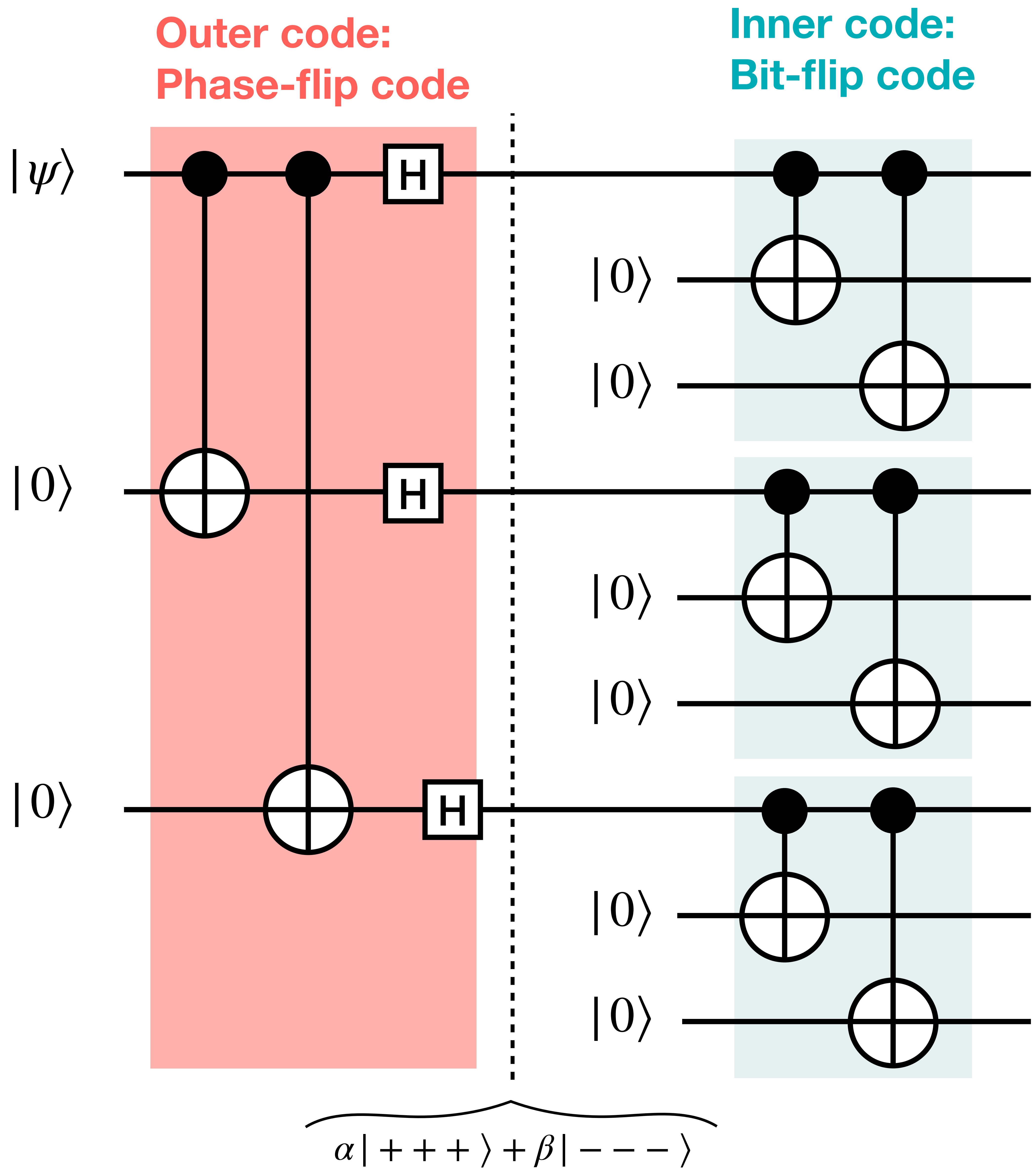

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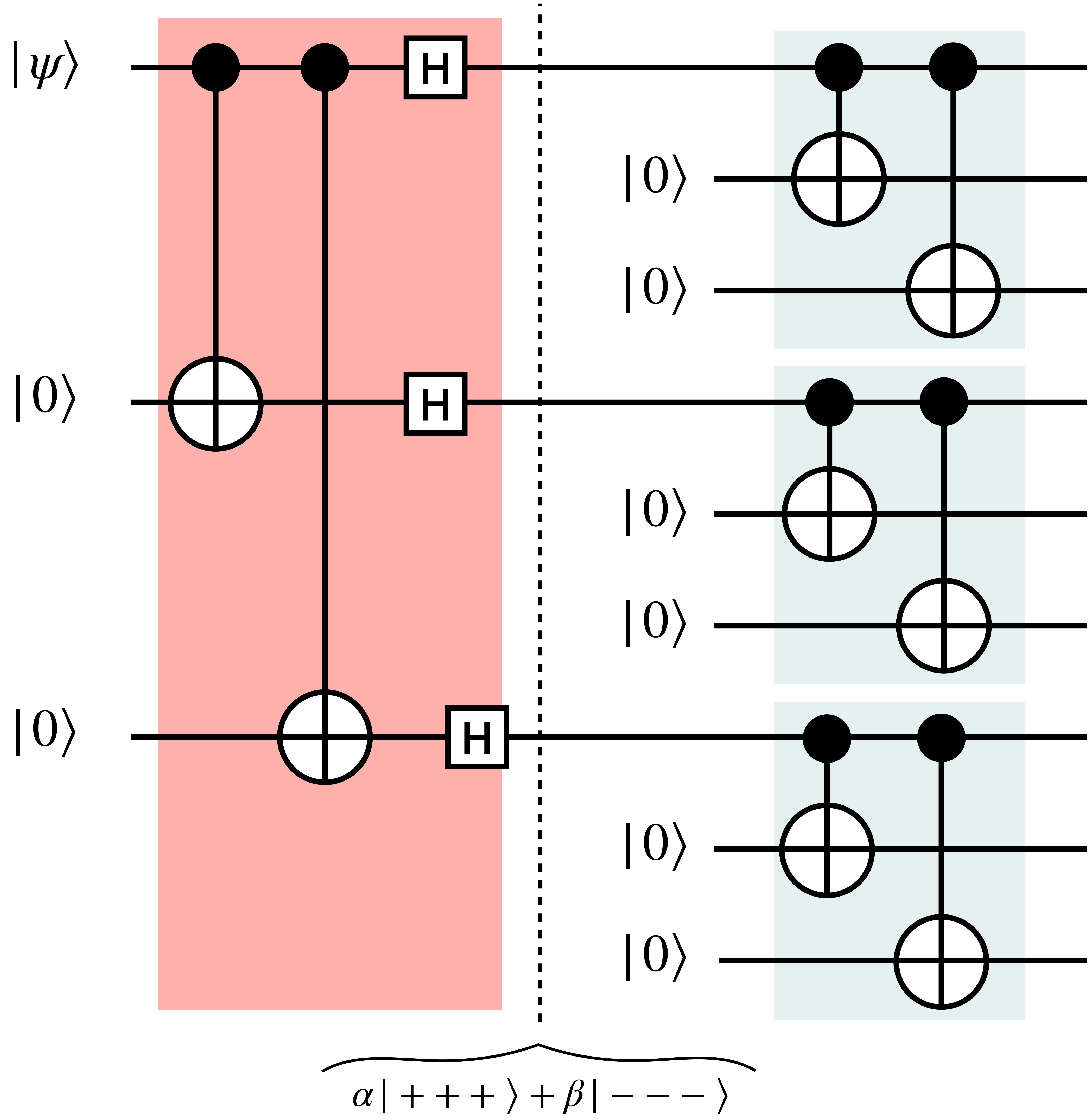
The Shor Code

Outer code:
Phase-flip code

Inner code:
Bit-flip code



We now <u>concatenate</u> the bit- and phase-flip code.

$$|\psi\rangle_{\text{Shor}} = \alpha|\bar{+}\,\bar{+}\,\bar{+}\rangle + \beta|\bar{-}\,\bar{-}\,\bar{-}\rangle$$

Outer code:
Phase-flip code

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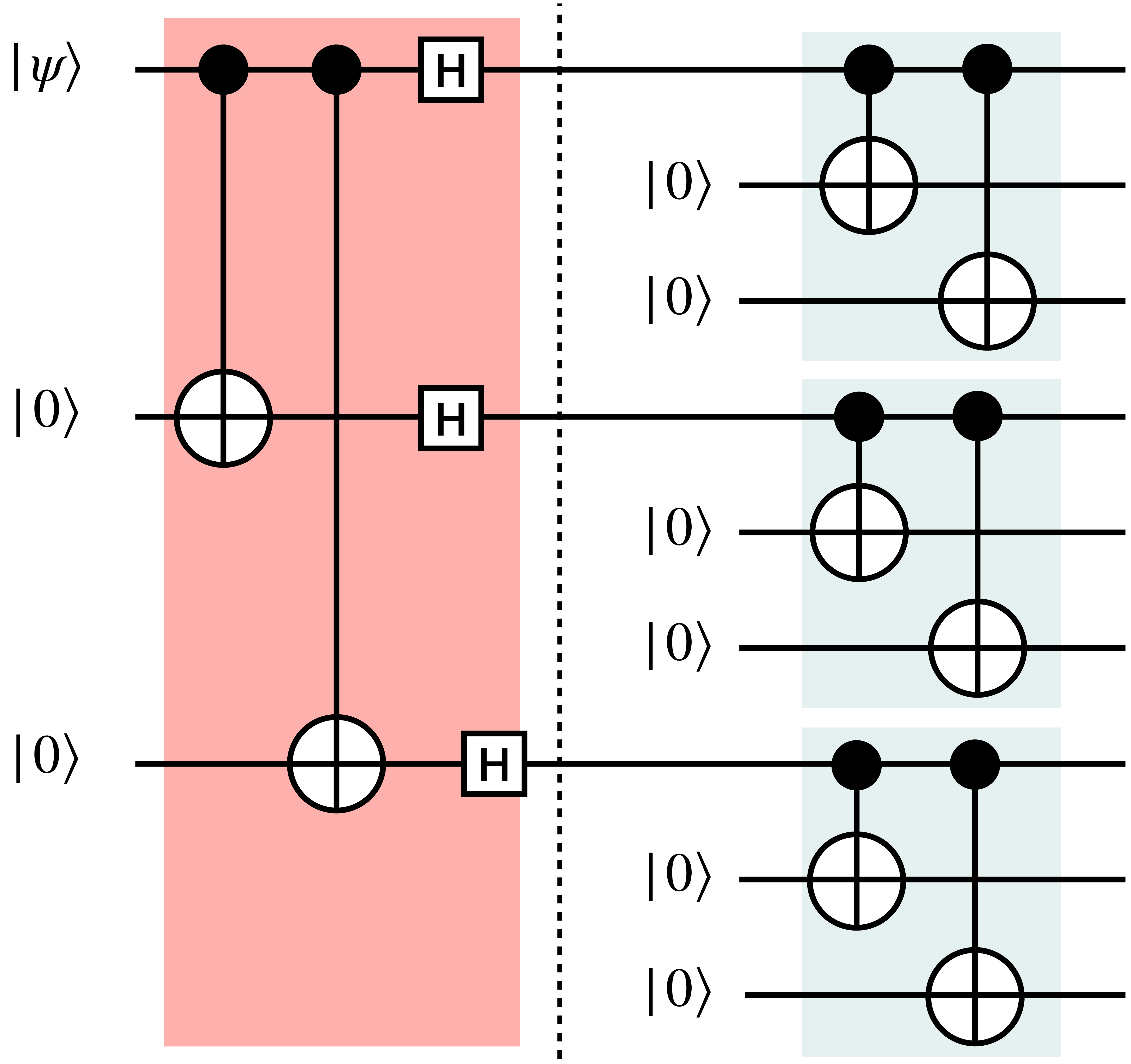
The Shor Code

Outer code: Phase-flip code

Inner code: Bit-flip code



$$\alpha|+++\rangle + \beta|---\rangle$$

We now concatenate the bit- and phase-flip code.

$$|\bar{-}\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle - |111\rangle\right)$$

Inner code: Bit-flip code

$$|\psi\rangle_{\text{Shor}} = \alpha|\bar{+}\bar{+}\bar{+}\rangle + \beta|\bar{-}\bar{-}\bar{-}\rangle$$

Outer code: Phase-flip code

$$|\bar{+}\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$

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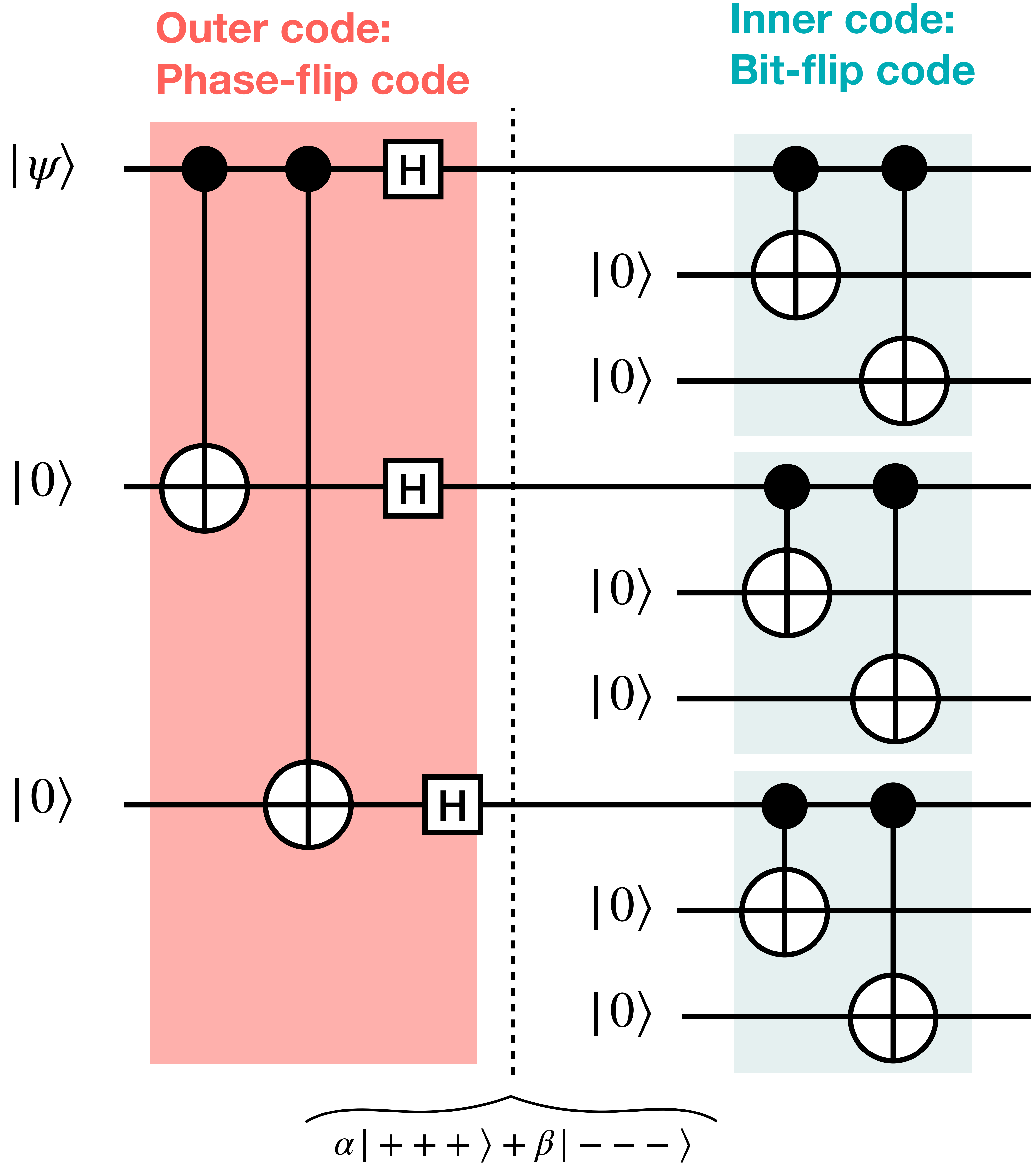

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The Shor code can correct a single bit- and phase-flip!

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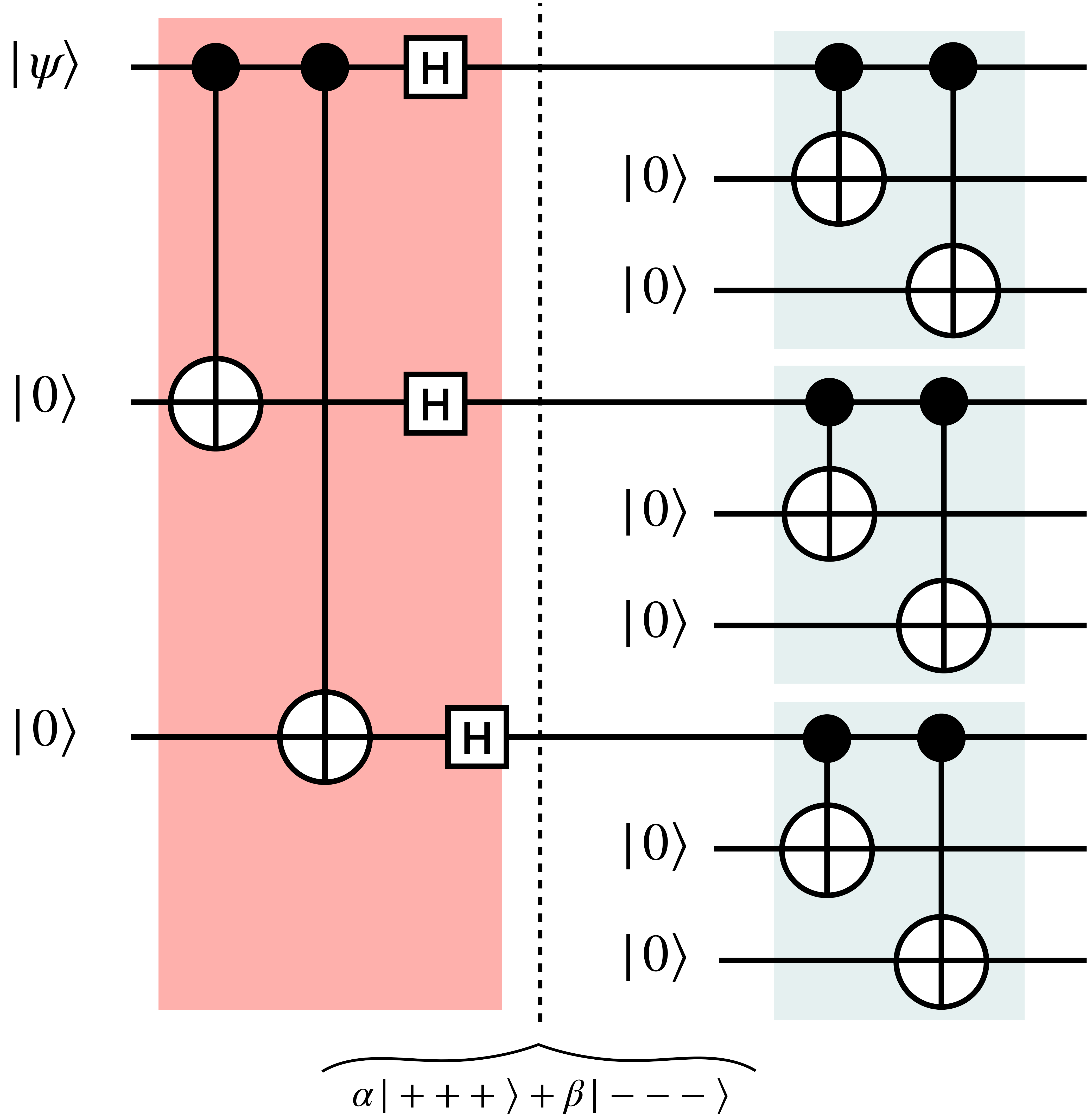

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Parity measurements:

$Z_1Z_2\,,Z_2Z_3$ $Z_4Z_5\,,Z_5Z_6$ $Z_7Z_8\,,Z_8Z_9$ **Checks of inner code**

$X_1X_2X_3X_4X_5X_6$ $X_4X_5X_6X_7X_8X_9$ **Checks of outer code**

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The Miracle: Discretisation of Errors

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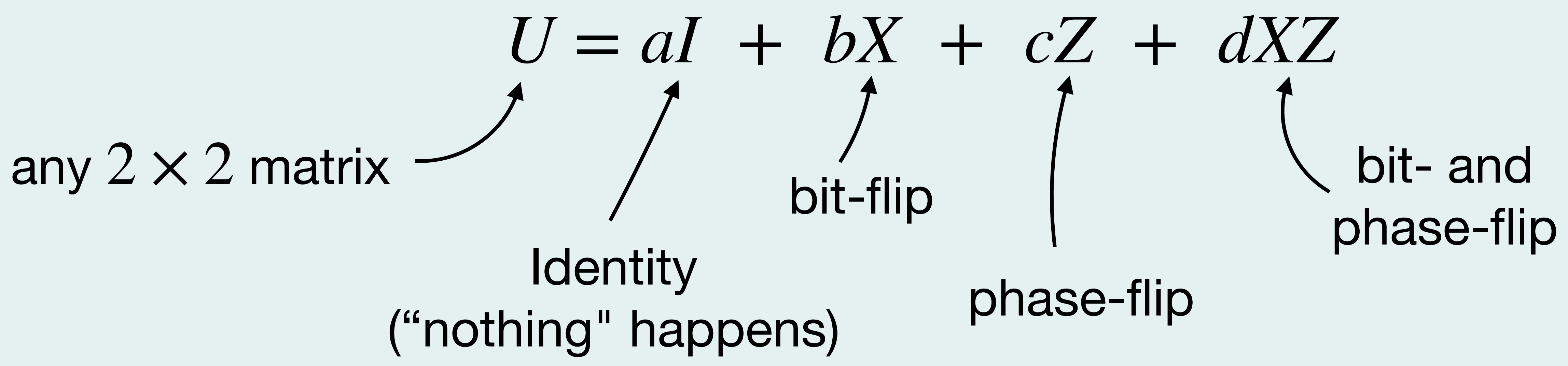
The Miracle: Discretisation of Errors

Miraculously, correcting bit- and phase-flips is enough!

The essential insight: Paulis form a basis of single-qubit operators



$$U = aI \ + \ bX \ + \ cZ \ + \ dXZ$$

any 2×2 matrix

Identity
(“nothing" happens)

bit-flip

phase-flip

bit- and
phase-flip

Informally
Anything that happens to a single qubit is a superposition of *nothing*, *a* *bit-flip*, *a* *phase-flip*, *and a* *bit- and phase-flip together*

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The Miracle: Discretisation of Errors

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$$\underset{\text{any } 2\times 2 \text{ matrix}}{U} = \underset{\text{Identity (“nothing" happens)}}{aI} + \underset{\text{bit-flip}}{bX} + \underset{\text{phase-flip}}{cZ} + \underset{\text{bit- and phase-flip}}{dXZ}$$

Informally

Anything that happens to a single qubit is a superposition of *nothing*, *a* *bit-flip*, *a* *phase-flip*, *and a* *bit- and phase-flip together*

After application of arbitrary single-qubit operator

$$\begin{aligned} U|\psi\rangle_{\text{Shor}} = a|\psi\rangle_{\text{Shor}} &+ \\ b\ X\ |\psi\rangle_{\text{Shor}} &+ \\ c\ Z\ |\psi\rangle_{\text{Shor}} &+ \\ d\ XZ\ |\psi\rangle_{\text{Shor}} & \end{aligned}$$

All correspond to different set of measurement outcomes!

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All correspond to different set of measurement outcomes!

Measuring the code checks will collapse the erroneous state into a discrete set of outcomes:

nothing, bit-flip, phase-flip, or both flips

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Preliminary Summary

The Shor Code
(and also the Steane Code!)
can correct an arbitrary single-qubit error.

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Preliminary Summary

The Shor Code
(and also the Steane Code!)
can correct an arbitrary single-qubit error.

Remember that this is highly non-trivial:

Fault tolerant “random access machines” do not exists!

$\Longrightarrow$ **QEC is possible because quantum mechanics is not just “wave mechanics”**

It is a dance of a continuous (entangled) quantum states and discrete (projective) measurements!

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Have Mercy with our Colleagues in the Lab!

**The Shor code is intuitive,
but not very practical:**

For example size-5 Shor code has checks

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The Shor code is intuitive, but not very practical:

For example size-5 Shor code has checks

$$Z_1Z_2\ ,\ Z_2Z_3\ ,\ Z_3Z_4\ ,\ Z_4Z_5$$

...

$$Z_{21}Z_{22}\ ,\ Z_{22}Z_{23}\ ,\ Z_{23}Z_{24}\ ,\ Z_{24}Z_{25}$$

$$X_1X_2X_3X_4X_5X_6X_7X_8X_9X_{10}$$

$$X_6\ldots X_{15}$$

...

$$X_{16}\ldots X_{25}$$

Checks of outer code get harder and harder to measure

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For example size-5 Shor code has checks

Z_1Z_2 , Z_2Z_3 , Z_3Z_4 , Z_4Z_5

…

$Z_{21}Z_{22}$, $Z_{22}Z_{23}$, $Z_{23}Z_{24}$, $Z_{24}Z_{25}$

$X_1X_2X_3X_4X_5X_6X_7X_8X_9X_{10}$

$X_6 \ldots X_{15}$

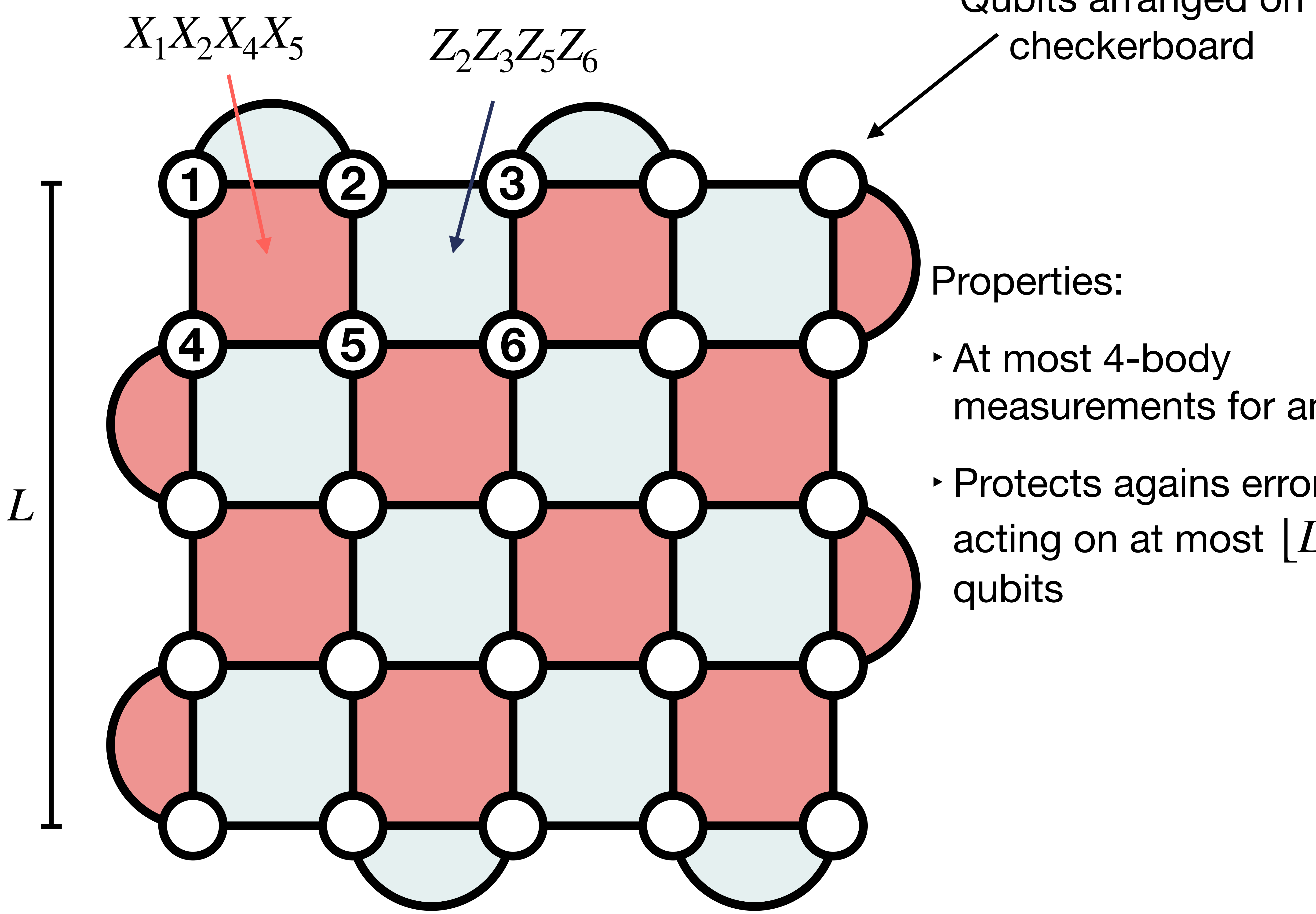
…

$X_{16} \ldots X_{25}$

Checks of outer code get harder and harder to measure

Better: Low-Density Parity Check (LDPC) codes

Most well-studied example: the surface code



Qubits arranged on checkerboard

Properties:

- At most 4-body measurements for any size
- Protects agains errors acting on at most $\lfloor L/2 \rfloor$ qubits

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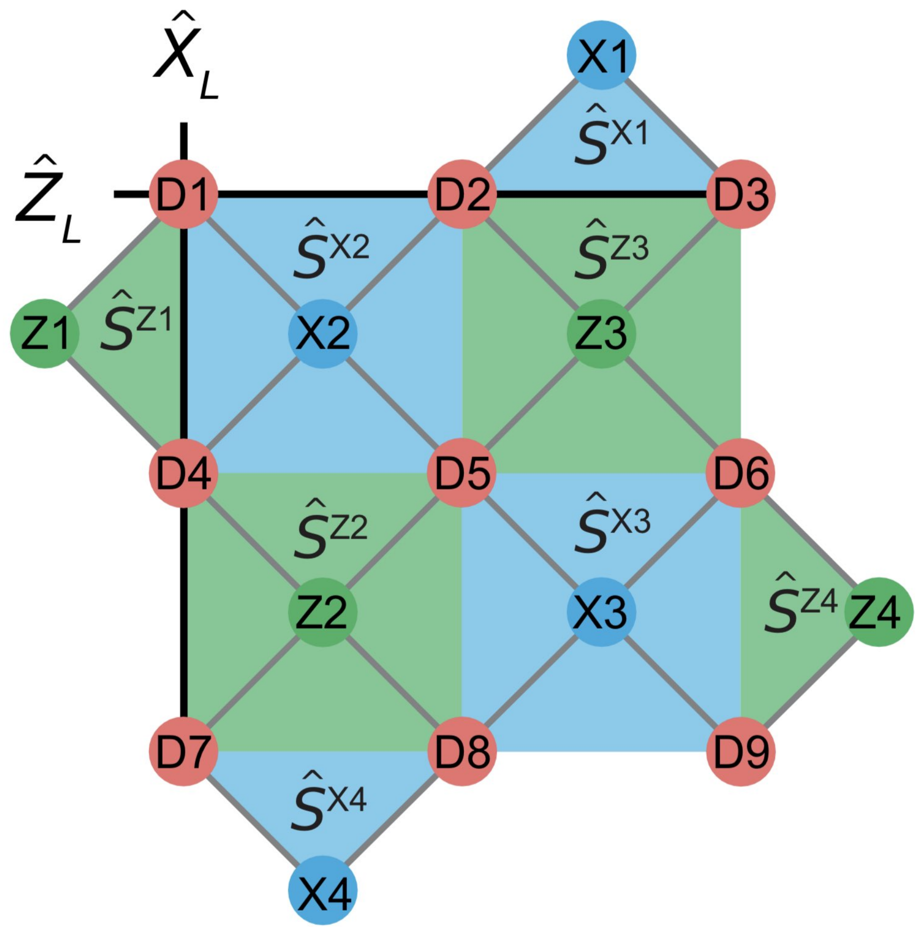
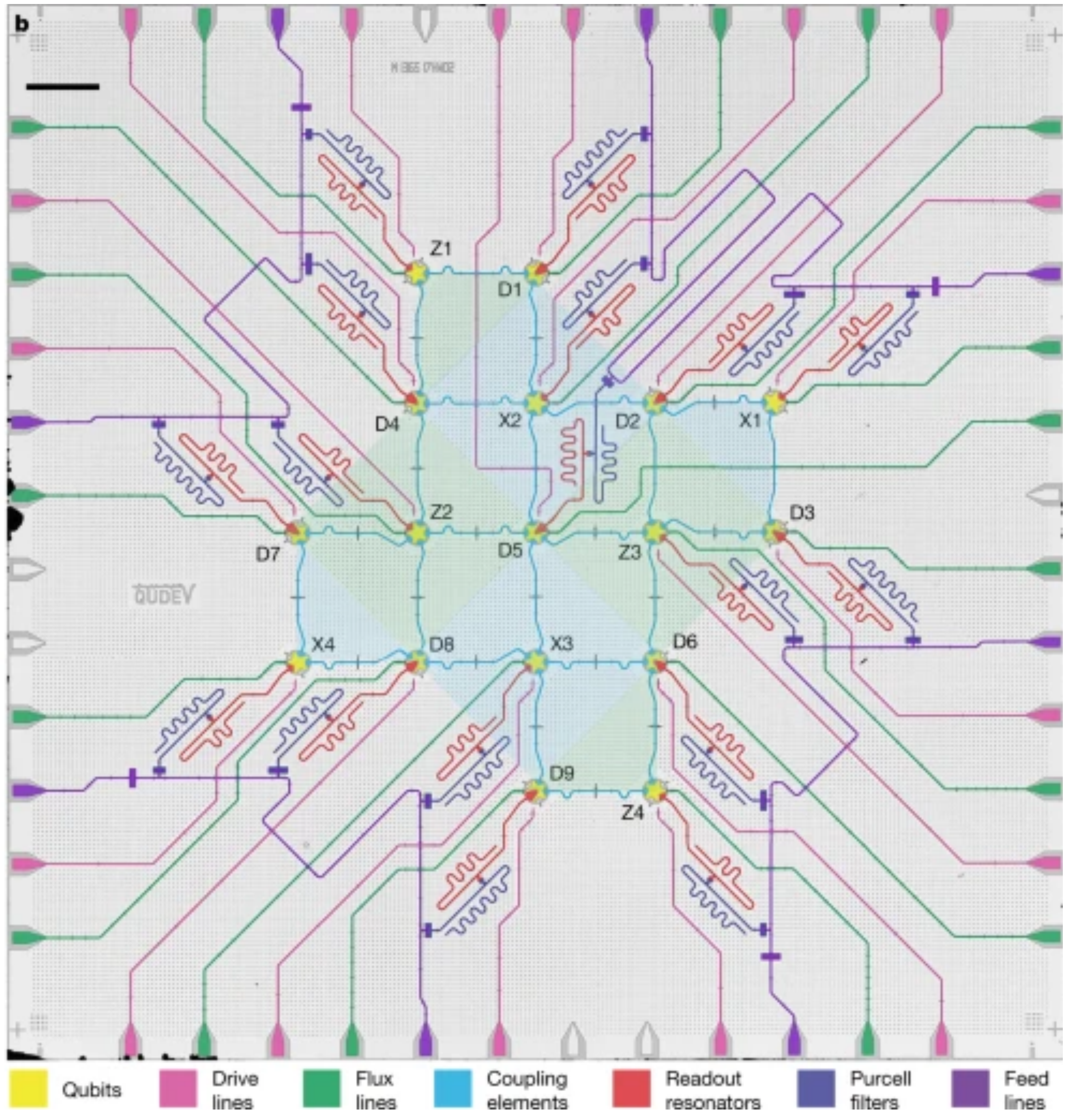
This has been built!

As a sketch …



… and as a photograph



- $L = 3$ surface code built by the Walraff Group at ETH
- Because it is an academic group, we even get a picture of the device!

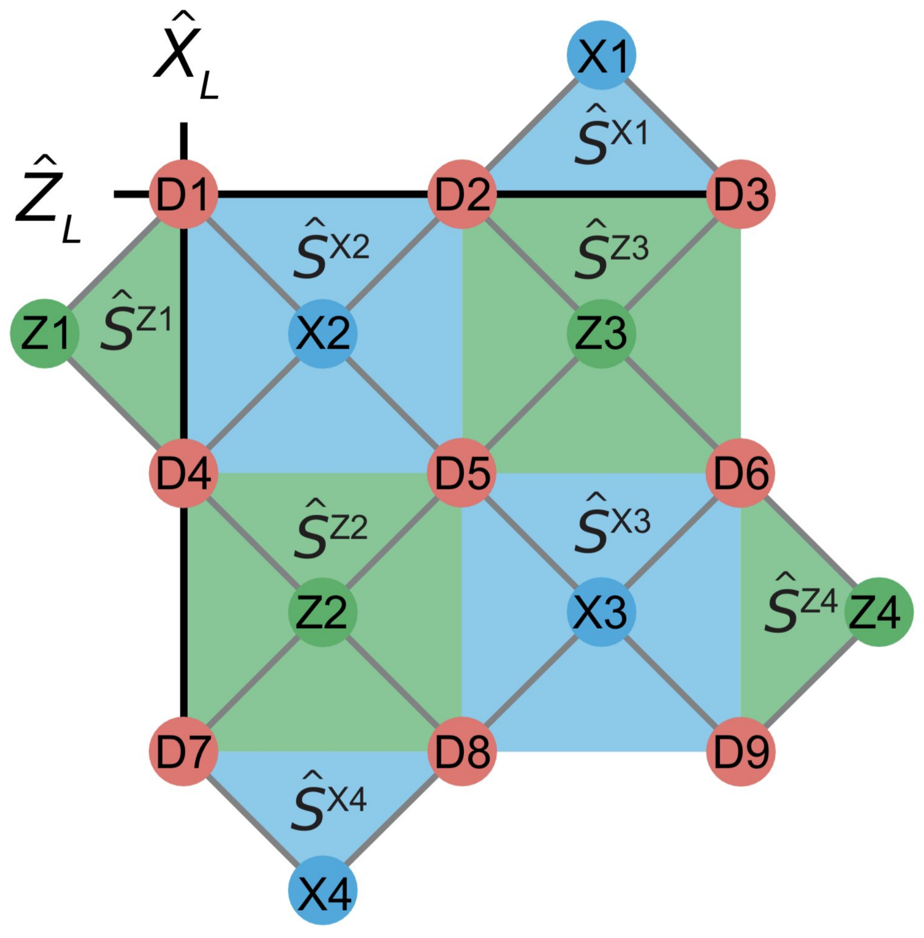
QUDEV

From Krinner et. al Nature (2022)
https://doi.org/10.1038/s41586-022-04566-8

This has been built!

As a sketch …

… and as a photograph



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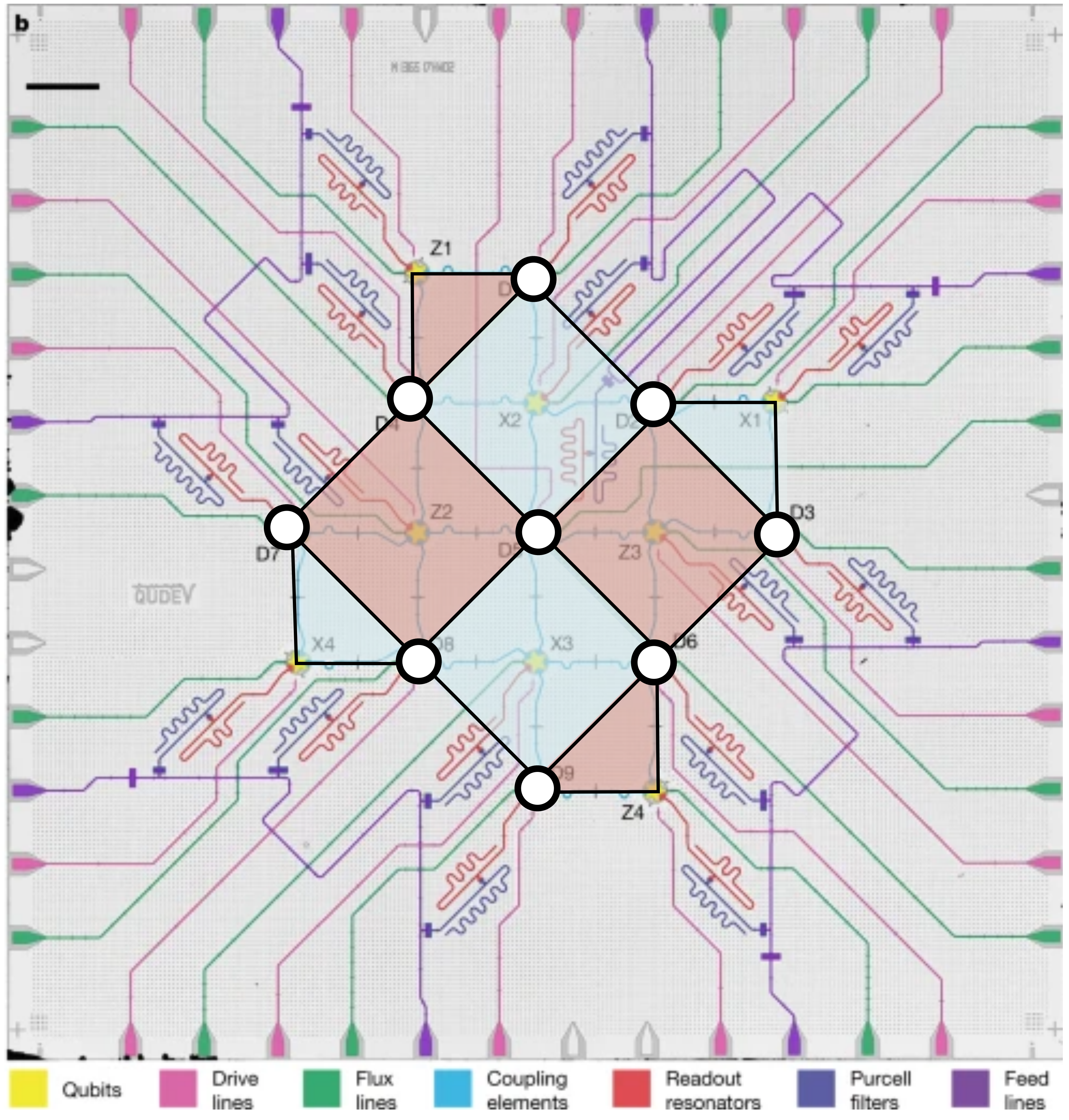

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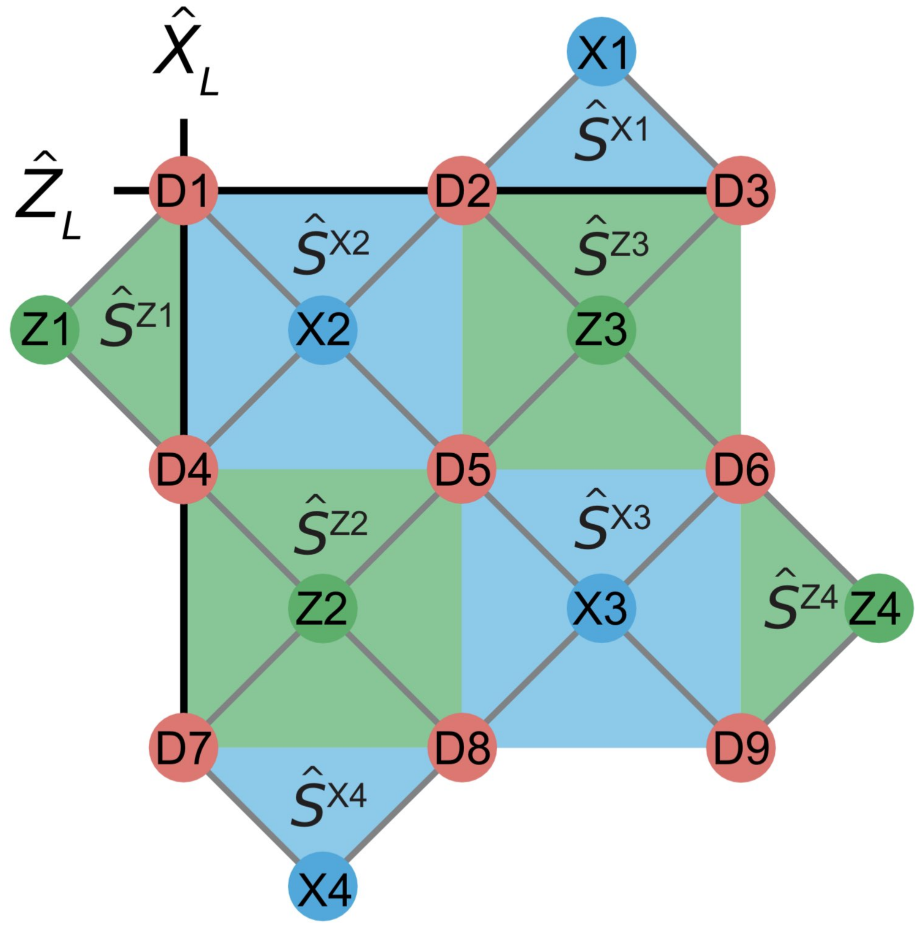

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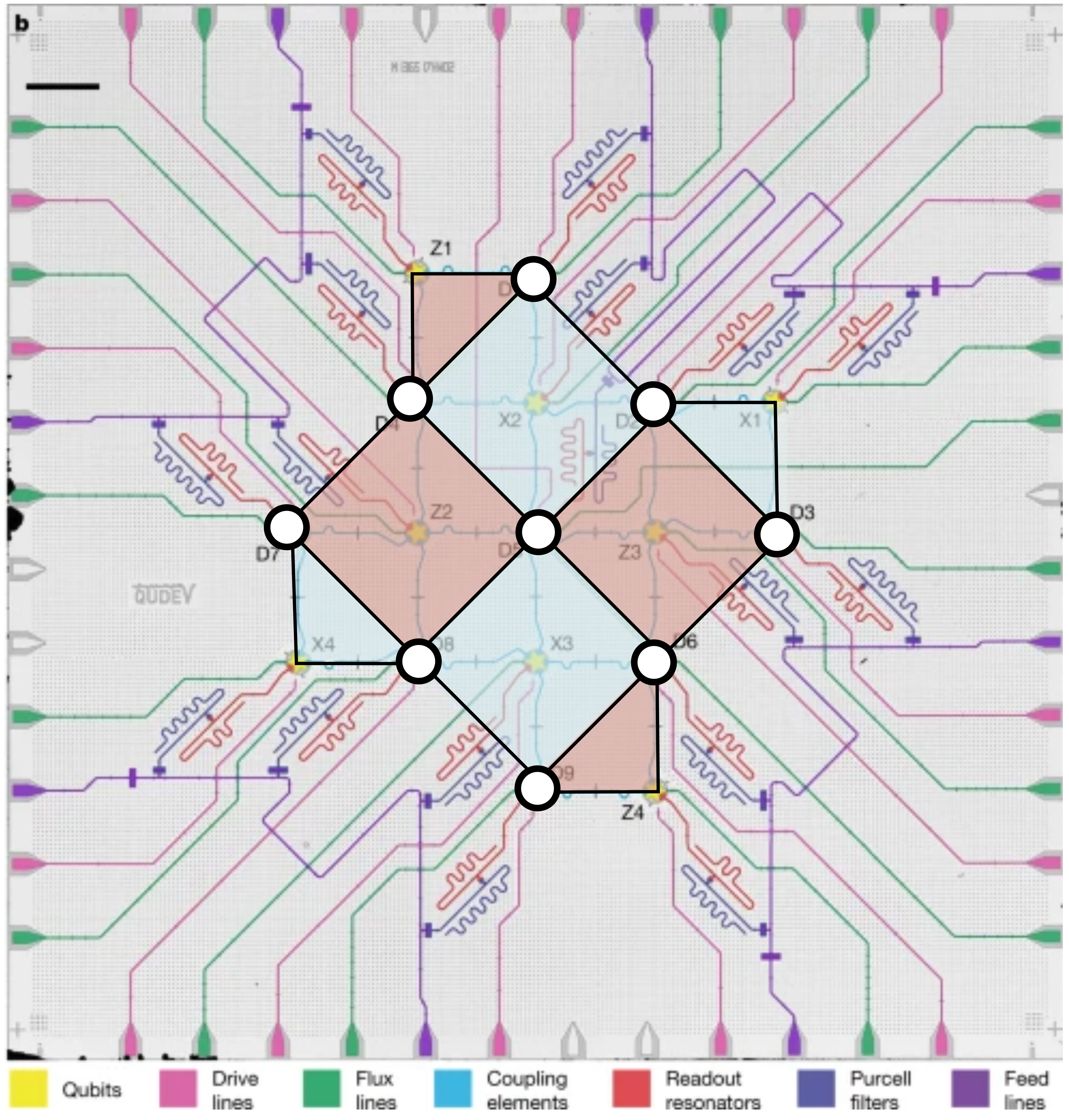

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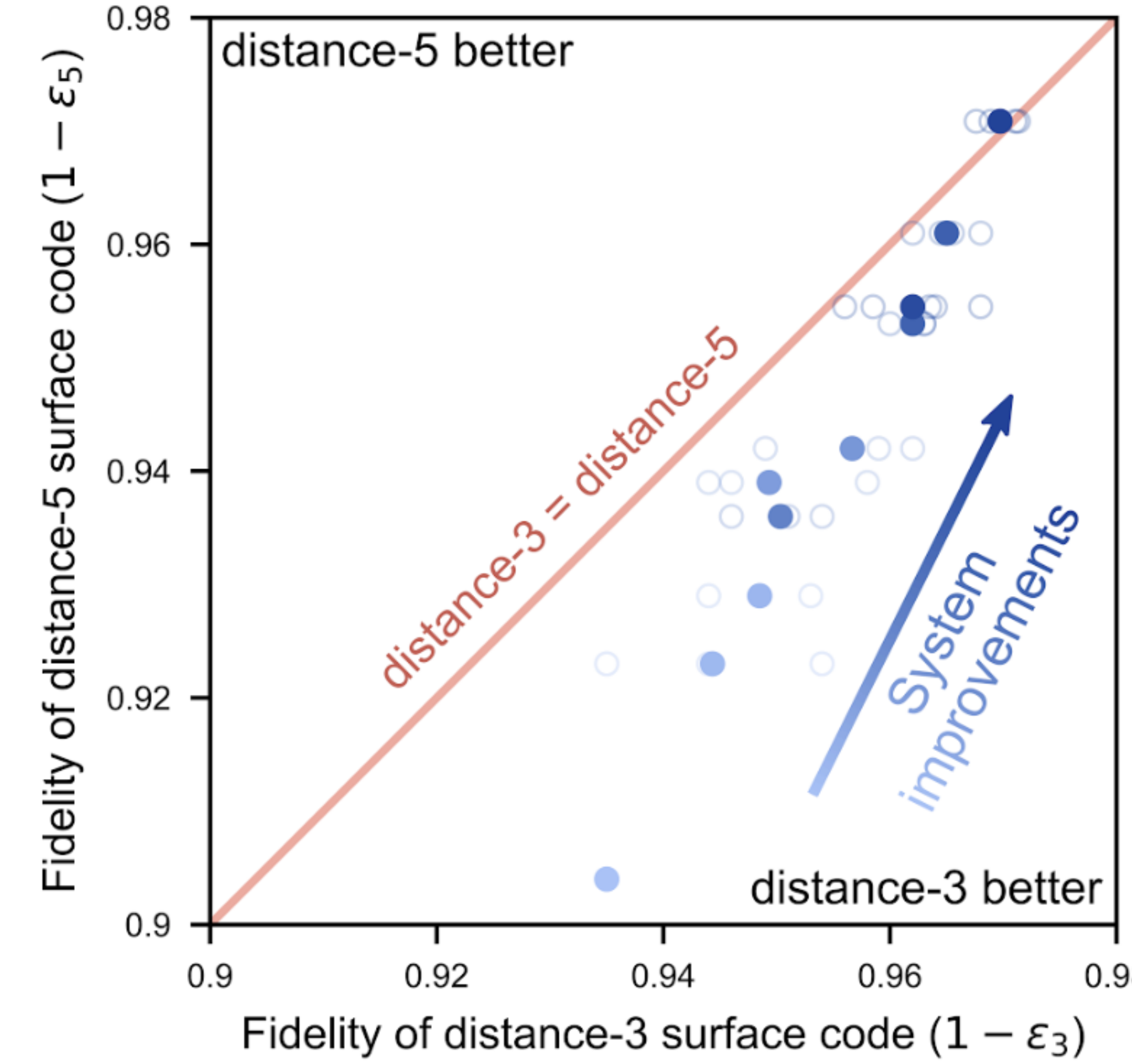
Google even built two!

$L = 5$ and $L = 3$



- No picture :(
- Only the "best" $L = 5$ code is better than $L = 3$?

How many errors are too much?

The “threshold” of a code

For QEC to work, the constituents have to be good enough!

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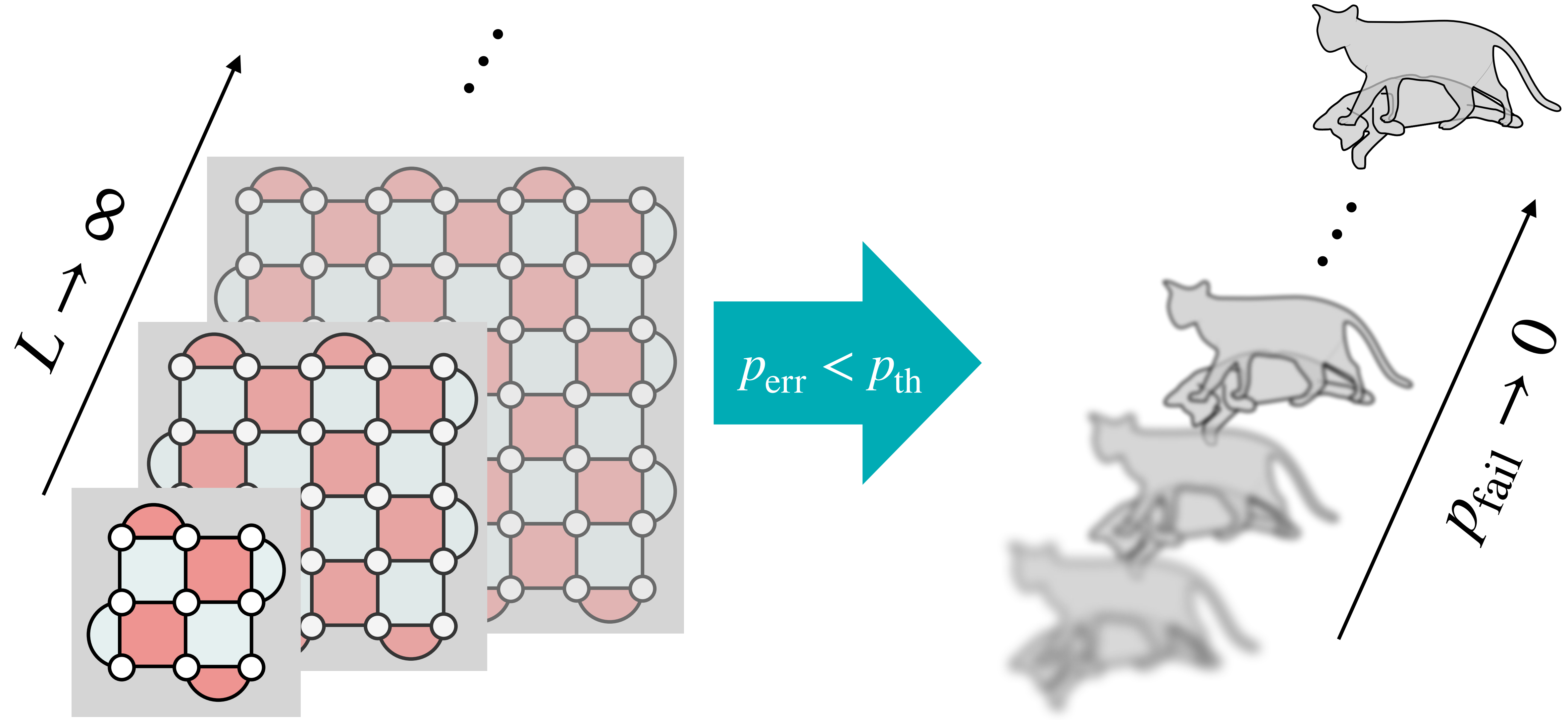
The "threshold" of a code

For QEC to work, the constituents have to be good enough!

Formally: error rate $p_{\text{err}} < p_{\text{th}}$, where p_{th} is the threshold rate

$$p_{\text{fail}} \to 0 \ \text{ as } L \to \infty \text{ for } \ p_{\text{err}} < p_{\text{th}}$$

$$p_{\text{fail}} \to \frac{1}{2} \ \text{ as } L \to \infty \text{ for } \ p_{\text{err}} > p_{\text{th}}$$



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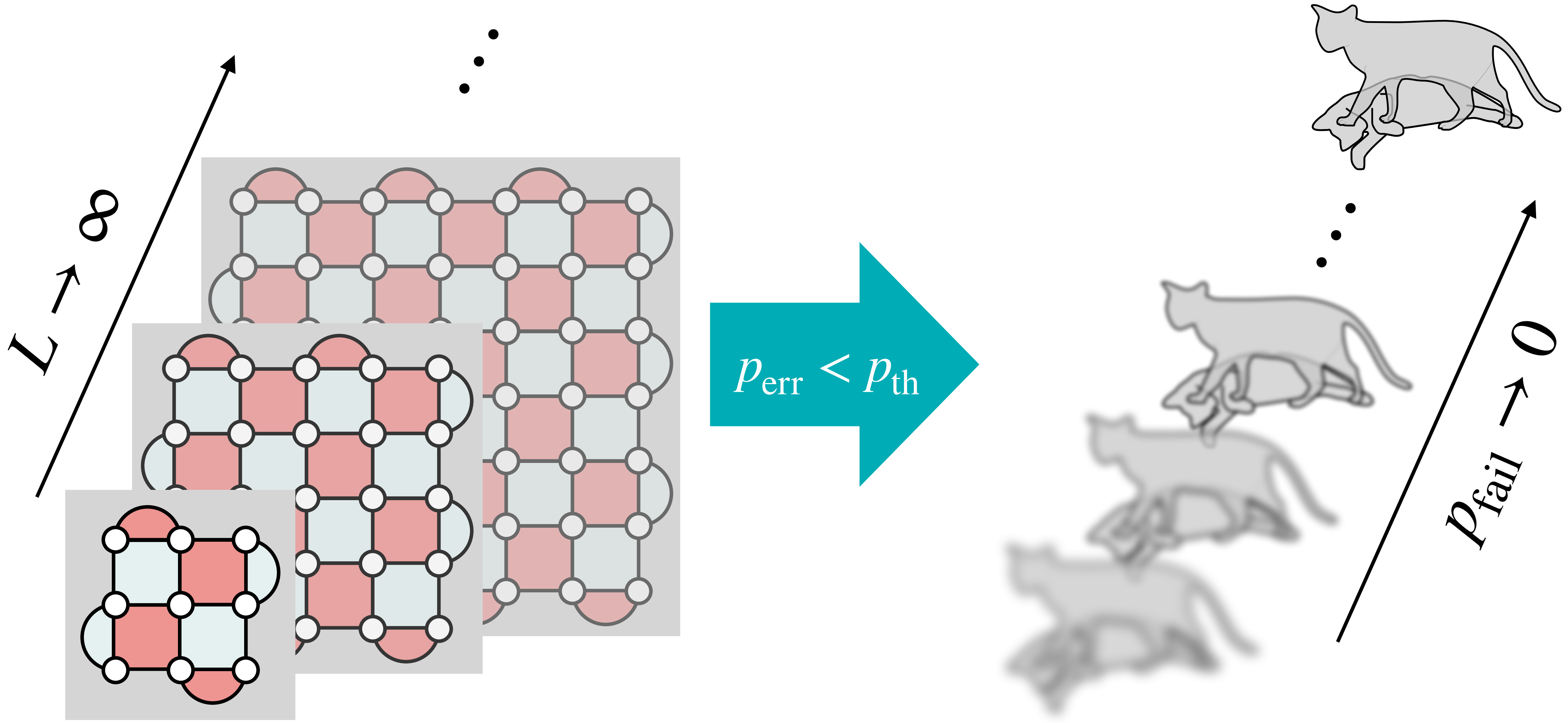

Modelling the threshold

- Error correction is a complicated statistical process
- Remarkably, with certain assumptions on the noise, it maps exactly on a well-known model of condensed matter physics: the random-bond Ising model [1,2]

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[1] *Dennis et al. (2001),* [2] *Wang et al (2002)* [3] *Chubb, Flammia (2019)*

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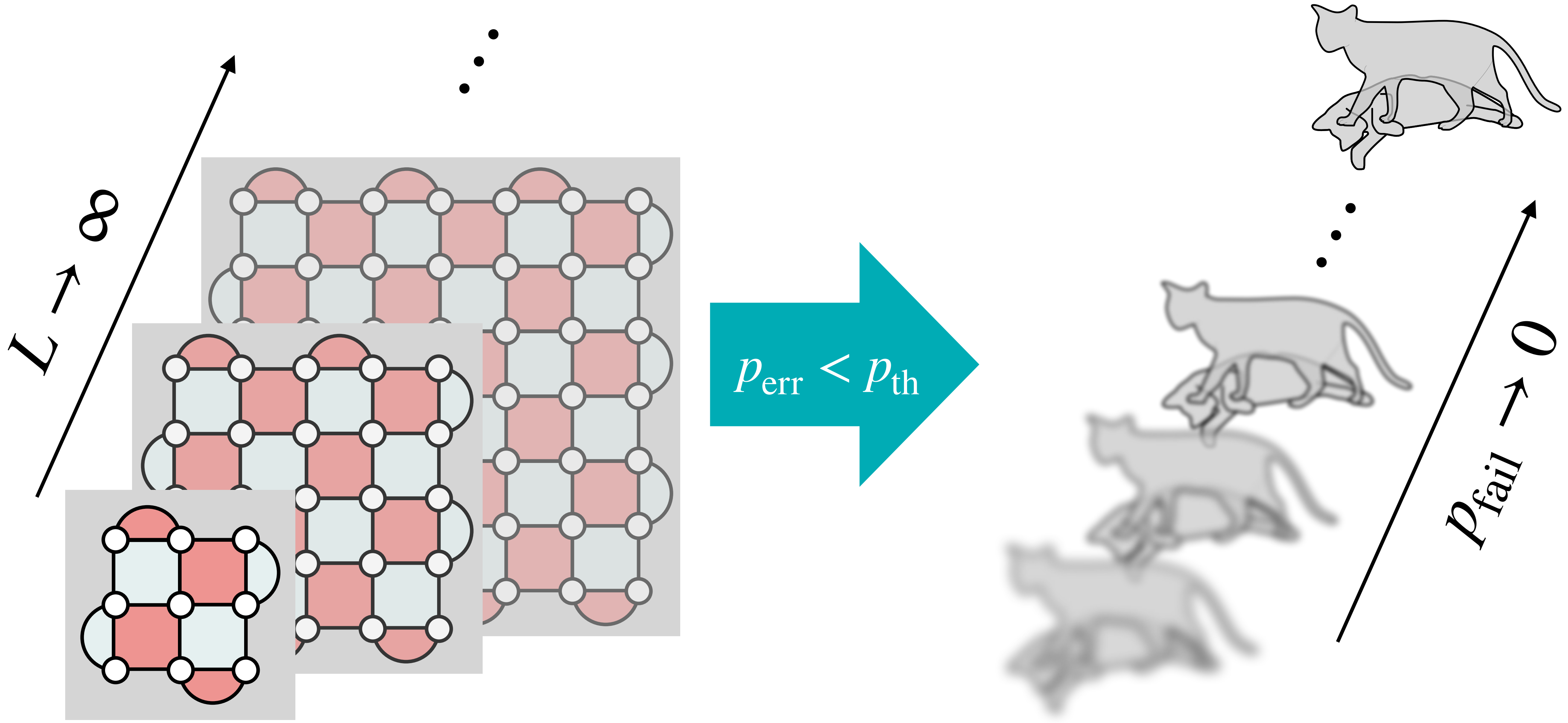

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Errors being correctible

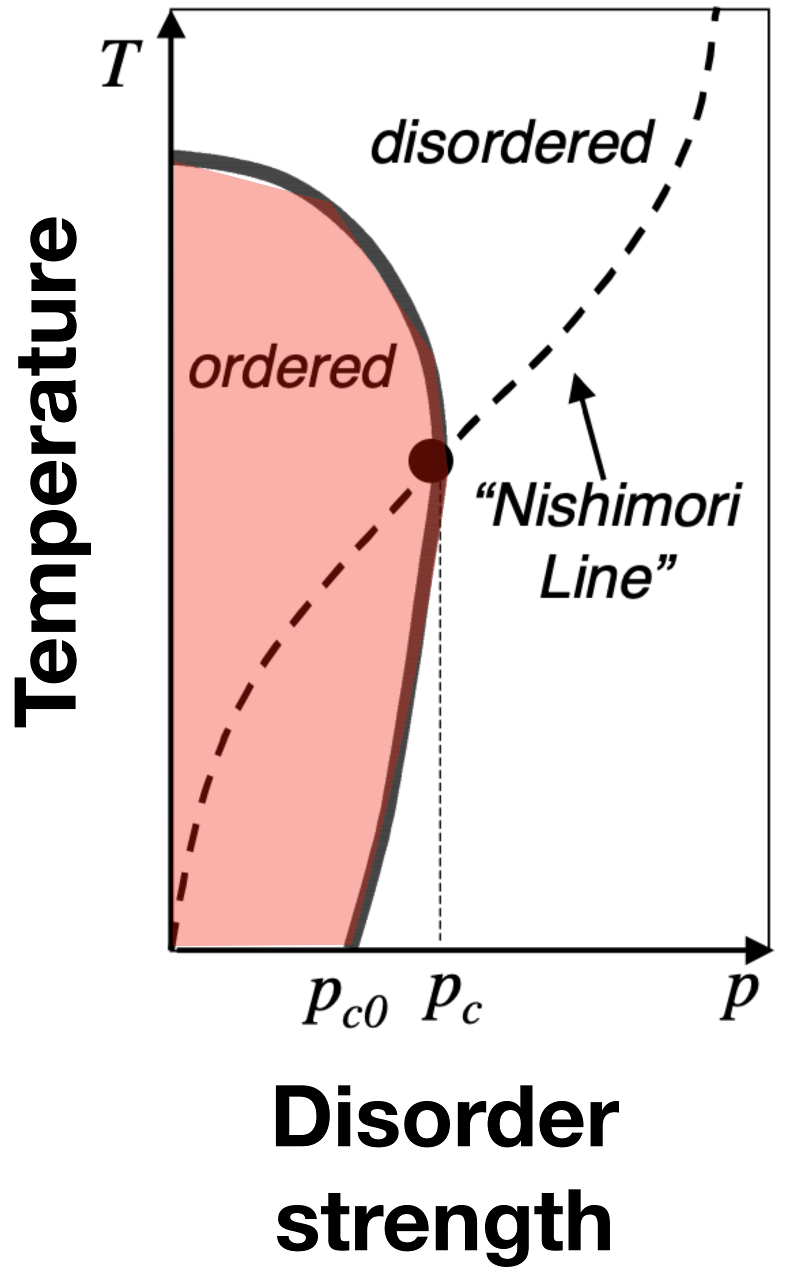
⇔

spin model is in ferromagnetic phase

Threshold of the surface code:

$$p_{\mathrm{th}} = p_c^{(RBIM)} \approx 11\ \%$$

Phase diagram of the the RBIM[3]



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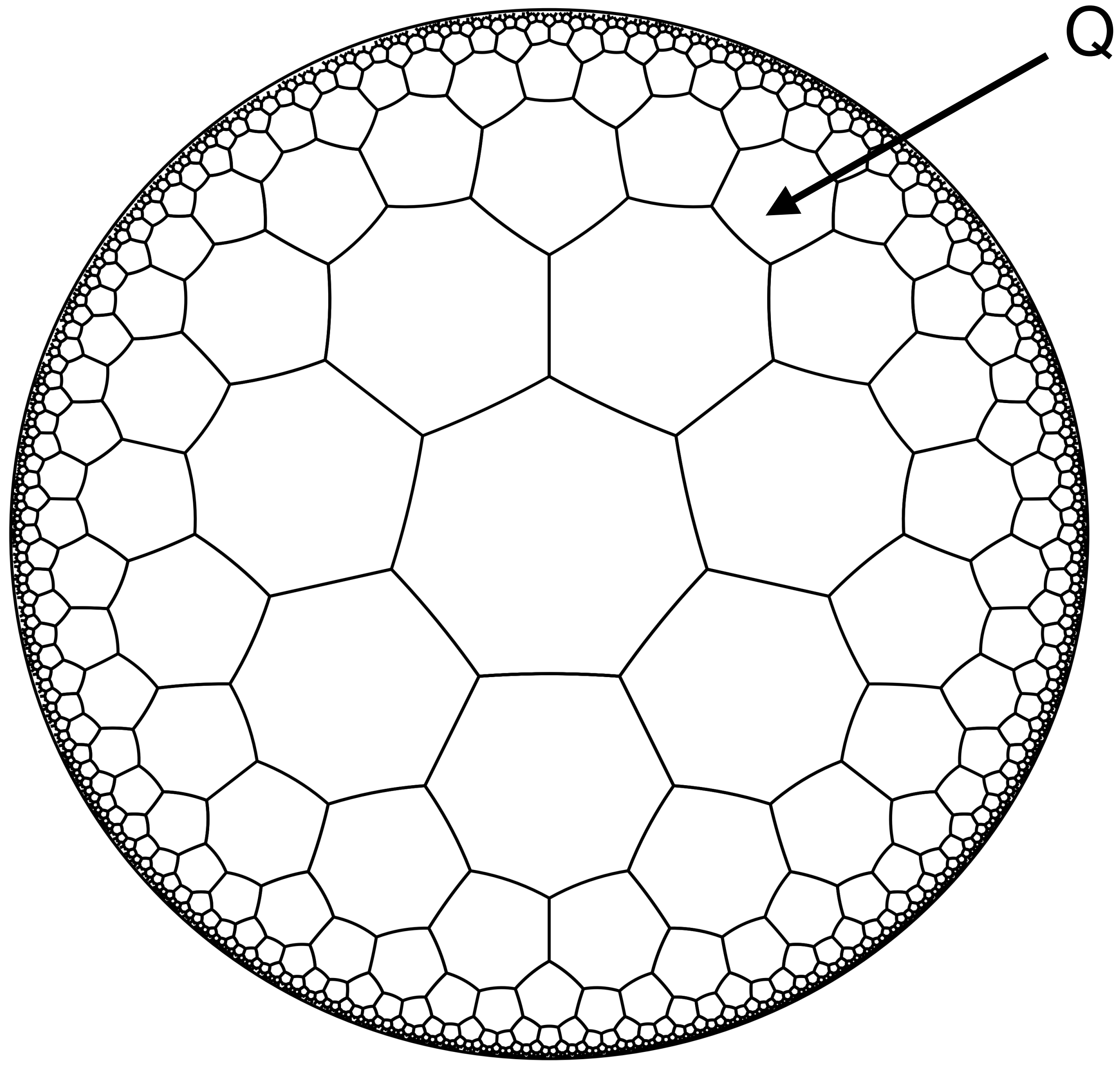
Active research: “better” codes:

Hyperbolic surface codes

Defined on regular tilings of the hyperbolic plane



M. C. Escher: Circle Limit III

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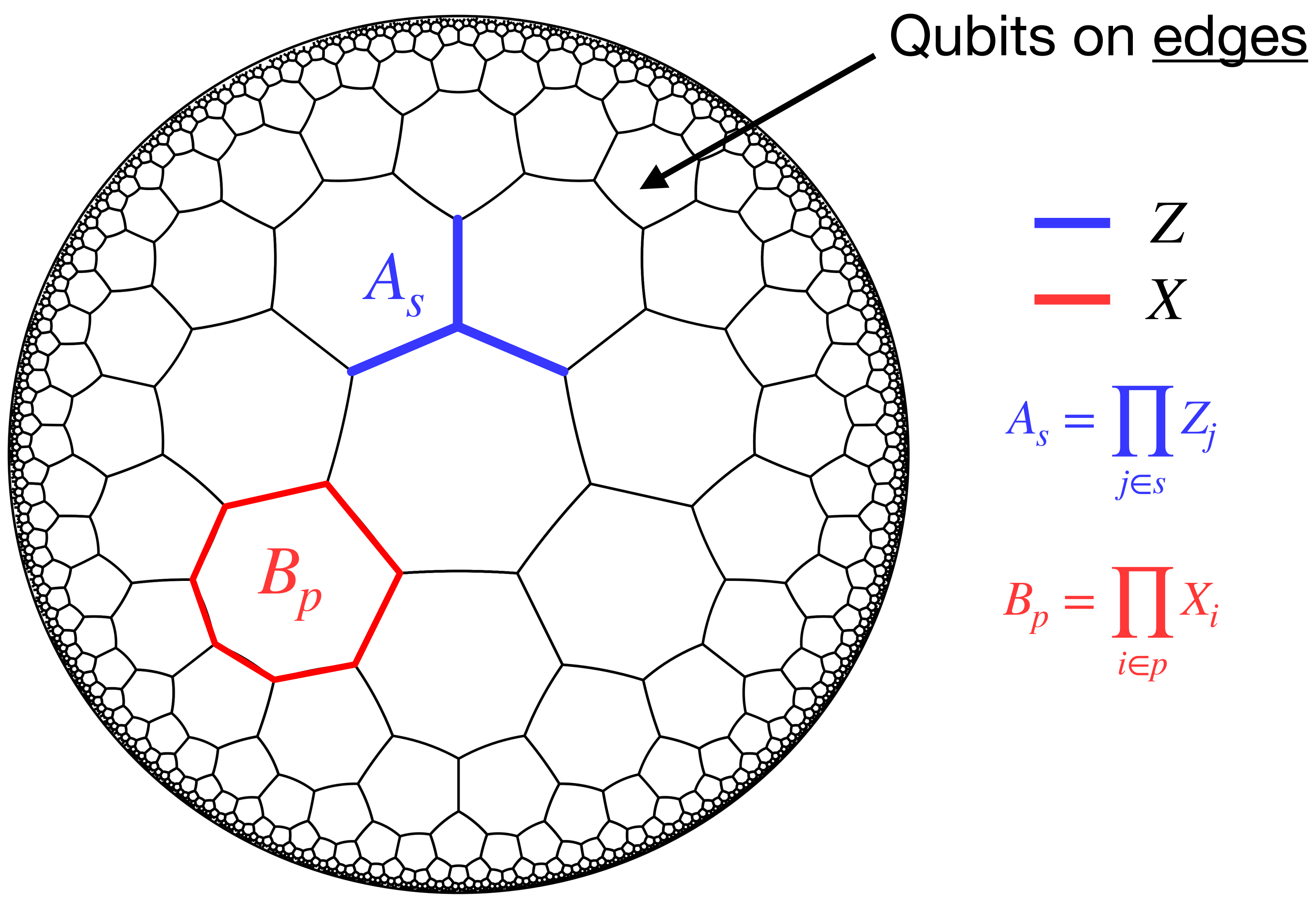

M. C. Escher: Circle Limit III

- Harder to built, since not naturally embedded into planar geometry
- But: if built has much reduced overhead (per qubit) compared to other codes

(#logical qubits) ∝ (#physical qubits)

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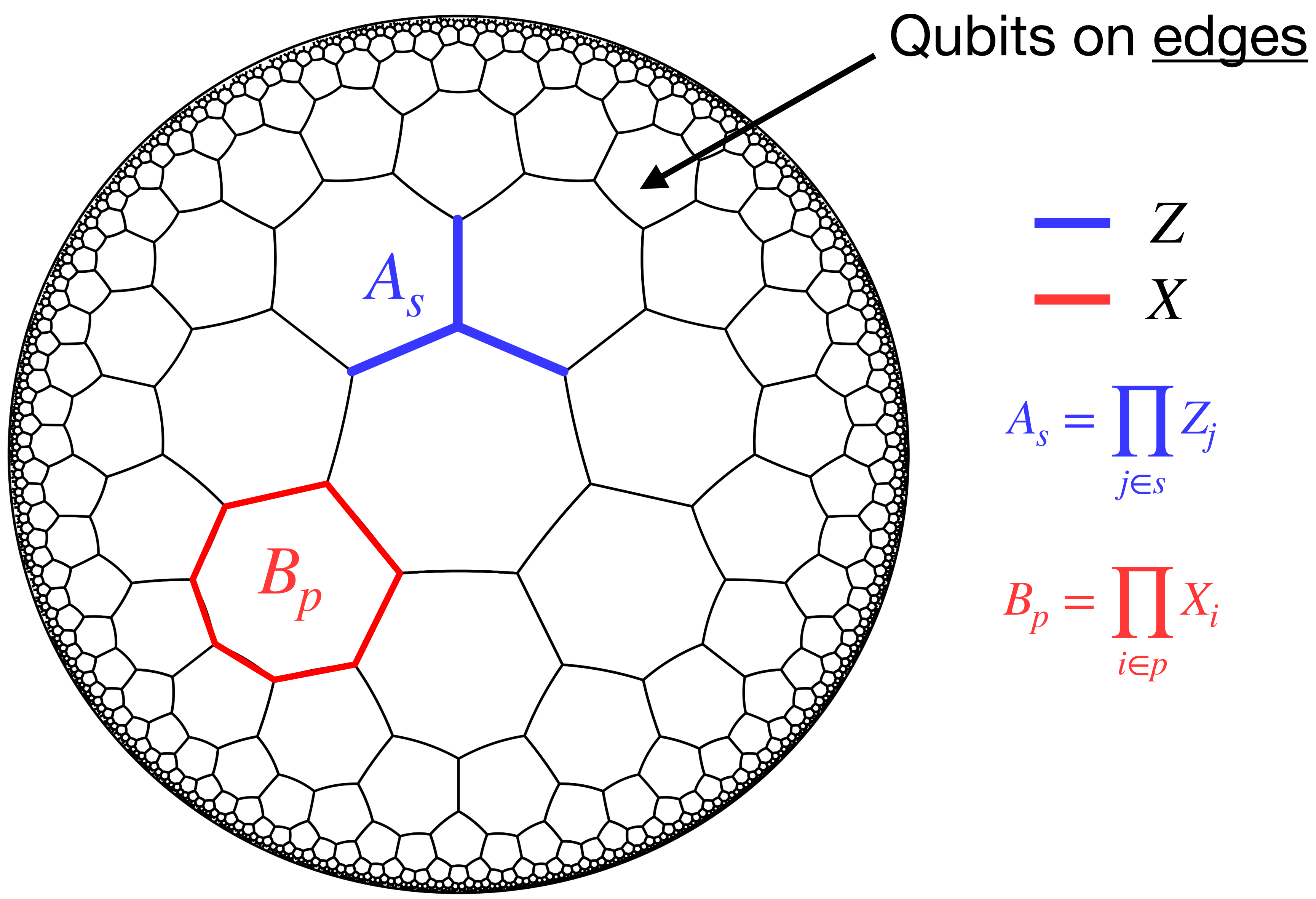

$$A_s = \prod_{j \in s} Z_j$$

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Such geometries can be built in principle!

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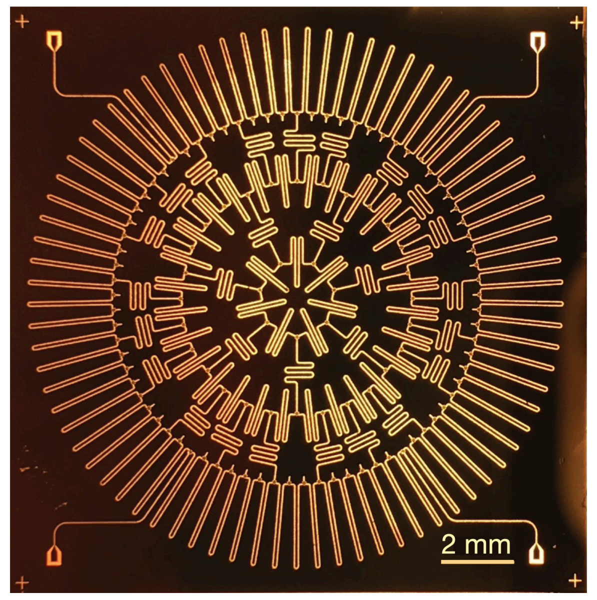

An artificial hyperbolic lattice

[Kollar et al. Nature (2019)]

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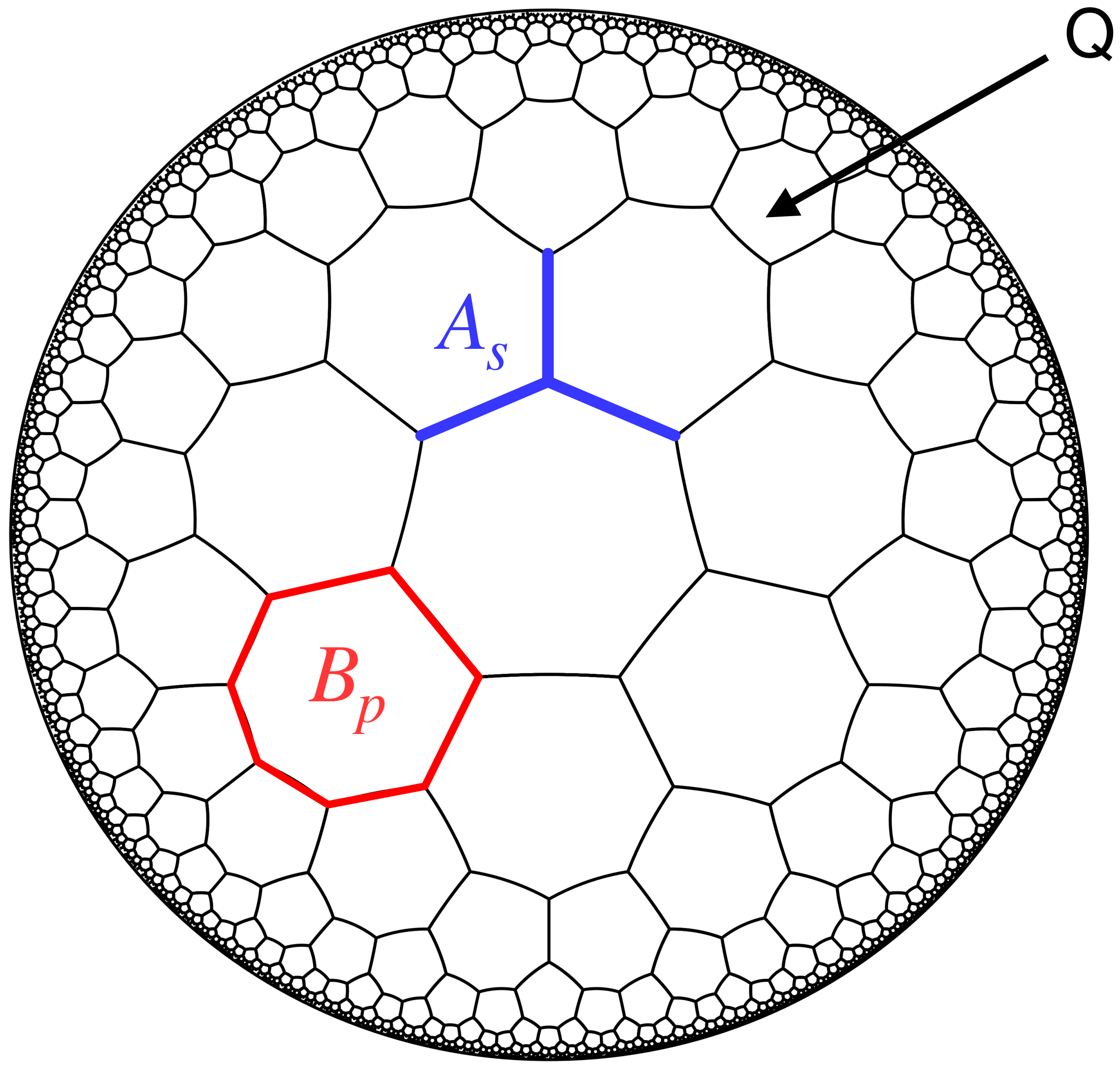

Qubits on edges

🟦 Z

🟥 X

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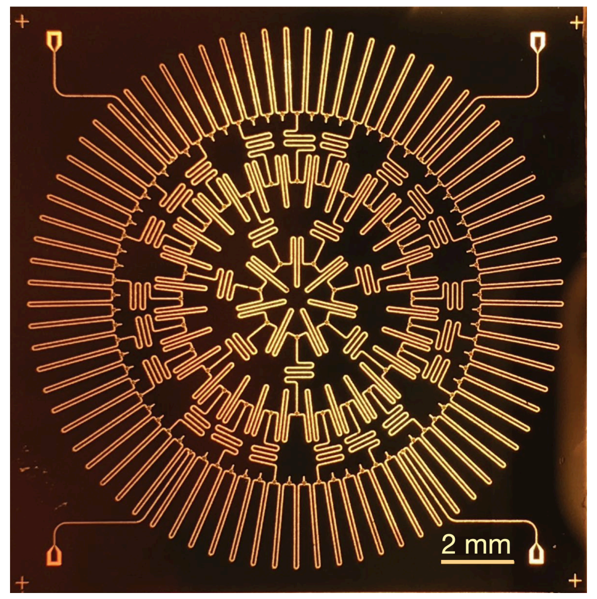
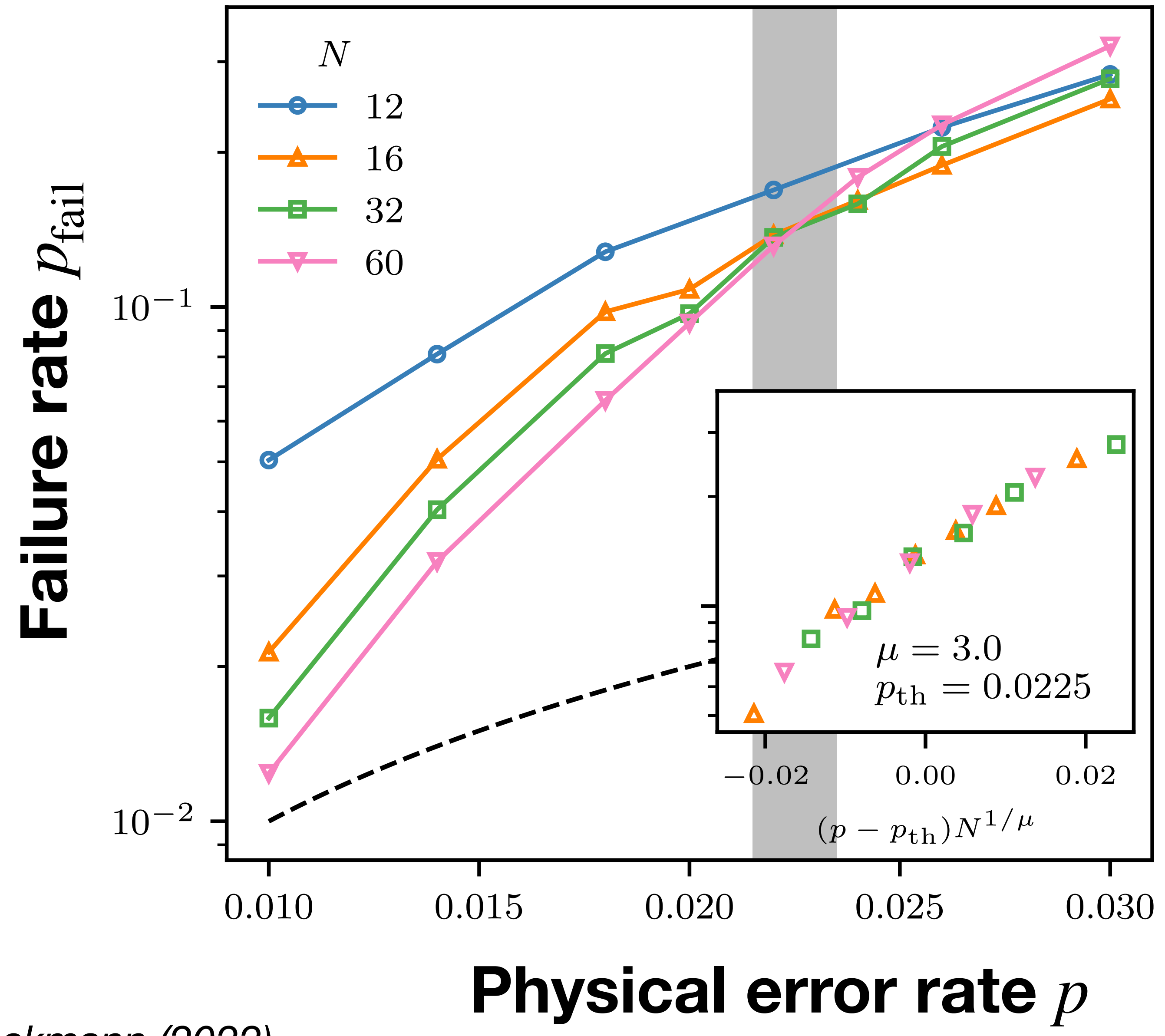


An artificial hyperbolic lattice

[Kollar et al. Nature (2019)]

Recent result: modelling the threshold of hyperbolic codes

- {5, 5} code, information-theoretic optimum performance
- Modelling also yielded new insights into statistical mechanics in curved geometries



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Roy, Placke, Breuckmann (2020); Placke, Breuckmann (2022)

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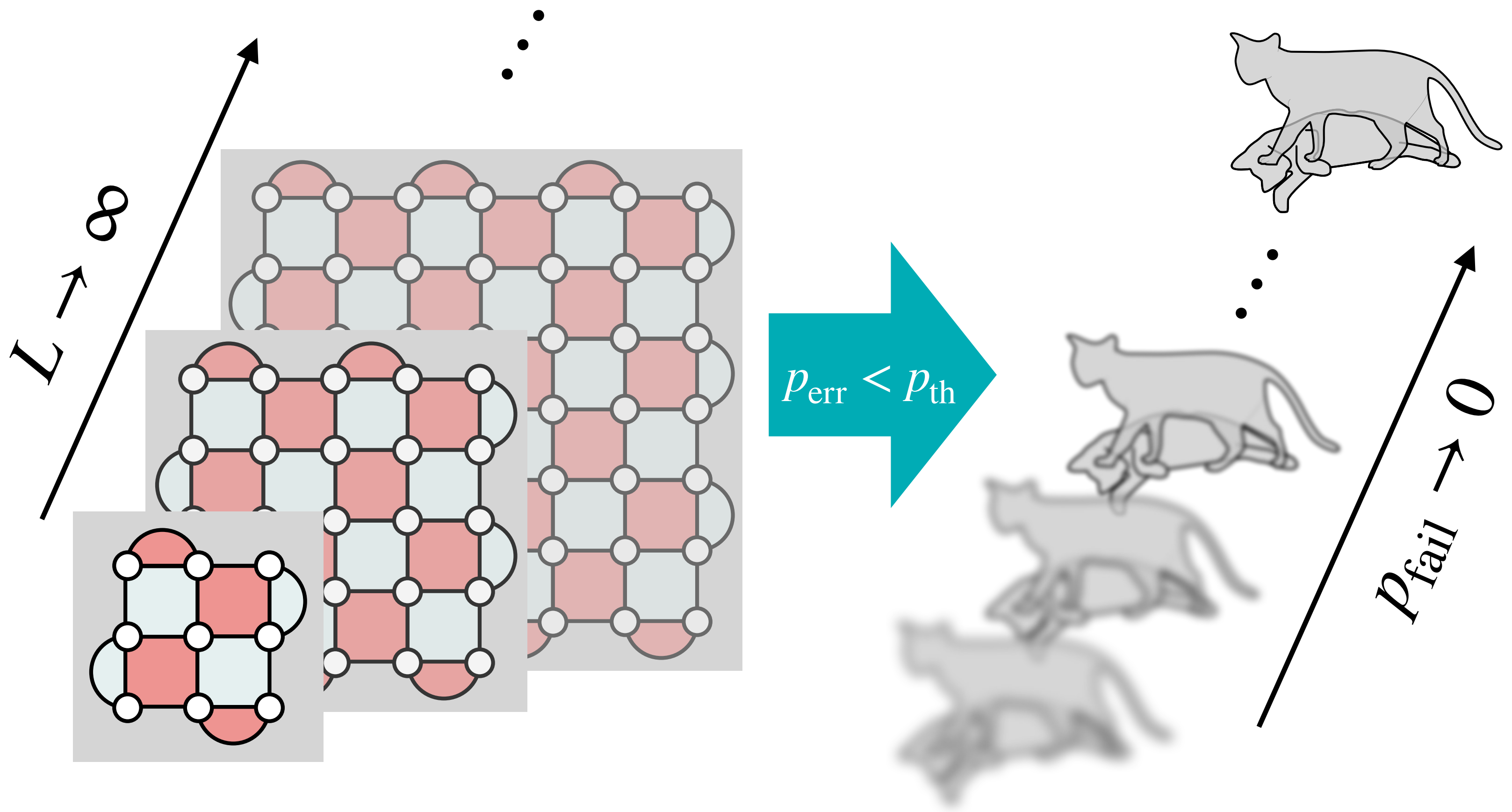
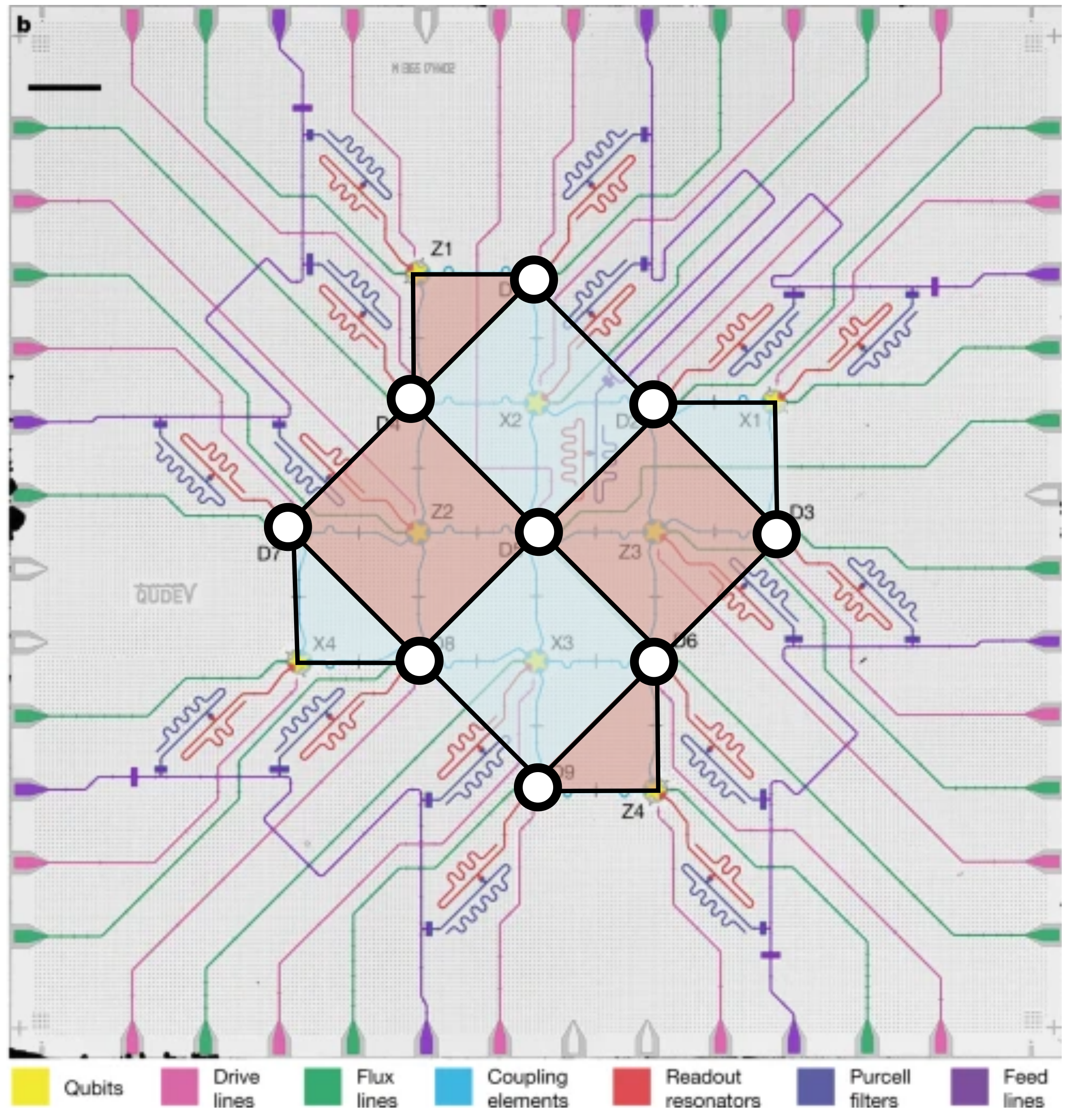
Summary & Conclusion

Quantum Error Correction is (surprisingly!) possible

- This is in contrast to other proposed alternative models of computation, like random access machines
- Constituents must have minimal fidelity for QEC to work (**threshold theorem**)





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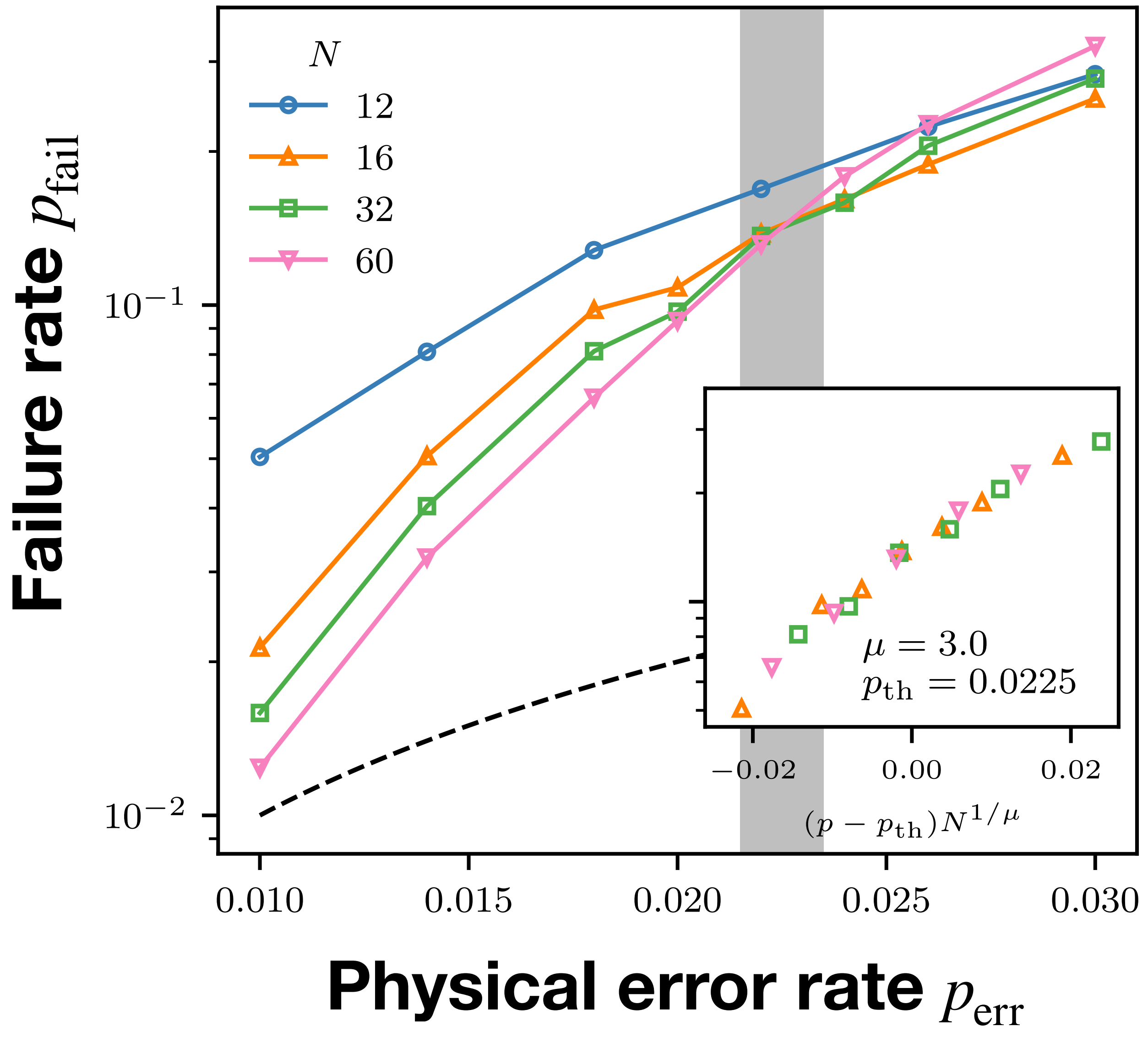
Experiments are "scratching the threshold"

Current research: finding better codes and modelling their error correction



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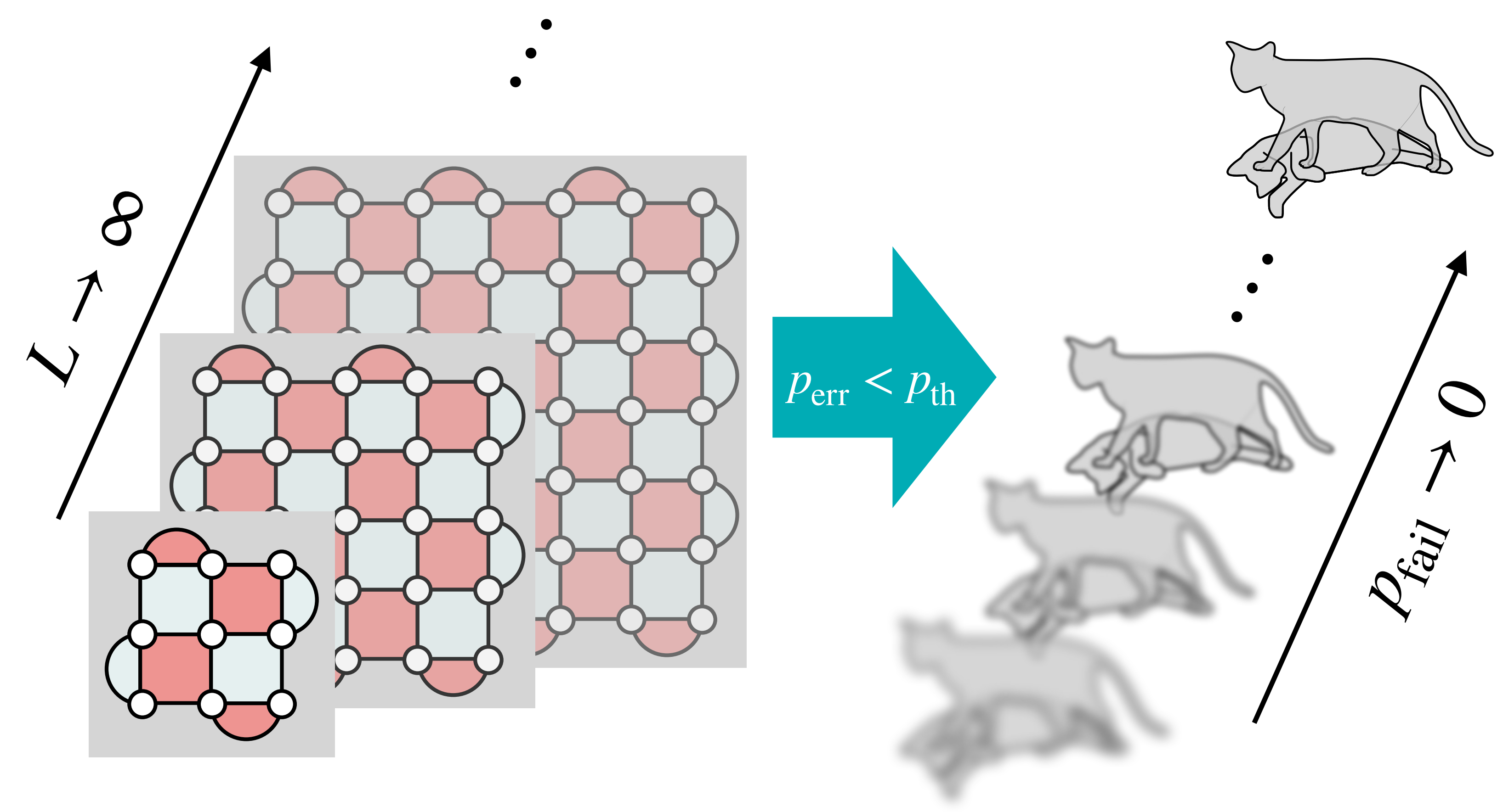

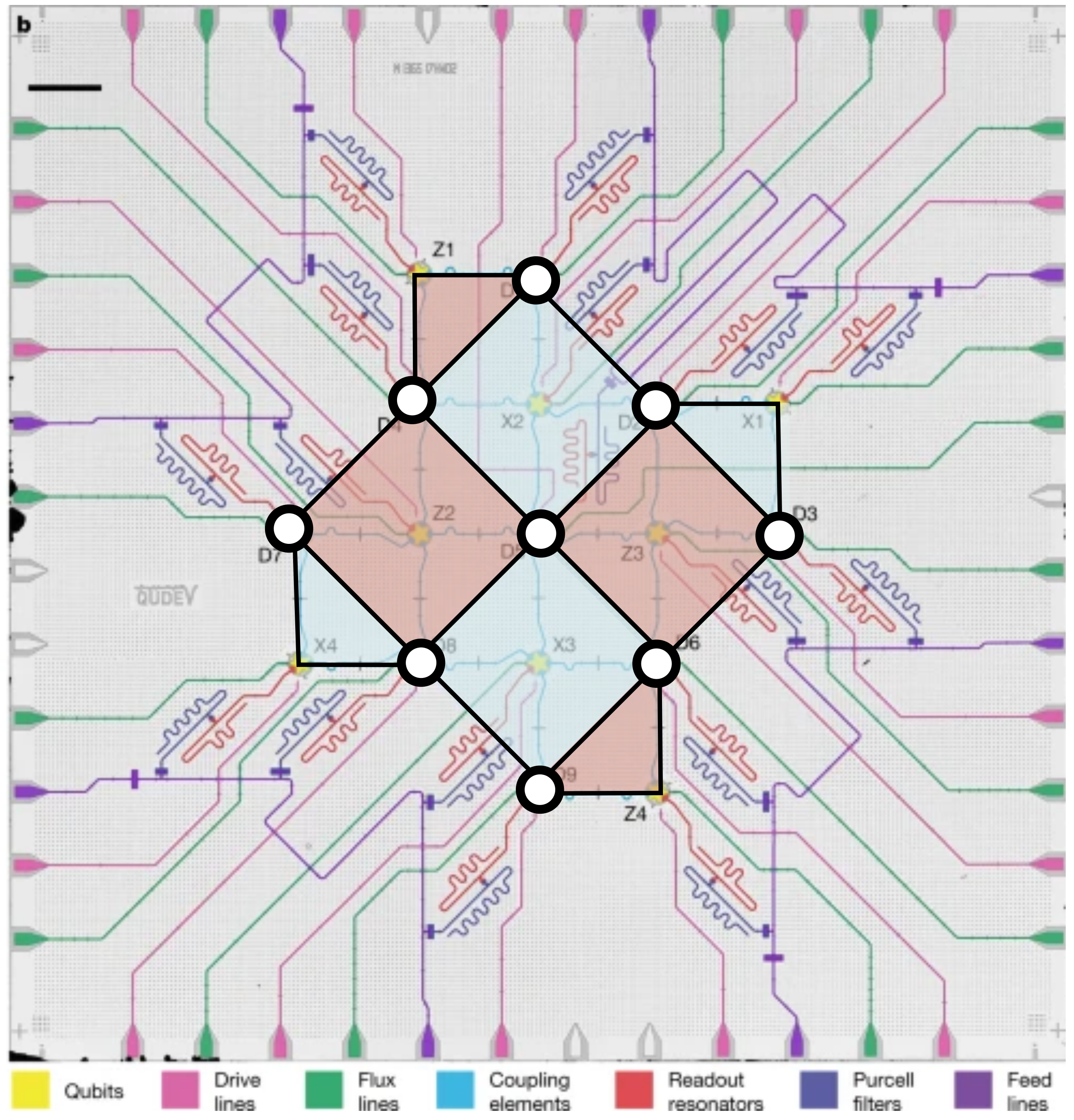

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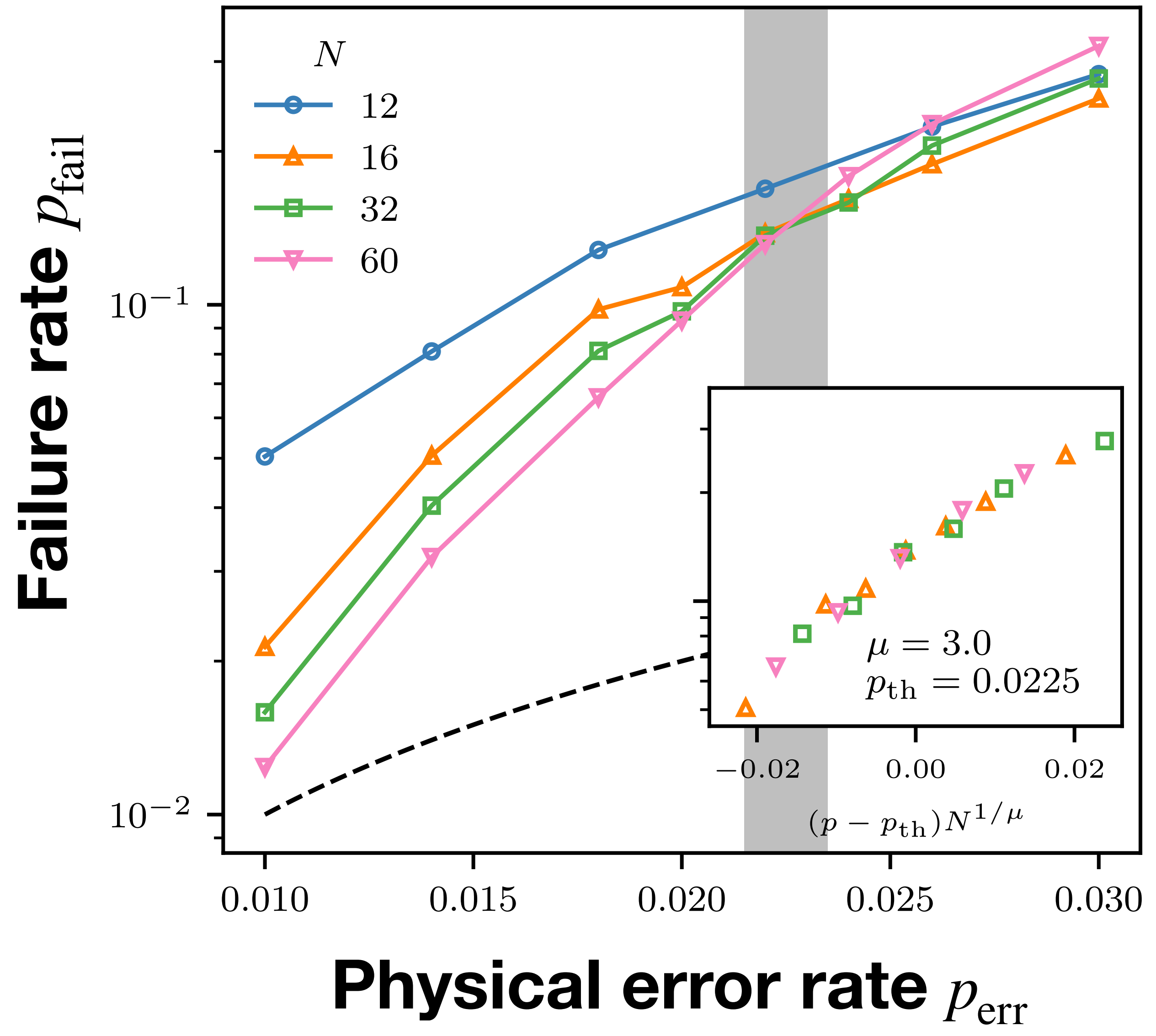
Thank you!

Questions?

Current research: finding better codes and modelling their error correction



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