The dynamics of galaxy discs

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20 January 2018
We live inside a disc galaxy:
Discs are easier to see from outside. Here is M51:

Notice: satellite, spiral arms, dust, young stars, pitch angle.
Spiral arms are ubiquitous in galaxy discs: the flocculent M63
What are spiral arms?

Just differential rotation? No, winds up too quickly!

Can magnetic fields stop spirals winding up?
- No, spirals present in old disc stars.
- On large scales \( \frac{1}{2\mu_0} B^2 \ll \frac{1}{2} \rho (\Delta v)^2 \).

Lin & Shu (1966) proposed that spirals are *density waves*. 
Outline

1. An idealised model of a galactic disc
2. Perturbation theory (analogy w/ ugrad QM/EM)
   - Disc is an elastic medium
3. Intro to dynamics of spiral arms
Understanding the stellar dynamics of discs

Simplifying assumptions:

1. Ignore magnetic fields, gas, cosmic rays, ...
2. Disc is 2d, isolated,
3. composed of identical stars,
4. moving in their own mean-field potential $\Phi(x)$.
5. These stars are eternal: no star formation or destruction.

Disc completely described by two functions $H = \frac{1}{2}v^2 + \Phi(x)$ and $f(x, v)$, coupled via

$$\frac{\partial f}{\partial t} + [f, H] = 0, \quad \nabla^2 \Phi = 4\pi G \int d\mathbf{v} f = \rho(x)$$

This talk: What happens when we poke a disc that is in dynamical (not thermal!) equilibrium?
Unperturbed disc is in steady, axisymmetric state with Hamiltonian $H_0(x, v) = \frac{1}{2} v^2 + \Phi(R)$.

Integrals of motion $(E, L)$, or $J = (J_r, J_\phi)$.

Location along an orbit given by $\theta = (\theta_r, \theta_\phi)$.
These increase at a rate $\dot{\theta} = \Omega \equiv \frac{\partial H_0}{\partial J} = (\kappa, \Omega)$.

Dynamical equilibrium requires phase-space DF $f = f(E, L) = f(J)$ (Jeans’ thm).
Almost-circular orbits \((J_r \approx 0)\) can be thought of as:

1. an underlying circular orbit at some \textbf{guiding center} radius \(R_g\), with
2. epicyclic oscillations superimposed.

Corresponding angular frequencies \(\Omega = \partial H / \partial J\):

1. \(\dot{\theta}_\phi = \Omega\),
2. \(\dot{\theta}_r = \kappa\).

\(J_r \propto \kappa A^2\), where \(A = \max(R - R_g)\)

Local “temperature” \(\sim \langle v_R^2 \rangle \sim \kappa^2 \langle A^2 \rangle\).
Perturbation theory

DF $f$ and Hamiltonian $H = \frac{1}{2}v^2 + \Phi$ coupled by CBE+Poisson:

$$\frac{\partial f}{\partial t} + [f, H] = 0, \quad \nabla^2 \Phi = 4\pi G \int d\mathbf{v} f.$$ 

Equilibrium $(f_0, H_0)$ boring.

From $t = 0$ apply perturbation $\epsilon \Phi_p(x, t)$. Then

$$f = f_0 + \epsilon f_1 + \cdots,$$

$$H = H_0 + \epsilon (\Phi_p + \Phi_s) + \cdots.$$

Linearized evolution equations are

$$\frac{\partial f_1}{\partial t} + [f_1, H_0] = -[f_0, \Phi_1], \quad \nabla^2 \Phi_s = 4\pi G \int d\mathbf{v} f_1.$$ 

Cf electrostatics:

$$\Phi_p \Leftrightarrow D,$$

$$\Phi_1 \Leftrightarrow \epsilon_0 E,$$

$$\Phi_s = \Phi_1 - \Phi_p \Leftrightarrow \epsilon_0 E - D = -P.$$
We want to understand $t > 0$ behaviour of

$$\frac{\partial f_1}{\partial t} + [f_1, H_0] = -[f_0, \Phi_1], \quad \nabla^2 \Phi_s = 4\pi G \int d\mathbf{v} f_1.$$ 

Choose an orthogonal basis $\Phi_\alpha(\mathbf{x})$ for $\Phi(\mathbf{x})$ and expand

$$\Phi_p(\mathbf{x}, t) = \sum_\alpha a_\alpha(t) \Phi_\alpha(\mathbf{x}),$$

$$\Phi_s(\mathbf{x}, t) = \sum_\alpha b_\alpha(t) \Phi_\alpha(\mathbf{x}).$$

Transform $(\mathbf{x}, \mathbf{v}) \rightarrow (\theta, \mathbf{J})$ and expand as Fourier series in $\theta$. Fourier transform $t \rightarrow \omega$ so that $F(t) = \frac{1}{2\pi} \int d\omega \tilde{F}(\omega)e^{-i\omega t}$. Then

$$\tilde{b}_\alpha(\omega) = \sum_\beta \tilde{P}_{\alpha\beta}(\omega)[\tilde{a}_\beta(\omega) + \tilde{b}_\beta(\omega)],$$

where $P_{\alpha\beta}$ is polarization matrix ($\Leftrightarrow -\chi$ in electrostatics).
Matrix mechanics: normal modes

We have just seen that

\[ \tilde{b}(\omega) = \tilde{P}(\omega)[\tilde{a}(\omega) + \tilde{b}(\omega)]. \]

Isolated galaxy: stimulus \( \Phi_p = 0 \), so \( \tilde{a} = 0 \).

Normal modes have \( \tilde{b} = \tilde{P}(\omega)\tilde{b} \).

Find them by solving \( |\tilde{P}(\omega) - I| = 0 \) for \( \omega \).

Time consuming numerical calculation. Difficult to gain insight.
Gain insight from **incomplete**, approximate set of $\Phi_{\alpha}(x)$.

Assume $|kR| \gg 1$ in

$$\Phi_1(R, \phi, t) = -\frac{2\pi G}{|k|} A(R)e^{i(kR+m\phi-\omega t)}e^{-k|z|},$$

$$\Sigma_1(R, \phi, t) \simeq A(R)e^{i(kR+m\phi-\omega t)}.$$

**Pattern speed** $\Omega_p = \omega / m$.

Increasing $|k|$ tightens.

Basis labels $\alpha = (m, k, R)$: **local** approximation.

Then $\tilde{P}_{\alpha\beta}(\omega)$ is diagonal with (after some manipulation)

$$\tilde{P}_{mkR}(\omega) = \frac{2\pi G\Sigma|k|F}{\kappa^2 - (\omega - m\Omega)^2},$$

where $F \leq 1$ depends on disc DF and on $(\omega, m, k, R)$. Self-consistent waves have $\tilde{P}_{mkR}(\omega) = 1$. 

**WKB approximation**
Whither that $[\kappa^2 - m^2(\Omega_p - \Omega)^2]$ denominator?

Consider a star at guiding centre radius $R$, frequencies $\Omega$, $\kappa$. To it, spiral perturbation comes by at rate $m(\Omega_p - \Omega)$.

If $m(\Omega - \Omega_p) = n\kappa$ this resonates with radial motion: self sustaining!

Most important resonances for discs:

- corotation $\Omega_p = \Omega$ (i.e., $n = 0$).
- “Lindblad” resonances $n = \pm 1$. 
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Most important resonances for discs:
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- “Lindblad” resonances $n = \pm 1$. 
Dispersion relation for waves $\Phi_1 \propto A(R)e^{i(kR+m\phi-\omega t)}$, assuming $|kR| \gg 1$, is

$$(\omega - m\Omega)^2 = \kappa^2 - 2\pi G\Sigma |k|F.$$ 

Take lukewarm disc. Apply spiral with $m = 2$ arms, $\Omega_p \equiv \frac{\omega}{m} = 1$.

Group velocity,

$$v_{g,R} = \frac{\partial \omega(R, k)}{\partial k}.$$ 

Collisionless dissipation as $k \to \infty$. Changes $f(J)$, makes $J_r \uparrow$.

Perturbation $\Phi_1$ lowers effective $\kappa$ around ILR/OLR.

[But what about that $k \sim 0$ stage?]
Beyond WKB: Swing amplification
(Toomre 1981)

WKB incomplete. Full calculation of spiral unwinding gives:
Peak spiral unwinding rate almost matches $\kappa$ for stars having $\Omega \simeq \Omega_p$: stars encouraged to join wave.

Discs are extremely responsive to perturbations.
Spiral arms redistribute disc material. Here is one example.

Consider star \((E, L)\) perturbed by spiral \(\Omega_p\).

\[ E - \Omega_p L = \text{const.} \]

\[ \Rightarrow \quad \Omega_p \Delta L = \Delta E \]

\[ = \frac{\partial E}{\partial J_r} \Delta J_r + \frac{\partial E}{\partial L} \Delta L \]

\[ = \kappa \Delta J_r + \Omega \Delta L. \]

So,

\[ \Delta J_r = \frac{\Omega_p - \Omega}{\kappa} \Delta L. \]

**Keeping cool:** spirals can shuffle disc stars in \(R\) while keeping them on almost circular orbits!
Galaxies can never achieve thermal equilibrium. How do disc galaxies satisfy their urge to increase entropy?

- local star-star encounters (as in globular clusters, $10^{10}$ yr);
- but stars “dressed” by polarisation wakes,
- with resonant absorption of collective oscillations across disc
  - $\sim 10^3$ x faster than local, undressed 2-body.

WIP, but modern surveys encouraging deeper thought.

Some important omissions

1. nonlinear response: mode mixing.
2. gas: star formation, molecular clouds, accordions.
3. Only a single DF $f$: blind to different stellar populations.
   - Seeing DFs in “colour” is best way of probing galaxies’ long memories.